MATHEMATICAL TABLES;

CONTAINING THE

LOGARITHMS OF NUMBERS,
LOGARITHMIC SINES, TANGENTS, AND SECANTS,
NATURAL SINES, AND TRAVERSE TABLE,

AND

VARIOUS TABLES USEFUL IN BUSINESS;

TO WHICH ARE PREFIXED,

THE CONSTRUCTION AND USE OF THE TABLES, PLANE AND
SPHERICAL TRIGONOMETRY, WITH THEIR APPLICATIONS;

ALSO

MENSURATION OF PLANE SURFACES AND OF SOLIDS.

FOR THE USE OF SCHOOLS.

By J. BROWN, Mathematician.

THE FIFTH EDITION,
IMPROVED AND ENLARGED,

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STIRLING & KENNEY,
EDINBURGH,
WHITTAKER, TREACHER, & ARNOT,
LONDON.

1830.

*Edinburgh.—*DUNCAN STEVENSON,
Printer to the University.

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THE additions and alterations which have been made in this fifth edition of Mr. BROWN'S Logarithmic Tables, are intended to render the work a useful sequel to the treatises commonly employed in teaching the Elements of Geometry. For this purpose, it has been the object of the Editor to exhibit a succinct view of the construction of the Logarithmic and Trigonometrical Tables; and, by a proper selection of Examples, to illustrate the various practical rules which the speculative truths of Elementary Geometry furnish. To every example given in the work, the answer is annexed.

J. W.

March 1830.

CONTENTS.

OF THE NATURE, CALCULATION, AND USE OF LOGARITHMS.

1 If the square root of any affirmative number be extracted, and the square root of the result be again taken, and if this process be carried on for a sufficient number of times, always extracting the root of the last result, there will, in every case, at length be obtained a number exceeding unity by a very small fraction. The integral powers of this last result, will constitute a geometrical series, of which any two successive terms (except among the very distant terms) will differ from one another by a very small quantity Hence it appears, that in this series, there will be found terms which will deviate but very little from the series of natural numbers, and which, for all the purposes of calculation, may be employed instead of them, so as to obtain results sufficiently accurate.

2 From these observations, it follows, that all affirmative numbers may be considered as powers of any one affirmative number, except of unity The powers of 2, for example, may become either exactly equal, or nearer than any assignable difference, to any number whatever, from 0 upwards

Thus

$$2^{0} = 1. \qquad 2^{2} = 4$$
$$2^{1} = 2 \qquad 2^{\frac{2325}{1000}} = 5$$
$$2^{\frac{1585}{1000}} = 3 \qquad 2^{\frac{2585}{1000}} = 6$$

&c.

In like manner may the powers of 10, be employed to express all numbers

Thus

$$10^{0} = 1 \qquad 10^{\frac{77815}{100000}} = 6$$
$$10^{\frac{30103}{100000}} = 2 \qquad 10^{\frac{84510}{100000}} = 7$$
$$10^{\frac{47712}{100000}} = 3 \qquad 10^{\frac{90309}{100000}} = 8$$
$$10^{\frac{60206}{100000}} = 4 \qquad 10^{\frac{95424}{100000}} = 9$$
$$10^{\frac{69897}{100000}} = 5 \qquad 10^{1} = 10$$

&c

3. In general, therefore, let a denote any number, and r any given number, a number x may be found, such that $r^x = a$.

Definition I. The number x is called the *Logarithm* of the number a.

Definition II. The given number r, by the powers of which all other numbers are expressed, is called the *Radical Number* of the logarithms, which are the indices of the powers.

Corollary. Since $r^0 = 1$, whatever be the value of r, it is evident that Log 1 is always equal to 0.

Also, since $r^1 = r$, it follows that Log $r = 1$. The logarithm of the radical number is therefore always equal to unity.

4. From the definition of a logarithm just given, it appears,

1*mo*, That the sum of the logarithms of any two numbers is equal to the logarithm of their product.

For, let a and b be any two numbers, and x and x' their logarithms, then $a = r^x$, $b = r^{x'}$, therefore, $a \times b = r^x \times r^{x'} = r^{x+x'}$; hence (by Def. I.) Log $ab = x + x' =$ Log a + Log b.

Corollary. If n be any number whole or fractional,

$$\text{Log } a^n = n \text{ Log } a.$$

2*do*, The logarithm of the quotient of any two numbers, is equal to the difference of their logarithms.

For a, b, x, x', denoting as before

$$\frac{a}{b} = r^x \div r^{x'} = r^{x-x'}$$

hence (by Def. I.) Log $\frac{a}{b} = x - x' =$ Log. a — Log b.

5. Let it now be required to find the logarithm of any number. With a view to the solution of this problem, we premise the following lemma.

6. LEMMA. If y and z be any two quantities, and n any whole positive number, then $y^n - z^n = (y - z) \times (y^{n-1} + y^{n-2}z + y^{n-3}z^2 + \cdots\cdots + yz^{n-2} + z^{n-1})$.

For, by actually multiplying the factors, we have

$$\begin{array}{llllll}
y^{n-1} & + y^{n-2}z & + y^{n-3}z^2 + & \cdots & + yz^{n-2} & + z^{n-1} \\
y & - z & & & & \\
\hline
y^n & + y^{n-1}z & + y^{n-2}z^2 + & \cdots & + y^2z^{n-2} & + yz^{n-1} \\
 & - y^{n-1}z & - y^{n-2}z^2 - & & - y^2z^{n-2} & - yz^{n-1} - z^n \\
\hline
y^n & \cdots & \cdots & \cdots & \cdots & - z^n
\end{array}$$

Corollary Hence it is evident, that

$$\frac{y^n - z^n}{y - z} = y^{n-1} + y^{n-2}z + y^{n-3}z^2 + \cdot \quad \cdot \cdot + yz^{n-2} + z^{n-1}.$$

7. Let us now suppose that N represents any number, whose logarithm is to be found Put $N = 1 + y$, and x for the logarithm required, so that $1 + y = r^x$

Assume Log $(1 + y) = A y + B y^2 + C y^3 + D y^4 +$ &c.*

Here, A, B, C, D &c represent quantities altogether independent of y, and which therefore involve only the powers of r, and de-

* It may be proper to prove, that the logarithmic series will actually have the form here assumed For this purpose, we remark, that Log 1 being equal to 0, the logarithm of any number N may be regarded as a function of the difference between that number and unity, that is, if $N = 1 + y$, then will Log N be a function of y With regard to the form of the function, it is evident, that it can contain no fractional power of y for if it does, let the term containing the fractional power have the form $U y^{\frac{m}{n}}$, so that $x = A y + B y^2 + C y^3 +$ &c . $+ U y^{\frac{m}{n}} +$ &c Then, since we know from the theory of equations, that a radical quantity has as many different values as there are units in its exponent, it follows, that $y^{\frac{m}{n}}$ or $\sqrt[n]{y^m}$ must have as many different values as there are units in n These values being successively substituted for $y^{\frac{m}{n}}$ in the series $A y + B y^2 + C y^3 +$ &c . $+ U y^{\frac{m}{n}} +$ &c, will give for x, n different values. Hence it appears, that the equation $N = r^x$ will give for N, (which is, in each particular case, a given number), n different values. But this conclusion is absurd It is equally obvious, that the series for Log N can contain no negative power of y for, if there occurred in it any term of the form $\frac{V}{y^m}$, that term when $y = 0$ would become infinite But when $y = 0$, $N = 1$ hence we would obtain Log 1 equal to infinity,—a conclusion which is inconsistent with what has already been shewn in Cor Def II § 3 Farther, it appears that the series for Log N may be multiplied by any constant quantity, integral, or fractional, without altering the mutual relations which subsist among the logarithms derived from it. For put Log $N = M(A y + B y^2 + C y^3 +$ &c$)$, M being any constant quantity, then the logarithmic equation becomes $N = r^{M(Ay + By^2 + \&c)} = (r^M)^{Ay + By^2 + \&c.}$, from which it is evident, that the logarithms derived from the formula Log $N = M(A y + B y^2 +$ &c$)$ will form a system differing in no respect, in relation to their mutual properties, from the logarithms obtained from the formula Log $N = A y + B y^2 + C y^3 +$ &c If, however, we suppose M a function of y, it appears, that in that case, the formula Log $N = M(A y + B y^2 +$ &c$)$ would give for the different values of N, logarithms belonging to different systems. Hence we conclude, that it would have been improper to have assumed for Log N a series affected by any function of y, integral or fractional, as a multiplier, and that upon the whole, the series for the Log N ought to have the form above assumed.

terminate numbers. If we suppose $y = 0$ we have Log. $1 = 0$, as it ought be.

Let $1 + z$ be any other number different from $1 + y$; then we have, in like manner,

$$\text{Log } (1 + z) = A z + B z^2 + C z^3 + D z^4 + \&c.$$

Now, subtracting the latter of these equations from the former, and observing that Log. $(1 + y)$ — Log $(1 + z) =$ Log. $\frac{1 + y}{1 + z}$, we obtain,

$$\text{Log. } (1 + y) - \text{Log. } (1 + z) \text{ or Log } \frac{1 + y}{1 + z} =$$
$$A (y - z) + B (y^2 - z^2) + C (y^3 - z^3) + D (y^4 - z^4) + \&c$$

But $\frac{1 + y}{1 + z} = 1 + \frac{y - z}{1 + z}$, and therefore Log. $\frac{1 + y}{1 + z} =$ Log. $\left(1 + \frac{y - z}{1 + z}\right) = A \frac{y - z}{1 + z} + B \frac{(y - z)^2}{(1 + z)^2} + C \frac{(y - z)^3}{(1 + z)^3} +$ &c

Hence putting these two expressions of Log $\frac{1 + y}{1 + z}$ equal to each other, we obtain

$$A (y - z) + B (y^2 - z^2) + C (y^3 - z^3) + D (y^4 - z^4) + \&c =$$
$$A \frac{y - z}{1 + z} + B \frac{(y - z)^2}{(1 + z)^2} + C \frac{(y - z)^3}{(1 + z)^3} + D \frac{(y - z)^4}{(1 + z)^4} + \&c$$

In this equation, both sides are divisible by $y - z$, therefore, (by the Lemma, § 6) we find,

$$A + B (y + z) + C (y^2 + yz + z^2) + D (y^3 + y^2 z + yz^2 + z^3) + \&c = \frac{A}{1 + z} + B \frac{y - z}{(1 + z)^2} + C \frac{(y - z)^2}{(1 + z)^3} + D \frac{(y - z)^3}{(1 + z)^4} + \&c$$

This equation must remain true, whatever be the values of y and z, let us therefore now suppose $y = z$, and the equation becomes $A + 2 B y + 3 C y^2 + 4 D y^3 + \&c = \frac{A}{1 + y}$. Or multiplying both sides by $(1 + y)$, and transposing all to one side, we have

$$\left.\begin{matrix} - A \\ + A \end{matrix}\right\} \left.\begin{matrix} + A \\ + 2B \end{matrix}\right\} y \left.\begin{matrix} + 2B \\ + 3C \end{matrix}\right\} y^2 \left.\begin{matrix} + 3C \\ + 4D \end{matrix}\right\} y^3 \left.\begin{matrix} + 4D \\ + 5E \end{matrix}\right\} y^4 + \&c = 0$$

From this resulting equation, we find by the *Method of Indeter-*

minate Coefficients,* the following equations for the determination of the coefficients A, B, C, D, &c.

$A - A = 0$, $A + 2B = 0$, $2B + 3C = 0$, $3C + 4D = 0$, $4D + 5E = 0$, &c.

Hence we obtain $A = A$, $B = -\frac{1}{2}A$, $C = -\frac{2}{3}B = +\frac{1}{3}A$, $D = -\frac{3}{4}C = -\frac{1}{4}A$, $E = -\frac{4}{5}D = +\frac{1}{5}A$, and so on.

Substituting these values in the series assumed for Log $(1 + y)$, we have

$$\text{Log } (1 + y) = A\left(y - \frac{1}{2}y^2 + \frac{1}{3}y^3 - \frac{1}{4}y^4 + \frac{1}{5}y^5 - \&c.\right)$$

Now, $N = 1 + y$, and $y = N - 1$. Substituting, therefore, these values for $1 + y$ and y the formula becomes,

$$\text{Log } N = A\left(N - 1 - \frac{1}{2}(N-1)^2 + \frac{1}{3}(N-1)^3 - \frac{1}{4}(N-1)^4 + \&c.\right)$$

8. The quantity A remains still to be determined. This may be accomplished, by considering, that if in the equation $1 + y = r^x$, we suppose $1 + y = r$, then x becomes equal to 1, that is Log $(1 + y) = 1$. But when $1 + y = r$, $y = r - 1$. Substitut-

* The Method of Indeterminate Coefficients, which is of the greatest utility in the higher branches of mathematics, depends upon the following theorem.

Theorem. Let x be any indeterminate quantity whatever, and let A, B, C, D, &c. P, Q, R, S, &c. be quantities whose values are altogether independent of the quantity x. If

$A + Bx + Cx^2 + Dx^3 + \&c = P + Qx + Rx^2 + Sx^3 + \&c.$ Or transposing all to one side, if

$$\left.\begin{matrix}A\\-P\end{matrix}\right\} + \left.\begin{matrix}B\\-Q\end{matrix}\right\}x + \left.\begin{matrix}C\\-R\end{matrix}\right\}x^2 + \left.\begin{matrix}D\\-S\end{matrix}\right\}x^3 + \&c = 0,$$

then also $A = P$, or $A - P = 0$, $B = Q$ or $B - Q = 0$, $C = R$ or $C - R = 0$, $D = S$, or $D - S = 0$, &c.

For since the values of A, B, C, &c. P, Q, R, &c. are altogether independent of the value of x, they must remain the same even when x is supposed equal to 0. But when $x = 0$, the above equation becomes $A = P$ or $A - P = 0$. Striking off these equal quantities, and dividing by x, we have $B + Cx + Dx^2 + \&c = Q + Rx + Sx^2 + \&c.$

$$\text{Or } \left.\begin{matrix}B\\-Q\end{matrix}\right\} + \left.\begin{matrix}C\\-R\end{matrix}\right\}x + \left.\begin{matrix}D\\-S\end{matrix}\right\}x^2 + \&c = 0.$$

If we again suppose $x = 0$ we have $B = Q$, or $B - Q = 0$. In the same manner, it may be demonstrated, that $C = R$ or $C - R = 0$, that $D = S$ or $D - S = 0$, and so on.

ing, therefore, these values in the above formula for Log. $(1 + y)$ we find

$$1 = A\left(r - 1 - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \&c.\right)$$

Hence $A = \dfrac{1}{r - 1 - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \&c.}$; from which result, it appears, that the value of A depends entirely upon the radical number.

Definition III. In any system of logarithms, the constant multiplier A, which depends entirely upon the radical number of the system, is called the *Modulus* of that system.

9. If we suppose such a value to be given to r, the radical number, that A, the modulus, may be equal to 1, then, in this case, the general formula found above for Log N, becomes,

$$\text{Log } N = N - 1 - \frac{1}{2}(N-1)^2 + \frac{1}{3}(N-1)^3 - \frac{1}{4}(N-1)^4 + \&c.$$

The system of logarithms, which results from this last supposition, is the most simple with respect to facility of computation. The logarithms of this system, are the same as those invented by Napier, and have likewise, though improperly, been called *Hyperbolic Logarithms*.

10. In the formula of last §, put $N = r$, then we obtain,

$$\text{Hyp. Log } r = r - 1 - \frac{1}{2}(r-1)^2 + \frac{1}{3}(r-1)^3 - \&c.$$

Comparing this result with the value of A, already found, it appears manifest, that $A = \dfrac{1}{\text{Hyp. Log } r}$, that is, the modulus of any system of logarithms, is equal to the reciprocal of the hyperbolic logarithm of the radical number of that system.

11. The series $\text{Log. } N = A\left(N - 1 - \frac{1}{2}(N-1)^2 + \frac{1}{3}(N-1)^3 - \&c.\right)$ can only be used when y is a small fraction, that is, when the given number is but a very little different from unity. In other cases, either the rate of convergency is too slow, or the series diverges.

12. In order to find a series that will always converge, let us

resume the formula, Log. $(1 + y) = A\left(y - \frac{1}{2}y^2 + \frac{1}{3}y^3 - \frac{1}{4}y^4 + \&c.\right)$, and let us suppose y to become negative, then

$$\text{Log. } (1 - y) = A\left(-y - \frac{y^2}{2} - \frac{y^3}{3} - \frac{y^4}{4} - \&c.\right)$$

But $\text{Log. } (1 + y) = A\left(y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \&c.\right)$

Hence, by subtracting the first of these equations from the last, we find

$$\text{Log. } (1 + y) - \text{Log. } (1 - y) \text{ or Log. } \frac{1+y}{1-y} = 2A\left(y + \frac{y^3}{3} + \frac{y^5}{5} + \frac{y^7}{7} + \&c.\right),$$

but putting $\frac{1+y}{1-y} = N$, y will be found equal to $\frac{N-1}{N+1}$, and the last series becomes,

$$\text{Log. } N = 2A\left(\frac{N-1}{N+1} + \frac{1}{3}\left(\frac{N-1}{N+1}\right)^3 + \frac{1}{5}\left(\frac{N-1}{N+1}\right)^5 + \frac{1}{7}\left(\frac{N-1}{N+1}\right)^7 + \&c.\right)$$

13 This series will always converge, whatever be the value of N, and by means of it, the logarithms of small numbers may be easily found If, however, the number be somewhat large, the series will evidently converge too slowly to be of any practical utility In the calculation of logarithms, it therefore becomes necessary, to derive the logarithm of one number from that of another. When a number is composite, its logarithm will most easily be found, by adding together the logarithms of its factors, but if it be a prime number, its logarithm may be derived from some convenient composite number, either greater or less Let n be a number, of which the logarithm is already found, then substituting $\frac{n+z}{n}$ for N in the formula of the preceding §, we have

$$\text{Log } \frac{n+z}{n} = 2A\left(\frac{z}{2n+z} + \frac{1}{3}\frac{z^3}{(2n+z)^3} + \frac{1}{5}\frac{z^5}{(2n+z)^5} + \&c.\right)$$

But $\text{Log. } \frac{n+z}{n} = \text{Log. } (n + z) - \text{Log. } n$, therefore,

$$\text{Log. } (n + z) = \text{Log. } n + 2A\left(\frac{z}{2n+z} + \frac{1}{3}\frac{z^3}{(2n+z)^3} + \frac{1}{5}\frac{z^5}{(2n+z)^5} + \&c.\right)$$

This series gives the logarithm of $n+z$ by means of the logarithm of n, and converges very fast when n is considerable.

14. Let L denote the hyperbolic logarithm of any number, and l, l' the logarithms of the same number, according to two other systems, whose moduli are A, and A′, then, from what has been said, $l = \text{AL}$, $l' = \text{A}'\text{L}$

therefore $\dfrac{l}{\text{A}} = \dfrac{l'}{\text{A}'}$, and $\text{A} : \text{A}' \ \ l . l'$,

That is, the logarithms of the same number, according to different systems, are directly proportional to the moduli of those systems, and are therefore to each other in a constant ratio.

Corollary. Hence it follows, that if the moduli of any two different systems, and the logarithms of one of the systems be given, the logarithms of the other system may be found.

15. We shall now exhibit, in one view, the different series that have here been investigated, and then proceed to exemplify their application, first, in the calculation of the hyperbolic logarithm of 10, from which we shall be able to determine, the modulus of the common system of logarithms, the determination of this number being the first step in the construction of the tables

$$\text{I. Log. N} = \text{A}\left(\text{N}-1-\frac{(\text{N}-1)^2}{2}+\frac{(\text{N}-1)^3}{3} - \frac{(\text{N}-1)^4}{4} + \&\text{c}\right)$$

$$\text{II. Log N} = 2\,\text{A}\left(\frac{\text{N}-1}{\text{N}+1}+\frac{1}{3}\left(\frac{\text{N}-1}{\text{N}+1}\right)^3+\frac{1}{5}\left(\frac{\text{N}-1}{\text{N}+1}\right)^5 + \&\text{c}\right)$$

$$\text{III Log.}\,(n+z) = \text{Log}\ n + 2\,\text{A}\left(\frac{z}{2n+z}+\frac{1}{3}\,\frac{z^3}{(2n+z)^3}+\frac{1}{5}\,\frac{z^5}{(2n+z)^5} + \&\text{c}\right)$$

16. In the application of these formulæ, the chief object is to make them converge as fast as possible, for which purpose a little management is sometimes necessary.* Here, we shall derive the hyperbolic logarithm of 10 from those of 2 and 5.

* In a paper by Professor Wallace, in the sixth volume of the Edin Phil Trans. the following series for the computation of logarithms are investigated, which have the remarkable property of being alike applicable whether the num-

To find the hyperbolic logarithms of 2 and 4, putting N, in the second formula, equal to 2, and $\frac{N-1}{N+1} = X$, (for the sake of abridging), we have $X = \frac{1}{3}$: hence

$2A$	$= 2.00000000$		
$2AX$	0.66666667	$2AX$	$= 0.66666667$
$2AX^3$	0.07407407	$\frac{2}{3}AX^3$	$= 0.02469136$
$2AX^5$	0.00823045	$\frac{2}{5}AX^5$	$= 0.00164609$
$2AX^7$	0.00091449	$\frac{2}{7}AX^7$	$= 0.00013064$
$2AX^9$	0.00010161	$\frac{2}{9}AX^9$	$= 0.00001129$
$2AX^{11}$	$= 0.00001129$	$\frac{2}{11}AX^{11}$	$= 0.00000103$
$2AX^{13}$	$= 0.00000125$	$\frac{2}{13}AX^{13}$	$= 0.00000009$
$2AX^{15}$	$= 0.00000014$	$\frac{2}{15}AX^{15}$	$= 0.00000001$
		Hyp. Log. 2	$= 0.6931472$
			2
		Hyp. Log. 4	$= 1.3862944$

her whose logarithm is required be large or small These series are farther remarkable, from this circumstance, that the terms of each approach continually to those of a geometrical series

Let N be any number, then

$$\frac{1}{\text{Log } N} = \frac{1}{2}\,\frac{N+1}{N-1} - \left(\frac{1}{4}\,\frac{N^{\frac{1}{2}}-1}{N^{\frac{1}{2}}+1} + \frac{1}{8}\,\frac{N^{\frac{1}{4}}-1}{N^{\frac{1}{4}}+1} + \frac{1}{16}\,\frac{N^{\frac{1}{8}}-1}{N^{\frac{1}{8}}+1} + \frac{1}{32}\,\frac{N^{\frac{1}{16}}-1}{N^{\frac{1}{16}}+1} + \&c\right)$$

In this series the terms approach continually to those of a geometrical series, whose common ratio is $\frac{1}{4}$, so that the sum of all the terms which follow any assigned term approaches the nearer to $\frac{1}{3}$ of that term according as it is more advanced in the series.

Again, putting N for any number, let a series of quantities n, n', n'', n''', &c be found such that

$$n = \frac{1}{2}\left(N + \frac{1}{N}\right),\ n' = \sqrt{\tfrac{1}{2}(n+1)},\ n'' = \sqrt{\tfrac{1}{2}(n'+1)},\ n''' = \sqrt{\tfrac{1}{2}(n''+1)},\ \&c$$

Then will

$$\frac{1}{l^2 N} = \frac{N}{(N-1)^2} + \frac{1}{12} - \left(\frac{1}{4^2}\,\frac{n'-1}{n'+1} + \frac{1}{4^3}\,\frac{n''-1}{n''+1} + \frac{1}{4^4}\,\frac{n'''-1}{n'''+1} + \&c\right)$$

In this series the terms approach continually to those of a geometrical series, whose common ratio is $\frac{1}{16}$, so that the sum of all the terms which follow any assigned term approaches the nearer to $\frac{1}{15}$ of that term according as it is more advanced in the series

The quantities n, n', n'', &c. being formed from N, as specified above, we have this other series,

To find the hyperbolic logarithms of 5 and 10. In the third formula, put $n = 4$, $z = 1$, and $\frac{z}{2n + z} = X$, then we find $X = \frac{1}{9}$. Hence

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$$\frac{1}{\text{Log}^4 N} = \frac{N(N^2 + 4N + 1)}{6(N-1)^4} - \frac{1}{8 \cdot 9 \cdot 10} + \frac{1}{3 \cdot 16^2} \cdot \frac{n + 12n' - 13}{n + 4n' + 3} + \frac{1}{3 \cdot 16^3} \cdot \frac{n' + 12n'' - 13}{n' + 4n'' + 3} + \frac{1}{3 \cdot 16^4} \cdot \frac{n'' + 12n''' - 13}{n'' + 4n''' + 3} + \&c.$$

In this series the terms approach continually to those of a geometrical series, whose common ratio is $\frac{1}{64}$, so that the sum of all the terms which follow any assigned term is the nearer to $\frac{1}{63}$ of that term, according as it is the more advanced in the series.

As an example of the application of this last series, let it be required to compute from it the modulus of the common system of logarithms, which is equal to the reciprocal of the hyperbolic logarithm of 10

$N = 10$, $n = 5.05$ $\qquad n''' = 1.0417078207$
$n' = 1.739252713093$ $\qquad n'''' = 1.01037315$
$n'' = 1.17031036761$ $\qquad n''''' = 1.0025899$

$$\frac{N(N^2 + 4N + 1)}{6(N-1)^4} = .0358177107148$$

$$\frac{1}{3 \cdot 16^2} \cdot \frac{n + 12n' - 13}{n + 4n' + 3} = .0011210934214$$

$$\frac{1}{3 \cdot 16^3} \cdot \frac{n' + 12n'' - 13}{n' + 4n'' + 3} = .0000240411229$$

$$\frac{1}{3 \cdot 16^4} \cdot \frac{n'' + 12n''' - 13}{n'' + 4n''' + 3} = .0000004092394$$

$$\frac{1}{3 \cdot 16^5} \cdot \frac{n''' + 12n'''' - 13}{n''' + 4n'''' + 3} = .0000000065357$$

$$\frac{1}{3 \cdot 16^6} \cdot \frac{n'''' + 12n''''' - 13}{n'''' + 4n''''' + 3} = .0000000001027$$

$\frac{1}{63}$ of last term = sum of the remaining terms nearly, $\Big\}$ .0000000000016

From sum of positive terms = .0369632611385

Subtract $\frac{1}{8 \cdot 9 \cdot 10}$ = .0013888888889

There remains $\frac{1}{\text{Log}^4 10}$ = .0355743722496

$\frac{1}{\text{Log}^2 10}$ = .1886116970113

Modulus = $\frac{1}{\text{Log. } 10}$ = .434294481903

$2A = 2.00000000$	
$2AX = 0.22222222$	$2AX = 0.22222222$
$2AX^2 = 0\ 02469136$	
$2AX^3 = 0.00274348$	$\frac{2}{3}AX^3 = 0.00091449$
$2AX^4 = 0.00030483$	
$2AX^5 = 0.00003387$	$\frac{2}{5}AX^5 = 0.00000677$
$2AX^6 = 0.00000376$	
$2AX^7 = 0.00000042$	$\frac{2}{7}AX^7 = 0.00000006$
	0.2231435
	Hyp. Log. 4. $= 1.3862944$
	Hyp. Log. 5 $= 1.6094379$
	Hyp. Log. 2 $= 0.6931472$
	Hyp. Log. 10 $= 2.3025851$

17. From the hyperbolic logarithm of 10, which we have thus determined, we find the modulus of the common system of logarithms, or $\frac{1}{\text{Hyp. Log. } 10}$ equal to .4342945.

18. The modulus of the common system of logarithms being now found, let it next be required to calculate the common logarithms of the first twelve natural numbers.

It is evident, that we need only seek the logarithms of the prime numbers. These are 2, 3, 5, 7, 11. The common logarithms of 2 and 5, might be determined, by multiplying their hyperbolic logarithms, already found, by the modulus .4342945. We prefer, however, to derive them immediately from the above formulas.

To find the common logarithm of 2. In formula II, put $N = 2$, $A = .43429448$, and $\frac{N-1}{N+1} = X$. Then $X = \frac{1}{3}$.

Hence we obtain,

$2A = 0.86858896$	
$2AX = 0.28952965$	$2AX = 0.28952965$
$2AX^3 = 0.03216996$	$\frac{2}{3}AX^3 = 0\ 01072332$
$2AX^5 = 0.00357444$	$\frac{2}{5}AX^5 = 0.00071489$
$2AX^7 = 0.00039716$	$\frac{2}{7}AX^7 = 0.00005674$
$2AX^9 = 0.00004413$	$\frac{2}{9}AX^9 = 0.00000490$
$2AX^{11} = 0.00000490$	$\frac{2}{11}AX^{11} = 0.00000045$
$2AX^{13} = 0.00000054$	$\frac{2}{13}AX^{13} = 0.00000004$
	0.30102999
	Log. 2 $= 0.301030$

To find the common logarithm of 3; in formula III, put Log. n = Log. 2, and $z = 1$, then $\frac{z}{2n+z} = \frac{1}{5}$. For the sake of abridging, put $\frac{z}{2n+z} = X$. Then we have

$2A$	$= 0.86858896$		
$2AX$	$= 0.17371779$	$2AX$	$= 0.17371779$
$2AX^3$	$= 0.00694871$	$\frac{1}{3}AX^3$	$= 0.00231624$
$2AX^5$	$= 0.00027795$	$\frac{2}{5}AX^5$	$= 0.00005559$
$2AX^7$	$= 0.00001112$	$\frac{2}{7}AX^7$	$= 0.00000159$
$2AX^9$	$= 0.00000044$	$\frac{2}{9}AX^9$	$= 0.00000005$
			0.17609126
		Log. 2	$= 0.30102999$
			0.47712125
		Log. 3	$= 0.477121$

We proceed now to find the common logarithm of 5, which will be most conveniently determined from the logarithm of 4, or 2 Log. 2 = 0.60205998. In formula III, therefore, put $n = 4$ and $z = 1$, then $\frac{z}{2n+1} = \frac{1}{9}$. Let $\frac{z}{2n+1}$ be represented by X. Hence,

$2A$	$= 0.86858896$		
$2AX$	$= 0.09650988$	$2AX$	$= 0.09650988$
$2AX^2$	$= 0.01072332$		
$2AX^3$	$= 0.00119148$	$\frac{1}{3}AX^3$	$= 0.00039716$
$2AX^4$	$= 0.00013239$		
$2AX^5$	$= 0.00001471$	$\frac{2}{5}AX^5$	$= 0.00000294$
$2AX^6$	$= 0.00000163$		
$2AX^7$	$= 0.00000018$	$\frac{2}{7}AX^7$	$= 0.00000002$
			0.09691000
		Log 4	$= 0.60205998$
			0.69896998
		Log. 5	$= 0.698970$

To find the common logarithm of 7. This logarithm will be most easily determined from that of $\frac{49}{50} = 1 - \frac{1}{50}$. In formula I, put $N = 1 - \frac{1}{50}$, then $N - 1 = -\frac{1}{50}$. Let X represent $1 - N$. Then

$$\begin{array}{lll|lll}
A & = & 0.43429448 & & & \\
A\,X & = & 0.00868589 & - & A\,X & = -\,0.00868589 \\
A\,X^2 & = & 0.00017371 & -\frac{1}{2} & A\,X^2 & = -\,0.00008685 \\
A\,X^3 & = & 0.00000347 & -\frac{1}{3} & A\,X^3 & = -\,0.00000116 \\
A\,X^4 & = & 0.00000007 & -\frac{1}{4} & A\,X^4 & = -\,0.00000001
\end{array}$$

$$\text{Log. } \tfrac{49}{50} = -\,0.00877391$$
$$\text{Log. } 50 = \text{Log. } 10 + \text{Log. } 5 = 1.69896996$$
$$\text{Log. } 49 = 1.69019605$$
$$\tfrac{1}{2}\text{ Log. } 49 = 0.84509802$$
$$\text{Log. } 7 = 0.845098$$

To find the common logarithm of 11 This logarithm, we shall determine from that of the fraction $\frac{121}{120}$. For this purpose, put N, in formula II, equal to $\frac{121}{120}$, then $\frac{N-1}{N+1} = \frac{1}{241}$. Representing $\frac{N-1}{N+1}$ by X, as before, we have

$$\begin{array}{lll|lll}
2A & = & 0.86858896 & & & \\
2A\,X & = & 0.00360410 & 2A\,X & = & 0.00360410 \\
2A\,X^3 & = & 0.00000006 & \frac{2}{3}\,A\,X^3 & = & 0.00000002
\end{array}$$

$$\text{Log. } \tfrac{121}{120} = 0.00360412$$
$$\text{Log. } 120 = \text{Log. } 10 + \text{Log. } 3 + \text{Log. } 4 = 2.07918120$$
$$\text{Log. } 121 = 2.08278532$$
$$\tfrac{1}{2}\text{ Log. } 121 = 1.04139266$$
$$\text{Log. } 11. = 1.041393$$

19. Collecting together the results of these calculations, we obtain,

Log. 1		=	0.000000
Log. 2		=	0.301030
Log. 3		=	0.477121
Log. 4	= 2 Log 2	=	0.602060
Log. 5		=	0.698970
Log. 6	= Log. 2 + Log. 3.	=	0.778151
Log. 7		=	0.845098
Log. 8	= 3 Log. 2	=	0.903090
Log 9	= 2 Log. 3	=	0.954243
Log 10		=	1.000000
Log. 11		=	1.041393
Log. 12	= Log. 2 + Log. 6	=	1.079181

20. By means of the following formulas, we are able to derive the logarithms of numbers from each other, and from the logarithms of numbers nearly equal to unity.

Let $n - 1$, n, $n + 1$ be three numbers having the common difference 1; then

$$\frac{}{(n-1)(n+1)} = \frac{n^2}{n^2-1} = \frac{2n^2}{2n^2-2} = \frac{(2n^2-1)+1}{(2n^2-1)-1} = \frac{1+\frac{1}{2n^2-1}}{1-\frac{1}{2n^2-1}}.$$

Putting $\frac{1+\frac{1}{2n^2-1}}{1-\frac{1}{2n^2-1}} = \text{P}$, and taking the logarithms, we obtain

$2 \text{ Log. } n - \text{Log. } (n-1) - \text{Log. } (n+1) = \text{Log P}$. Hence,

I. $2 \text{ Log. } n = \text{Log. } (n-1) + \text{Log. } (n+1) + \text{Log. P}$.

II. $\text{Log. } (n-1) = 2 \text{ Log. } n - \text{Log } (n+1) - \text{Log P}$.

III. $\text{Log. } (n+1) = 2 \text{ Log } n - \text{Log } (n-1) - \text{Log P}$.

21 From these formulæ, it appears, that the logarithms of any two of the three numbers $n - 1$, n, $n + 1$ being given, the logarithm of the remaining number may be readily found. For it will only be necessary to calculate the logarithm of the fraction P, and to combine it with the given logarithms But since P must always differ from unity by a small fraction, it is evident that series I. and II. of § 15, will always converge with sufficient rapidity.

For example. Supposing the logarithms of 18 and 20 known, it is required to compute the logarithm of 19

Here we put $n = 19$, then $n - 1 = 18$, and $n + 1 = 20$. Hence, by the first of the above formulas, we have,

$2 \text{ Log. } 19 = \text{Log } 18 + \text{Log } 20 + \text{Log } \frac{361}{360}$

It is necessary, therefore, to calculate the logarithm of $\frac{361}{360}$. In formula II. of § 15. Put $\text{N} = \frac{361}{360}$ and $\frac{\text{N}-1}{\text{N}+1} = \text{X}$, then $\text{X} = \frac{1}{721}$. Hence,

$$
\begin{array}{rl}
2\text{ A} & = 0.86858896 \\
2\text{ A X} & = \overline{0.00120470} = \text{Log.} \\
\text{Log. } 18 = \text{Log } 9 + \text{Log } 2 & = 1.25527249 \\
\text{Log. } 20 = \text{Log } 10 + \text{Log. } 2 & = 1.30102999 \\
\hline
2 \,) & 2.55750718 \\
\hline
 & 1.27875359 \\
\text{Log. } 19 & = 1.278754
\end{array}
$$

22. We now proceed to resolve this other problem. Having given the radical number of a system of logarithms, and a logarithm belonging to that system, to determine the corresponding natural number.

23. Let N represent the number, and x its logarithm; and let r be the radical number of the system, as before; then, $N = r^x$. Let us now assume $r^x = A + Bx + Cx^2 + Dx^3 + \&c.$
A, B, C, D, &c being quantities independent of x. Put, therefore, any other quantity z instead of x, and we shall have, in like manner,

$$r^z = A + Bz + Cz^2 + Dz^3 + \&c.$$

Subtracting the second of these equations from the first, and dividing the result by $x - z$, we obtain, (See Lemma, § 6.)

$$\frac{r^x - r^z}{x - z} = B + C(x + z) + D(x^2 + xz + z^2) + E(x^3 + x^2 z + x z^2 + z^3) + \&c.$$

In order to obtain another developement of the first member of this equation, let us write the numerator thus $r^z(r^{x-z} - 1)$. Put now $r = 1 + b$ in the quantity r^{x-z}, and let the quantity be expanded into a series proceeding according to the powers of b, by means of the *Binomial Theorem*,* then we have,

$$(1 + b)^{x-z} = 1 + \frac{(x - z)}{1} b + \frac{(x - z)\ (x - z - 1)}{1 \cdot 2} b^2 + \frac{(x - z)\ (x - z - 1)\ (x - z - 2)}{1 \cdot 2 \cdot 3} b^3 + \&c.$$ Whence we derive,

$$r^z(r^{x-z} - 1) = r^z \left(\frac{(x - z)}{1} b + \frac{(x - z)\ (x - z - 1)}{1 \cdot 2} b^2 + \&c.\right)$$

* For a demonstration of the *Binomial Theorem*, see Wood's *Algebra*, p 117. This theorem gives the expansion of any power, whatever integral or fractional, positive, or negative, of any binomial quantity. The most simple form of the theorem, is the following.

$$(1 + y)^n = 1 + ny + \frac{n(n-1)}{2} y^2 + \frac{n(n-1)(n-2)}{2 \cdot 3} y^3 + \frac{n(n-1)(n-2)(n-3)}{2\ 3\ 4} y^4 + \&c.$$

Dividing the last member of this equation by $x - z$, and putting the result equal to the expression for $\frac{r^x - r^z}{x - z}$ formerly found, we have,

$$r^z \left(b + \frac{(x-z-1)}{1 \cdot 2} b^2 + \frac{(x-z-1)(x-z-2)}{1 \cdot 2 \cdot 3} b^3 + \&c.\right)$$
$$= B + C(x+z) + D(x^2 + xz + z^2) + E(x^3 + x^2 z + x z^2 + z^3)$$
$$+ \&c.$$

This equation must be true, whatever be the value of x and z. Suppose, therefore, $x = z$, and the equation becomes,

$$r^x \left(b - \frac{1}{2} b^2 + \frac{1}{3} b^3 - \&c\right) = B + 2\,C\,x + 3\,D\,x^2 + \&c.$$

Putting in order to abridge, $b - \frac{1}{2} b^2 + \frac{1}{3} b^3 - \&c. = k$, and substituting instead of r^x, the series $A + B\,x + C\,x^2 + D\,x^3 + \&c.$, we obtain,

$$A\,k + B\,k\,x + C\,k\,x^2 + D\,k\,x^3 + E\,k\,x^4 + \&c. =$$
$$B + 2\,C\,x + 3\,D\,x^2 + 4\,E\,x^3 + 5\,F\,x^4 + \&c.$$

Hence we derive, according to the method of indeterminate coefficients,

$$B = A\,k,\ C = \frac{1}{2} B\,k,\ D = \frac{1}{3} C\,k,\ E = \frac{1}{4} D\,k,\ F = \frac{1}{5} E\,k,$$
&c.

From these equations, all the coefficients may be determined except A. But, it is to be observed, that when $x = 0$, the equation $r^x = A + B\,x + C\,x^2 + \&c$ becomes $1 = A$, hence it follows, that

$$A = 1,\ B = \frac{k}{1},\ C = \frac{k^2}{1 \cdot 2},\ D = \frac{k^3}{1 \cdot 2 \cdot 3},\ E = \frac{k^4}{1 \cdot 2 \cdot 3 \cdot 4},$$

$F = \frac{k^5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5}$, &c., and, therefore, we obtain

$$N = r^x = 1 + \frac{k\,x}{1} + \frac{k^2\,x^2}{1 \cdot 2} + \frac{k^3\,x^3}{1 \cdot 2 \cdot 3} + \frac{k^4\,x^4}{1 \cdot 2 \cdot 3 \cdot 4} + \&c.$$

The quantity k remains still to be considered. It is equal to $b - \frac{1}{2} b^2 + \frac{1}{3} b^3 - \frac{1}{4} b^4 + \&c.$ But since $b = r - 1$, we have $k = r - 1 - \frac{1}{2} (r - 1)^2 + \frac{1}{3} (r - 1)^3 - \&c$ Comparing this result with formula I. of § 15, and representing the modulus of the system by A (as before), it is evident, that $k = \text{Hyp. Log. } r = \frac{1}{A}$. Hence the above series becomes,

$$N = 1 + \frac{x}{A} + \frac{x^2}{1 \cdot 2 \cdot A^2} + \frac{x^3}{1 \cdot 2 \cdot 3 \cdot A^3} + \frac{x^4}{1 \cdot 2 \cdot 3 \cdot 4 \cdot A^4} + \&c.,$$

which last is the series we proposed to investigate.

24. By means of this series, the radical number of the hyperbolic logarithms may be found, for in this system, $\frac{1}{A} = 1$, and it is evident, that the logarithm of the radical number is also 1, therefore the radical number or $r = 1 + 1 + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4} + \frac{1}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} + \&c. = 2.718,281,828,46$

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25. Having thus given a short account of the nature and construction of logarithms, we shall now proceed to shew their practical applications

26. From the properties of logarithms which have been demonstrated in § 4, it follows, that if we possess tables by which we can assign the logarithm corresponding to any given number, and also the number corresponding to any given logarithm, we shall thus be enabled to shorten the operations of multiplication, division, &c. in common arithmetic.

27 To answer this end, tables of logarithms have accordingly been made for all numbers under 10,000 or 100,000, &c. from which the logarithm of any number may be found, and the number corresponding to any logarithm, to five, six, or seven figures, according to the extent of the tables.

28. Since, in the common system of logarithms, the logarithm of 1 is 0, and the logarithm of 10 is 1, it is evident, that the logarithm of any number between 1 and 10 must be a decimal. Again, because the logarithm of 10 is 1, and that of 100 is 2, the logarithm of any number between 10 and 100, must be the integer 1 with a decimal annexed. In like manner, the logarithm of any number between 100 and 1000, is the integer 2, with a decimal annexed. and so on. The integral part of a logarithm, which is usually called its *Index* or *Characteristic*, is thus, in the case of the logarithms of whole or mixed numbers, always a unit less than the number of figures in the integral part of the corresponding natural number.

29. From formula I of § 15, it is evident that the logarithms of all proper fractions must be negative. Thus the logarithm of $\frac{49}{50}$, which was calculated in § 18, was found to be — 0.008774. The logarithms of fractions may, however, be expressed in such a manner as to have only their indices negative, but their decimal parts positive. For example, since $\frac{49}{50} = \frac{98}{100} = .98$, we have Log $\frac{49}{50}$ or Log 0 98 = Log. 98 — Log. 100 Let a represent the decimal part of the logarithm of 98, so that Log. 98 = 1 + a, then we have Log. 0.98 = 1 + a — 2 = 1 — 2 + a = — 1 + a. Hence it is evident that we may make the decimal part of Log $\frac{98}{100}$ the same with that of Log. 98, and put — 1 for the index. In like manner Log. $\frac{98}{1000}$ = Log. 98 — Log. 1000 = 1 + a — 3 = 1 — 3 + a = — 2 + a, that is, we may make the decimal part of Log. 0.098 the same with that of Log. 98 and prefix — 2 for the index. Thus it appears that we may always make the decimal part of the logarithm of a fraction the same with that of the logarithm of the numerator of the fraction in its decimal form, and prefix a negative index whose numerical value is a unit greater than the number of ciphers between the decimal point and the first significant figure. It is sometimes convenient in adding and subtracting logarithms to avoid negative indices, by using their complements to 10.

30. We here give an example of a natural number, with its corresponding logarithm, in several variations.

Number.	*Logar.*	*Logar.*
2651	3.423410	
265.1	2.423410	
26.51	1.423410	
2 651	0 423410	
2651	— 1 423410 or	9.423410
.02651	— 2.423410 or	8.423410
.002651	— 3.423410 or	7.423410

Note.—The negative sign is usually written over the index instead of before it, thus, $\overline{1}.423410$.

31. The arrangement and use of the table of logarithms come now to be explained.

EXPLANATION OF THE TABLE OF LOGARITHMS.

1. *To find the Logarithm of any whole number under* 100.

Look for the number under N, in the first page of the logarithmic table, then immediately on the right of it is the logarithm sought, with its proper index. Thus, the log. of 64 is 1.806180, and the log. of 72 is 1.857332.

2. *To find the Logarithm of any number between* 100 and 1000.

Find the given number in some of the following pages of the table, in the first column, marked N, and immediately on the right of the number stands the decimal part of the logarithm, in the column marked 0 at top and bottom, to which decimal prefix the proper index. Thus the log. of 364 is 2.561101, and the log of 3.33 is 0 522444.

3. *To find the Logarithm of any number consisting of four places.*

Seek the first three figures in one of the left-hand columns, as in the last article, and the fourth figure at the top or bottom of the table, then the logarithm directly under the fourth figure, and in a straight line with the three figures found in the column on the left, with its proper index, will be the logarithm sought.—Thus, the log. of 7464 is 3.872972, and that of 378.5 is 2.578066, and that of 3 132 is 0.495822.

4. *To find the Logarithm of any number consisting of* 5 *or* 6 *places*

Find the logarithm of the first four left-hand figures, as in the last article, to which prefix the index according to the figures in the natural number, then from the right-hand column marked D,

take the number opposite to that logarithm, and multiply this number (which is called the Tabular Difference) by the remaining figures of the natural number, point off in the product as many figures to the right-hand as there are figures in the multiplier, then add the rest of the product to the logarithm before found, and the sum is the logarithm required.

Ex. 1. Required the logarithm of 36548.

Log. of 36540 is	4.562769	Diff 119
Add	95	× 8
Log. of 36548 =	4.562864	95|2

2. What is the logarithm of 508793?

Log. of 508700 is	5.706462	Diff. 85
Add	79	× 93
Log. of 508793 =	5.706541	255
		765
		79|05

In like manner the log. of 56789.8 will be found to be 4 754270.

5. *To find the Logarithm of a Vulgar Fraction, or of a mixt number.*

Reduce the vulgar fraction to a decimal, then find the decimal part of its logarithm by the preceding rules, and prefix the proper index with its negative sign, § 29.

Or, from the logarithm of the numerator subtract the logarithm of the denominator, and the remainder is the logarithm of the fraction sought. (See rule for division by logarithms.) A mixt number may be reduced to an improper fraction, and its logarithm found in the same manner.

Ex. 1. Required the logarithm of $\frac{3}{16}$ or .1875.

From log. of 3 = 0.477121
Take log. of 16 = 1.204120

Rem. log. of $\frac{3}{16}$ or .1875 = $\bar{1}$.273001

2. Required the logarithm of $13\frac{3}{4}$ or $\frac{55}{4}$.

From log. of 55 = 1.740363
Take log. of 4 = 0.602060

Rem. log. of $\frac{55}{4}$ or 13.75 = 1.138303

6. *To find the natural Number answering to any given Logarithm.*

Look for the decimal part of the given logarithm, in the different columns until you find either it exactly, or the next less. Then in a line with the logarithm found, in the left-hand column marked N, you have three figures of the number sought, and on the top of the column in which the logarithm found stands, you have one figure more, which is to be annexed to the other three; place the decimal point according as the index of the given logarithm directs, and if the logarithm was found exactly you have the number required.

If the logarithm be not found exactly in the table, subtract the logarithm found in the table from the given one, and divide the remainder by the tabular difference, annexing one cypher to the dividend for each figure in the number, above four, which is required to be found, the quotient annexed to the former figures, gives the number answering to the logarithm.

Ex. Required the number answering to the logarithm 3.233568.

The given logarithm = 3.233568
The next less tab log. is the log. of 1712 = 3.233504

Remainder = 64
Tab. Diff. = 253) 64.00 (25

Hence the number sought is 1712.25, making four places of integers for the index 3.

In like manner the natural numbers corresponding to the following logarithms may be found.

Number corresponding to the Log.	1.234568	is	17.162
————————	3.734300	is	5423.76
————————	$\bar{1}$.092141	is	0.123635
————————	4.612300	is	40954.4

LOGARITHMIC ARITHMETIC.

1. *Multiplication by Logarithms.*

Add together the logarithms of all the factors, and the sum will be the logarithm of the product. If there be negative and affirmative indices, their difference, with the proper sign prefixed, is to be taken for the index of the logarithm of the product.

Observe to add, to the sum of the affirmative indices, what is carried from the sum of the decimal parts of the logarithms.

Ex. 1. Required the product of 23.14 and 5.062

$$\begin{aligned} \text{Log. of } 23.14 &= 1.364363 \\ \text{Log. of } 5.062 &= 0.704322 \\ \hline \text{Log. of Prod. } 117.134 &= 2.068685 \end{aligned}$$

2. What is the product of 2.58193 and 3.45729?

$$\begin{aligned} \text{Log. of } 2.58193 &= 0.411944 \\ \text{Log. of } 3.45729 &= 0.538736 \\ \text{Log. of Prod. } 8.92647 &= 0.950680 \end{aligned}$$

3. What is the continued product of 3.902, and 597.16, and .031473?

$$\begin{aligned} \text{Log. } 3.902 &= 0.591287 \\ \text{Log. } 597.16 &= 2.776091 \\ \text{Log. } 0.031473 &= \bar{2}.497938 \\ \hline \text{Log. of Prod. } 73.3357 &= 1.865316 \end{aligned}$$

The —2 cancels the affirmative 2, and the 1 carried from the decimal is put down as the index of the product.

4. Required the continued product of 3.586, 2.1046, .8872, and .0294.

$$\begin{aligned} \text{Log } 3.586 &= 0.554610 \\ \text{Log. } 2.1046 &= 0.323170 \\ \text{Log. } 0.8872 &= \bar{1}.922829 \\ \text{Log. } 0.0294 &= \bar{2}.468347 \\ \text{Log. of Prod. } 0.185761 &= \bar{1}.268956 \end{aligned}$$

Here the 2 which is carried from the decimal cancels the —2, and —1 remains.

2. *Division by Logarithms.*

From the logarithm of the dividend subtract the logarithm of the divisor; the remainder is a logarithm, whose corresponding natural number is the quotient. But first, observe to change the sign of the index of the logarithm of the divisor, from negative to affirmative, or from affirmative to negative, then take the sum of the indices, if they be of the same sign, prefixing the common sign, or their difference when of different signs, prefixing the sign of the greater, for the index of the logarithm of the quotient. Farther, when 10 is borrowed in the left-hand place of the decimal part of the logarithm, observe that the 1 which is to be carried must be added to the index of the logarithm of the divisor when that index is affirmative, but subtracted from it when the index is negative, then changing the sign of the result thus found, we are to work as before.

Ex. 1. Divide 24163 by 4567.

$$\begin{aligned} \text{Dividend } 24163, \text{ its log.} &= 4.383151 \\ \text{Divisor } 4567 \quad \text{———} &= 3.659631 \\ \text{Quot. } 5.29078 \quad &= 0.723520 \end{aligned}$$

2. Required the quotient of 37.149 by 523.76.

$$\begin{aligned} \text{Dividend } 37.149, \text{ its log.} &= 1.569947 \\ \text{Divisor } 523.76 \quad \text{———} &= 2.719132 \\ \text{Quot. } 0.0709275 \quad &= \bar{2}.850815 \end{aligned}$$

3. Divide .06314 by .007241.

Dividend .06314, its log. = $\bar{2}.800305$
Divisor .007241 ——— = $\bar{3}.859799$

Quot. 8.71978 ——— = 0.940506

Here 1 carried to —3 makes —2, which being subtracted from -2 in the dividend, 0 remains as the index of the quotient.

4. Divide .7438 by 12.9476.

Dividend .7438, its log. = $\bar{1}.871456$
Divisor 12.9476 ——— = 1.112189

Quot. .0574469 ——— = $\bar{2}.759267$

In this example, the affirmative 1 being changed into negative, and added to the —1 above, the index of the quotient is —2.

3. *Rule of Three, or Proportion*

From the sum of the logarithms of the second and third terms, subtract the logarithm of the first, the remainder will be the logarithm of the fourth term Or, in any compound proportion, add together the logarithms of all the terms to be multiplied, and from that sum take the sum of the logarithms of the other terms, the remainder will be the logarithm of the term sought.

Note.—Instead of subtracting any logarithm, if its arithmetical complement be added, the result will be the same. By the arithmetical complement of a logarithm, is meant the logarithm of the reciprocal of the corresponding natural number, or the difference between its logarithm and that of unity —To find the arithmetical complement, begin at the left hand, and subtract each figure from 9, except the last significant figure, which must be taken from 10 But when the index is negative, it is to be added to 9, and the rest subtracted as before. In taking the sum of the logarithms, it is to be observed, that for every arithmetical complement employed, 10 must be subtracted from the sum of the indices, in order to obtain the proper index of the result.

Ex. 1. If 72.34 lbs cost L.2 519, what will 357.486 lbs. cost?

As 72 34	Logarithm	1.859379
Is to 2.519	....	0.401228
So is 357 486	.. ..	2.553259
To 12.1483=L 12 8 . 11½		1.095108

Or, employing the arithmetical complement of the logarithm of the first term:

As 72.34	ar. comp. log.	8.140621
Is to 2.519	log.	0.401228
So is 357.486	....	2.553259
To 12.4483	....	1.095108

2. To find a third proportional to 12.796 and 3.24718.

As 12.796	ar. comp. log.	8.892926
Is to 3.24718	log.	0.511506
So is 3.24718	....	0.511506
To .824021	....	$\bar{1}$.915938

3. To find a number which shall be to .379145, as .85132 is to .0649.

As .0649	ar. comp. log.	11.187755
Is to .85132	log.	$\bar{1}$.931051
So is .379145	....	$\bar{1}$.578805
To 4.9844	....	0.697611

4. What is the interest of L.279, 5*s*. for 274 days, at the rate of $4\frac{1}{2}$ per cent. per annum?

As $\begin{cases}100\\365\end{cases}$	ar. comp. log.	$\begin{cases}8.000000\\7.437707\end{cases}$
Is to 4.5	log.	0.653213
So is 279.25	....	2.445993
274	....	2.437751
To 9.4333 = L.9 : 8 . 8		0.974664

4. *Involution, or the Raising of Powers.*

Multipy the logarithm of the given number by the index of the power, and the product will be the logarithm of the power sought.

Note.—In multiplying a logarithm having a negative index by an affirmative number, the product of the index is negative, but what is carried from the decimal of the logarithm is affirmative, therefore their difference will be the index of the product, and is of the same sign with the greater.

Ex. 1. Raise 25.791 to the 2d power or square.

Root 25.791, its log.		1.411468
	Index	2
Square 665.175		2.822936

2. Required the cube of 30.7146.

Root 30.7146, log.		1.487345
	Index	3
Cube 28975.7		4.462035

3. What is the fourth power of .09163?

Root .09163, log.		$\bar{2}.962038$
	Index	4
Biquadrate 0 0000704938		$\bar{5}.848152$

The 3 carried, taken from —8, leaves —5, the index of the product.

4 Raise 1.0045, to the 365th power.

Root 1.0045, its log.	0.001950
	365
	9750
	11700
	5850
Power 5 149	0.711750

5. *Evolution, or Extraction of Roots.*

Divide the logarithm of the power, or given number, by the index of the root, and the quotient will be the logarithm of the root sought.

Note.—When the index of the logarithm is negative, and the divisor is not exactly contained in it without a remainder, increase it by such a number as will make it exactly divisible; and carry the units borrowed, as so many tens, to the left-hand figure of the decimal part of the logarithm, which divide by the index of the root.

Ex. 1. To find the square root of 365.

$$2\,)\,2.562293 = \text{the log. of } 365.$$

$$\text{Root } 19.105 \quad 1.281146$$

2. To find the cube root of 12345.

$$3\,)\,4.091491 = \text{the log. of } 12345.$$

$$\text{Root } 23.1116 \quad 1.363830$$

3. Extract the 10th root of 2?

$$10\,)\,0.301030 = \text{the log. of } 2.$$

$$\text{Root } 1.071773 \quad 0.030103$$

4. Extract the 365th root of 1.045?

$$365\,)\,0.019116 = \text{the log. of } 1.045$$

$$\text{Root } 1.00012 \quad 0.000052$$

5 Required the square root of .093?

$$2\,)\,\bar{2}.968483 = \text{the log. of } .093.$$

$$\text{Root } .304959 \quad \bar{1}.484241$$

6 Find the cube root of .00048?

$$3\,)\,\bar{4}.681241 = \text{the log. of } .00048.$$

$$\text{Root } 0.0782973 \quad \bar{2}.893747$$

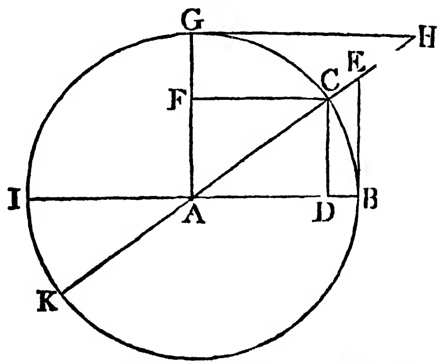
In the last example we add —2 to —4, and then 3 the divisor measures it exactly, —2 is therefore the index of the logarithm of the root, and, the 2 borrowed, being carried as so many tens to the left-hand figure of the decimal part, and the result then divided by the index, gives the decimal part of the logarithm of the root as above.

THE NATURE AND CALCULATION OF SINES, TANGENTS, &c.

1. Let BAC be any angle, and upon A, as a centre, with any radius whatever, let the circle BGK be described. Through A draw AG perpendicular to IB, also, through the points G and C draw GH, CF perpendicular to AG, and through B and C draw BE and CD perpendicular to AB.

Definitions.

I. The circumference of every circle is divided into 360 equal parts, called degrees, each degree is divided into 60 equal parts, called minutes, each minute is divided into 60 equal parts, called seconds, &c. marked thus. $25^{\circ}\,14'\,19''\,24'''$.



II. The arch BC is called the measure of the angle BAC at the centre of the circle, and the angle BAC is said to be an angle of as many degrees and parts of a degree, as the arch BC contains. For an angle at the centre of a circle has the same ratio to four right angles, that the arch intercepted between its sides has to the whole circumference, so that the angle will be determined, when it is known, what part the arch is of the whole circumference.

III. The straight line CD drawn from one extremity of the arch BC perpendicular to the diameter passing through the other extremity, is called the sine of the arch BC, or of the angle BAC.

IV. The segment BD of the diameter IB intercepted between B the extremity of the arch and the sine CD, is called the versed-sine of the arch BC, or of the angle BAC.

V. The straight line BE drawn at right angles to the diameter IB passing through one extremity of the arch BC and terminated by AE drawn from the centre through the other extremity, is called the tangent of the arch BC, or of the angle BAC.

VI. The line AE from the centre, terminating the tangent, is called the secant of the arch BC, or of the angle BAC.

VII. A quadrant is one-fourth part of a circle, and contains 90°.

VIII. The complement of an arch is its difference from a quadrant.

IX The tangent, sine, versed-sine, and secant of the complement of an arch, are called the co-tangent, co-sine, co-versed-sine, and co-secant of that arch.

Corollary I. to Def. III. IV. V. VI. and IX. Since ADC is a right angled triangle, we have $CD^2 = AC^2 - AD^2$, also $AD^2 = AC^2 - CD^2$. Hence, if we put the arch BC $=$ A, and the Radius $=$ R, we obtain, Sin. A $= \sqrt{R^2 - \text{Cos.}^2 A}$, and Cos A $= \sqrt{R^2 - \text{Sin.}^2 A}$. Again, from the similar triangles ADC, ABE, it appears, that AD : DC :: AB : BE and AD : AC :: AB : AE, hence we have also Tan. A $= \frac{\text{Sin. A} \times \text{R}}{\text{Cos. A}}$, and Sec. A $= \frac{R^2}{\text{Cos. A}}$. It is farther evident, that BD $=$ AB $-$ AD, that is, Ver. Sin. A $=$ R $-$ Cos. A.

Corollary II. It is evident, that the sine of 90°, the versed-sine of 90°, the tangent of 45°, and the secant of 0°, are each equal to radius.

X. The supplement of an arch is its difference from a semicircle, or 180°.

Corollary. The sine, co-sine, tangent, co-tangent, secant, and co secant of the supplement of an arch are equal to the sine, co-sine, tangent, co-tangent, secant and co-secant of the arch itself. The sine and co-secant of the supplement have also the same direction

with the sine, and co-secant of the arch: but the co-sine, tangent, co-tangent, and secant of the supplement have an opposite direction from the co-sine, tangent, co-tangent, and secant of the arch. When these lines are numerically expressed this difference of position is indicated by the positive and negative signs, so that if we consider as positive, or as affected by the sign $+$, the co-sines, tangents, co-tangents and secants of arches less than a quadrant, we must consider as negative, or as affected by the sign $-$, the co-sines, tangents, co-tangents and secants of arches greater than a quadrant.

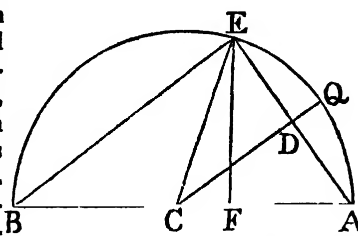
2. By means of the lines in and about a circle, that have here been defined, we are able to express the relations between the sides and angles of a triangle, and, upon these magnitudes, the solutions of the cases in trigonometry depend.

A *Trigonometrical Canon* is a table exhibiting the length of the sine, tangent, &c. to every degree and minute of the quadrant, radius being supposed unity, and conceived to be divided into 10000000 or more decimal parts.

3. One method of constructing the trigonometrical table, is contained in the following propositions

PROPOSITION I.

The cosine of an arch A being given to find the cosine of half that arch.



Let AE be any arch, of which CF is the cosine. From A and B, the extremities of the diameter AB, draw the chords AE, BE, and let CQ bisect the arch AE in Q, and, consequently, its chord AE perpendicularly in D. Then, since the right angled triangles ABE, BEF are equiangular, and similar, AB . BE : BE BF, and therefore $BE^2 = AB \times BF$. But since CD is parallel to BE, AC . CD : AB : BE, and alternately AC : AB . CD BE. Now $AC = \frac{1}{2}AB$, therefore, also $CD = \frac{1}{2}BE$, and $CD^2 = \frac{1}{4}BE^2$. But BE^2 has been shewn to be equal to $AB \times BF$, hence $CD^2 = \frac{1}{4}AB \times BF = \frac{1}{2}AC \times (AC + CF)$ Or putting the radius equal to R, and the arch $AE = A$, we have (since $CF = \text{Cos. } AE$, and $CD = \text{Cos. } AQ$) $\text{Cos. } \frac{1}{2} A = \sqrt{\frac{1}{2}R\,(R + \text{Cos. } A)}$.

PROPOSITION II.

Given the radius of a circle equal find the cosine minute.

In the formula obtained in last proposition, let us suppose the arch $A = 30^0$ Then, since the sine of an arch is equal to half the chord of double the arch, it follows, that the sine of 30^0 must be equal to half the chord of 60^0. But the chord of 60^0 is equal to radius, therefore, the sine of 30^0 is equal to half the radius, or equal to $\frac{1}{2}$ Hence, by the formula (Cor. I. Def. 3. 4 5. 6 and 9.) Cos. $A = \sqrt{R^2 - \text{Sin}^2 A}$, we have Cos. $30^0 = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = 0.8660254$. Suppose now, that in the formula of the preceding proposition, namely, Cos $\frac{1}{2} A = \sqrt{\frac{1}{2} R (R + \text{Cos. } A)}$, we substitute for A, R, and Cos. A their respective values, and it becomes Cos. $15^0 = \sqrt{\frac{1}{2}(1 + 0.8660254)} = 0.9659258$. In like manner, by supposing next that A is equal to 15^0, we obtain Cos. $7^0\ 30' = \sqrt{\frac{1}{2}(1 + \text{Cos. } 15^0)} = \sqrt{\frac{1}{2}(1 + 0.9659258)} = 0.9914449$. In this manner, Cos $3^0\ 45'$, Cos. $1^0\ 52'\ 30''$, and so on, may be successively computed, till after eleven bisections of the arch of 30^0 the cosine of the small arch $52''\ 44'''\ 3''''\ 45'''''$ is found. But by the formula (Cor. I. Def. 3. 4. 5. 6. and 9.) Sin. $A = \sqrt{R^2 - \text{Cos}^2 A}$, we obtain, Sin. $(52''\ 44'''\ \&c.) = \sqrt{1 - \text{Cos}^2 (52''\ 44'''\ \&c.)}$; and hence the sine of $52''\ 44'''\ 3''''\ 45'''''$ may be determined.

Now, it is to be observed, that the sines of very small arches nearly coincide with, and have therefore to one another nearly the same ratio as the arches themselves. Hence we have,

$52''\ 44'''\ 3''''\ 45''''' : 1' \,.\,.\, \text{Sin. } (52''\ 44'''\ 3''''\ 45''''') : \text{Sin. } 1' = 0.000290888$.

The sine of $1'$ being thus found, the cosine may easily be determined, for Cos. $1' =$

$\sqrt{1 - \text{Sin}^2 1'} = \sqrt{1 - (0.000290888)^2} = 0.999999958$.

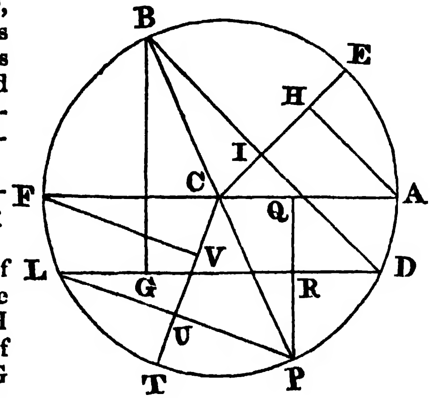
PROPOSITION III.

Twice the rectangle under the sine of half the sum, and the cosine of half the difference of two arches, is equal to the rectangle

under radius and the sum of their sines. Also twice the rectangle under the cosine of half the sum, and sine of half the difference of two arches is equal to the rectangle under radius, and the difference of their sines.

Let AB, AD, be two arches of a circle, of which C is the centre, and AF a diameter, draw DL parallel to AF, and BG perpendicular to DL; also draw CE perpendicular to the line joining B, D, and AH parallel to BD.

Then the triangles BGD, CHA, which have the angles at G and H right angles and the angles at D and A equal, by reason of parallel lines, are equiangular, therefore BD . BG CA . CH, and consequently, BD × CH, or 2BI × CH = CA × BG.



Now, BI is the sine of BE, half the sum of the arches AB and AD, CH is the cosine of AE, half their difference, and BG the sum of their sines, whence it is evident, that 2 Sin. $\frac{1}{2}$ (AB + AD) × Cos. $\frac{1}{2}$ (AB — AD) = R × (Sin. AB + Sin. AD).

Again, let the diameter BCP be drawn, from P draw PQ perpendicular to AF, and meeting DL in R, join PL, and let CT be drawn perpendicular to PL and FV parallel to it. Then, from the similar triangles PLR, CVF, we have PL, or 2PU . PR ,. FC . CV, hence 2PU × CV = PR × FC. But PU is evidently equal to the sine of half the difference of PF and LF, or to the sine of half the difference of AB and AD, and CV is equal to the cosine of half the sum of the same two arches, also PR is equal to the difference of their sines.

Hence, 2Cos. $\frac{1}{2}$ (AB + AD) × Sin. $\frac{1}{2}$ (AB — AD) = R × (Sin. AB — Sin. AD).

Corollary. Suppose A and B to represent any two arches, and let the arch AB = A + B, and AD = A — B. Then $\frac{1}{2}$ (AB + AD) = A, and $\frac{1}{2}$ (AB — AD) = B. Hence, by substituting, and putting rad. = 1, we have,

I. Sin. A × Cos. B = $\frac{1}{2}$ Sin. (A + B) + $\frac{1}{2}$ Sin. (A — B).
II. Cos. A × Sin. B = $\frac{1}{2}$ Sin. (A + B) — $\frac{1}{2}$ Sin. (A — B).

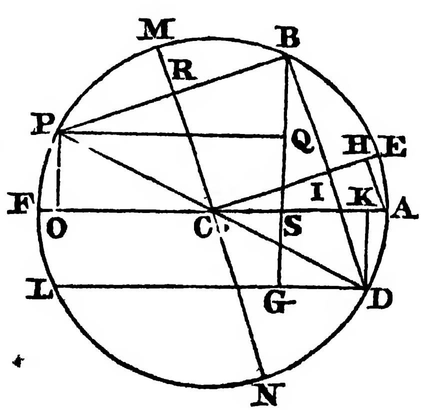
By adding and subtracting these two formulæ, we obtain the two following.

III. Sin. A $\times$ Cos. B + Cos. A $\times$ Sin. B = Sin. (A + B).
IV. Sin. A $\times$ Cos. B — Cos. A $\times$ Sin. B = Sin. (A — B).

PROPOSITION IV.

Twice the rectangle under the cosine of half the sum, and the cosine of half the difference of any two arches, is equal to rectangle under radius, and the sum of their cosines. Also twice the rectangle under the sine of half the sum, and the sine of half the difference of any two arches, is equal to the rectangle under radius, and the difference of their cosines.

Let AB, AD, be two arches of a circle, whose centre is C, and of which AF is a diameter, draw DL parallel and DK perpendicular to AF, and BG perpendicular to DL; also draw CE perpendicular to the line joining B, D, and AH and MCN parallel to BD. Farther, draw the diameter DP, join BP, and through P draw PQ parallel and PO perpendicular to AF. Then from the similar triangles BPQ, ACH, we have BP or 2BR : PQ : : CA : CH; therefore 2BR $\times$ CH = CA $\times$ PQ. But it is manifest that BR = Cos. BE = Cos $\frac{1}{2}$ (BA + AD), that CH = Cos. $\frac{1}{2}$ (BA — AD), and that PQ = CO + CS = CS + CK = Cos. AB + Cos. AD. Hence 2Cos. $\frac{1}{2}$ (BA + AD) $\times$ Cos. $\frac{1}{2}$ (BA — AD) = R $\times$ (Cos. AB + Cos. AD).



Again, from the similar triangles BGD, CAH, we obtain BD or 2BI . DG . : CA . AH, therefore 2BI $\times$ AH = CA $\times$ DG. But BI = Sin. $\frac{1}{2}$ (AB + AD), AH = Sin. $\frac{1}{2}$ (AB — AD) and DG = CK — CS = Cos. AD — Cos. AB, whence we have 2 Sin. $\frac{1}{2}$ (AB + AD) $\times$ Sin. $\frac{1}{2}$ (AB — AD) = R $\times$ (Cos. AD — Cos. AB).

Corollary. Let A and B be any two arches, and let the arch AB = A + B, and the arch AD = A — B, then $\frac{1}{2}$ (AB + AD) = A, and

$\frac{1}{2}$ (AB — AD) = B. Hence, by substituting and putting radius equal to unity, we have,

$$\text{I. Cos. A} \times \text{Cos. B} = \tfrac{1}{2}\,\text{Cos. (A} - \text{B)} + \tfrac{1}{2}\,\text{Cos. (A} + \text{B)}$$
$$\text{II. Sin. A} \times \text{Sin. B} = \tfrac{1}{2}\,\text{Cos (A} - \text{B)} - \tfrac{1}{2}\,\text{Cos. (A} + \text{B).}$$

By adding and subtracting these two formulæ, we obtain the two following:

$$\text{III. Cos. A} \times \text{Cos B} + \text{Sin. A} \times \text{Sin. B} = \text{Cos. (A} - \text{B).}$$
$$\text{IV Cos A} \times \text{Cos B} - \text{Sin A} \times \text{Sin. B} = \text{Cos. (A} + \text{B)}$$

PROPOSITION V

The sine and cosine of the arch of 1 minute being given, to find the sine and cosine of every arch of an exact number of minutes, from 1 minute to 90 degrees

The sine and cosine of the arch of 1 minute being known, the sines of all the multiple arches of 1 minute may be successively deduced, by putting in formula I of Cor to Prop III, B = 1′, and by making successively A equal to 1′, 2′, 3′, 4′, &c. Thus, we obtain,

$$\begin{aligned}
\text{Sin } 2' &= 2\text{Cos } 1' \times \text{Sin. } 1' - \text{Sin. } 0' \\
\text{Sin } 3' &= 2\text{Cos } 1' \times \text{Sin } 2' - \text{Sin. } 1' \\
\text{Sin } 4' &= 2\text{Cos } 1' \times \text{Sin } 3' - \text{Sin. } 2' \\
\text{Sin. } 5' &= 2\text{Cos. } 1' \times \text{Sin. } 4' - \text{Sin. } 3' \\
\&c \quad & \quad \&c. \quad\quad \&c
\end{aligned}$$

In like manner, by putting in formula I of Cor. to Prop. IV., B = 1′, and by making A successively equal to 1′, 2′, 3′, 4′, &c., we have,

$$\begin{aligned}
\text{Cos. } 2' &= 2\text{Cos. } 1' \times \text{Cos. } 1' - 1 \\
\text{Cos. } 3' &= 2\text{Cos. } 1' \times \text{Cos } 2' - \text{Cos. } 1' \\
\text{Cos. } 4' &= 2\text{Cos. } 1' \times \text{Cos. } 3' - \text{Cos. } 2' \\
\text{Cos. } 5' &= 2\text{Cos. } 1' \times \text{Cos. } 4' - \text{Cos. } 3' \\
\&c. \quad & \quad \&c. \quad\quad \&c.
\end{aligned}$$

Thus, the sines and cosines of all arches of an exact number of minutes, may be successively derived from one another, but having proceeded in this manner as far as the sine and cosine of 30°,

the sines and cosines of all the multiple arches of 1 minute beyond 30°, may be found by addition and subtraction only. This will appear manifest, if in the formulæ,

$$\text{Sin. } A \times \text{Cos. } B = \tfrac{1}{2} \text{Sin. } (A + B) + \tfrac{1}{2} \text{Sin. } (A - B)$$
$$\text{Sin. } A \times \text{Sin. } B = \tfrac{1}{2} \text{Cos. } (A - B) - \tfrac{1}{2} \text{Cos. } (A + B)$$

we suppose $A = 30^{\circ}$, for then Sin. $A =$ Sin. 30°, becomes equal to $\frac{1}{2}$, and we obtain

$$\text{Cos. } B = \text{Sin. } (30^{\circ} + B) + \text{Sin. } (30^{\circ} - B)$$
$$\text{Sin. } B = \text{Cos. } (30^{\circ} - B) - \text{Cos. } (30^{\circ} + B)$$

Hence it follows, that

$$\text{Sin. } (30^{\circ} + B) = \text{Cos. } B - \text{Sin. } (30^{\circ} - B).$$
$$\text{Cos. } (30^{\circ} + B) = \text{Cos. } (30^{\circ} - B) - \text{Sin. } B.$$

In these two last expressions, let B be made successively equal to $1'$, $2'$, $3'$, $4'$, &c. then we shall have

$$\text{Sin. } 30^{\circ}\ 1' = \text{Cos. } 1' - \text{Sin. } 29^{\circ}\ 59'$$
$$\text{Sin. } 30^{\circ}\ 2' = \text{Cos. } 2' - \text{Sin. } 29^{\circ}\ 58'$$
$$\text{Sin. } 30^{\circ}\ 3' = \text{Cos. } 3' - \text{Sin. } 29^{\circ}\ 57'$$
$$\text{Sin. } 30^{\circ}\ 4' = \text{Cos. } 4' - \text{Sin. } 29^{\circ}\ 56'$$
&c. &c. &c.

And

$$\text{Cos. } 30^{\circ}\ 1' = \text{Cos. } 29^{\circ}\ 59' - \text{Sin. } 1'$$
$$\text{Cos. } 30^{\circ}\ 2' = \text{Cos. } 29^{\circ}\ 58' - \text{Sin. } 2'$$
$$\text{Cos. } 30^{\circ}\ 3' = \text{Cos. } 29^{\circ}\ 57' - \text{Sin. } 3'$$
$$\text{Cos. } 30^{\circ}\ 4' = \text{Cos. } 29^{\circ}\ 56' - \text{Sin. } 4'$$
&c. &c. &c.

In this manner, it is manifest, the sines and cosines of all arches of an exact number of minutes from 30° to 60°, are readily found from the sines and cosines of arches less than 30°.

In order to derive the sines and cosines of the arches between 60° and 90°, in the formulæ

$$\text{Cos. } A \times \text{Sin. } B = \tfrac{1}{2} \text{Sin. } (A + B) - \tfrac{1}{2} \text{Sin. } (A - B)$$
$$\text{Cos. } A \times \text{Cos. } B = \tfrac{1}{2} \text{Cos. } (A + B) + \tfrac{1}{2} \text{Cos. } (A - B)$$

let us put $A = 60^0$, so that Cos. $A =$ Cos. $60^0 = \frac{1}{2}$, then we obtain

$$\text{Sin. } B = \text{Sin. } (60^0 + B) - \text{Sin. } (60^0 - B).$$
$$\text{Cos. } B = \text{Cos. } (60^0 + B) + \text{Cos. } (60^0 - B).$$

Hence it follows, that

$$\text{Sin. } (60^0 + B) = \text{Sin. } B + \text{Sin. } (60^0 - B).$$
$$\text{Cos. } (60^0 + B) = \text{Cos. } B - \text{Cos. } (60^0 - B).$$

Let us now suppose B to become successively equal to 1′, 2′, 3′, 4′, &c. and we find

$$\text{Sin. } 60^0\ 1' = \text{Sin. } 1' + \text{Sin. } 59^0\ 59'$$
$$\text{Sin. } 60^0\ 2' = \text{Sin. } 2' + \text{Sin. } 59^0\ 58'$$
$$\text{Sin. } 60^0\ 3' = \text{Sin. } 3' + \text{Sin. } 59^0\ 57'$$
$$\text{Sin. } 60^0\ 4' = \text{Sin. } 4' + \text{Sin. } 59^0\ 56'$$

&c. &c &c.

And,

$$\text{Cos. } 60^0\ 1' = \text{Cos } 1' - \text{Cos. } 59^0\ 59'$$
$$\text{Cos. } 60^0\ 2' = \text{Cos. } 2' - \text{Cos. } 59^0\ 58'$$
$$\text{Cos. } 60^0\ 3' = \text{Cos. } 3' - \text{Cos. } 59^0\ 57'$$
$$\text{Cos. } 60^0\ 4' = \text{Cos. } 4' - \text{Cos. } 59^0\ 56'$$

&c. &c. &c.

By this method, a table of sines and cosines may be calculated, for every degree and minute of the quadrant. But, it is to be remarked, that the sines and cosines of the arches between 0^0 and 45^0, comprehend the sines and cosines of the arches between 45^0 and 90^0, for, it is evident, that Sin. $(45^0 + A) =$ Cos. $(90^0 - (45^0 + A)) =$ Cos. $(45^0 - A)$, and that Cos. $(45^0 + A) =$ Sin. $(90^0 - (45^0 + A)) =$ Sin. $(45^0 - A)$. Hence, it follows, that when we have proceeded in the calculation of the sines and cosines, as far as the sine and cosine of 45^0, the table of sines and cosines is completed.

With regard to those arches, which do not consist of an exact number of minutes, for the odd seconds in each, a proportional part of the difference between the sines or cosines of the next greater and next less arches may be taken, and added to or subtracted from the sine or cosine of the less arch, and that will give the sine or cosine of such an arch nearly.

PROPOSITION VI.

. The sine and cosine of every arch of the quadrant being given, to find the tangents, cotangents, secants, cosecants, versed-sines, and coversed-sines of these arches.

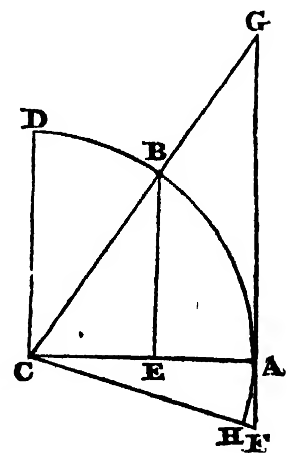
Let A represent any arch of the quadrant, then, since by hypothesis, Sin. A and Cos. A are given, we obtain at once from Cor. I. to Def. 3. 4. 5. 6. and 9., the following expressions (radius being unity),

$$\text{Tan. A} = \frac{\text{Sin A}}{\text{Cos. A}}, \text{ Cot. A} = \frac{\text{Cos. A}}{\text{Sin. A}},$$

$$\text{Sec. A} = \frac{1}{\text{Cos. A}}, \text{ Cosec. A} = \frac{1}{\text{Sin. A}},$$

$$\text{Ver. Sin. A} = 1 - \text{Cos. A, Cover. Sin.} = 1 - \text{Sin. A.}$$

The secants and cosecants will be found most conveniently from the tangents and cotangents. For the secant of any arch, is equal to its tangent, added to the tangent of half its complement. Let AB be any arch of the quadrant AD, and let AG be the tangent, and CG the secant of the arch AB. Produce GA to F, making GF equal to GC, and let CF be joined. Then, since the angles ACG, CGA are together equal to a right angle, and the angles ACF, AFC are also together equal to a right angle, the angles ACG, CGA are together equal to the angles ACF, AFC. But the angle AFC is equal to FCG, therefore, the angles ACG, CGA, are together equal to ACF, FCG, that is, to the angle ACG, together with twice the angle ACF. Taking from each of these equals the common angle ACG, there remains the angle CGA equal to twice the angle ACF. Hence, it appears, that the angle ACF is equal to half the angle AGC, or to half the angle BCD, and, therefore, the arch AH, is also equal to half the arch BD. Now, BD is the comple-



ment of AB, and AF is the tangent of AH, wherefore, AF is the tangent of half the complement of AB. From this property of the secant, we derive the following formulæ. Let A represent, as before, any arch of the quadrant, then

$$\text{Sec. } A = \text{Tan. } A + \text{Tan. } 45^{\circ} - \tfrac{1}{2} A).$$
$$\text{Cosec. } A = \text{Cot. } A + \text{Tan. } \tfrac{1}{2} A.$$

From these expressions, the secant and cosecant of every second arch, of which the tangent and cotangent are known, are derived by addition only. Thus, supposing the tangents to be given for every minute of the quadrant, we have,

$$\begin{aligned} \text{Sec. } 2' &= \text{Tan. } 2' + \text{Tan. } 44^{\circ}\,59' \\ \text{Sec. } 4' &= \text{Tan. } 4' + \text{Tan. } 44^{\circ}\,58' \\ \text{Sec. } 6' &= \text{Tan. } 6' + \text{Tan. } 44^{\circ}\,57' \\ \text{Sec. } 8' &= \text{Tan. } 8' + \text{Tan. } 44^{\circ}\,56' \\ &\&c. \qquad\qquad \&c. \end{aligned}$$

4. Upon the principles laid down in the preceding propositions, the trignometrical tables may be constructed But there is another method of computing the natural sine, cosine, &c of any arch, immediately from the length of the arch being given, by means of series.

5. For investigating the formulæ necessary for this purpose, let us again suppose the radius equal to 1, then, it is evident, that A being any arch, $\text{Cos.}^2 A + \text{Sin}^2 A = 1$. The first member of this equation may be regarded as the product of the two imaginary factors, Cos. A $+ \sqrt{-1}$ Sin A, and Cos. A $- \sqrt{-1}$ Sin. A. If we multiply together the two similar factors, Cos. A $+ \sqrt{-1}$ Sin. A, and Cos. B $+ \sqrt{-1}$ Sin. B, the product is found to be Cos. A Cos. B $-$ Sin. A Sin. B $+$ (Sin A Cos B $+$ Sin B Cos. A) $\sqrt{-1}$. But, by Cor. Prop. IV. § 3, we have Cos. A Cos B $-$ Sin. A Sin. B $=$ Cos. (A $+$ B), and by Cor. Prop. III, Sin A Cos. B $+$ Sin. B Cos. A $=$ Sin (A $+$ B). Hence the above product becomes Cos (A $+$ B) $+ \sqrt{-1}$ Sin (A $+$ B), an expression exactly similar in form to each of the factors, from which it has been

derived. We have therefore in general, $(\text{Cos. A} + \sqrt{-1}\ \text{Sin. A}) \times (\text{Cos. B} + \sqrt{-1}\ \text{Sin. B}) = \text{Cos.}(A+B) + \sqrt{-1}\ \text{Sin.}(A+B)$, and, it is remarkable, that the multiplication of this kind of quantities, is performed by simply adding together the arches themselves. This property is analogous to that of logarithms.

From the above formula, we derive successively,

$(\text{Cos. A} + \sqrt{-1}\ \text{Sin. A}) \times (\text{Cos. A} + \sqrt{-1}\ \text{Sin. A}) = \text{Cos } 2A + \sqrt{-1}\ \text{Sin. } 2A$.

$(\text{Cos. A} + \sqrt{-1}\ \text{Sin. A}) \times (\text{Cos. } 2A + \sqrt{-1}\ \text{Sin. } 2A) = \text{Cos. } 3A + \sqrt{-1}\ \text{Sin. } 3A$.

$(\text{Cos A} + \sqrt{-1}\ \text{Sin. A}) \times (\text{Cos. } 3A + \sqrt{-1}\ \text{Sin. } 3A) = \text{Cos. } 4A + \sqrt{-1}\ \text{Sin } 4A$.

&c. &c. &c.

The first product is equal to $(\text{Cos. A} + \sqrt{-1}\ \text{Sin. A})^2$, the second is equal to $(\text{Cos A} + \sqrt{-1}\ \text{Sin. A})^3$, and so on. Therefore, in general, we have

$$(\text{Cos A} + \sqrt{-1}\ \text{Sin. A})^n = \text{Cos. } nA + \sqrt{-1}\ \text{Sin } nA$$

Hence, by changing the sign of $\sqrt{-1}$, we obtain,

$$(\text{Cos. A} - \sqrt{-1}\ \text{Sin. A})^n = \text{Cos. } nA - \sqrt{-1}\ \text{Sin. } nA$$

From these two equations, which have been derived, the one from the other, we obtain the values of Sin nA, and Cos nA, for, by adding them together, we find Cos. $nA =$

$$\tfrac{1}{2}(\text{Cos. A} + \sqrt{-1}\ \text{Sin. A})^n + \tfrac{1}{2}(\text{Cos. A} - \sqrt{-1}\ \text{Sin. A})^n,$$

and by subtracting the one from the other, we have Sin. $nA =$

$$\frac{1}{2\sqrt{-1}}\Big((\text{Cos A} + \sqrt{-1}\ \text{Sin. A})^n - (\text{Cos A} - \sqrt{-1}\ \text{Sin. A})^n\Big)$$

If we now wish to express the same quantities by means of series, it will be necessary to expand by means of the Binomial Theorem, the expression $(\text{Cos A} + \sqrt{-1}\ \text{Sin. A})^n$, this will give us for a result,

$$\text{Cos}^n A + \frac{n}{1} \text{Cos}^{n-1} A \text{ Sin. } A \sqrt{-1} - \frac{n(n-1)}{1 \cdot 2} \text{Cos}^{n-2} A \text{ Sin}^2 A$$
$$- \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} \text{Cos}^{n-3} A \text{ Sin}^3 A \sqrt{-1} + \&c.$$

This expression being the value of Cos. $nA + \sqrt{-1}$ Sin. nA, let us put the real part equal to Cos. nA, and the imaginary part equal to $\sqrt{-1}$ Sin. nA. We therefore have for the sine and cosine of the multiple arch, the following formulæ:

$$\text{Cos. } nA = \text{Cos}^n A - \frac{n(n-1)}{1 \cdot 2} \text{Cos}^{n-2} A \text{ Sin}^2 A +$$
$$\frac{n(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3 \cdot 4} \text{Cos}^{n-4} A \text{ Sin}^4 A - \&c.$$

$$\text{Sin. } nA = n\text{Cos}^{n-1} A \text{ Sin. } A - \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} \text{Cos}^{n-3} A \text{ Sin}^3 A + \&c.$$

The law of these series, is easily perceived.

Since we know that Sin. A = Cos. A × Tan. A, the above series may also be put under the following form:

$$\text{Cos. } nA = \text{Cos}^n A \left(1 - \frac{n(n-1)}{1 \cdot 2} \text{Tan}^2 A + \frac{n(n-1)(n-2)(n-3)}{1 \cdot 2 \cdot 3 \cdot 4} \text{Tan}^4 A - \&c.\right)$$

$$\text{Sin. } nA = \text{Cos}^n A \left(\frac{n}{1} \text{Tan. } A - \frac{n(n-1)(n-2)}{1 \cdot 2 \cdot 3} \text{Tan}^3 A + \&c.\right).$$

Let us now suppose $n = \frac{x}{A}$; and we shall have, by substituting this value, retaining, however, the factor Cos^n A.

$$\text{Cos. } x = \text{Cos}^n A \left(1 - \frac{x(x-A)}{1 \cdot 2} \cdot \frac{\text{Tan}^2 A}{A^2} + \frac{x(x-A)(x-2A)(x-3A)}{1 \cdot 2 \cdot 3 \cdot 4} \cdot \frac{\text{Tan}^4 A}{A^4} - \&c.\right)$$

$$\text{Sin. } x = \text{Cos}^n A \left(\frac{x}{1} \cdot \frac{\text{Tan. } A}{A} - \frac{x(x-A)(x-2A)}{1 \cdot 2 \cdot 3} \cdot \frac{\text{Tan}^3 A}{A^3} + \&c\right).$$

In these formulas, we may take the arch A at pleasure; let us therefore suppose A to be very small, then $\frac{\text{Tan. A}}{\text{A}}$ will differ but very little from unity, because the tangent of a very small arch is very nearly equal to the arch itself. But, it is evident, that as long as the arch is not equal to nothing, we have, $\frac{\text{Tan. A}}{\text{A}} > 1$: and we have, at the same time, $\text{A} > \text{Sin. A}$, so that, $\frac{\text{Tan. A}}{\text{A}} < \frac{\text{Tan. A}}{\text{Sin. A}}$, or $\frac{\text{Tan. A}}{\text{A}} < \frac{1}{\text{Cos. A}}$. Hence it follows, that the ratio $\frac{\text{Tan. A}}{\text{A}}$ is always comprehended between the limits 1 and $\frac{1}{\text{Cos. A}}$.

Let A be equal to 0, then we shall have $\text{Cos. A} = 1$; therefore, since $\frac{\text{Tan. A}}{\text{A}}$ is comprehended between 1 and $\frac{1}{\text{Cos. A}}$, it necessarily follows, that when $\text{A} = 0$, we have exactly $\frac{\text{Tan. A}}{\text{A}} = 1$. Put, therefore, $\text{A} = 0$, and we obtain,

$$\text{Cos.}\,x = \text{Cos}^n\text{A}\left(1 - \frac{x^2}{1\cdot 2} + \frac{x^4}{1\cdot 2\cdot 3\cdot 4} - \frac{x^6}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6} + \&c.\right)$$

$$\text{Sin.}\,x = \text{Cos}^n\text{A}\left(x \quad x^3 \quad \frac{}{1\cdot 2\cdot 3\cdot 4\cdot 5} - \frac{}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} + \&c\right).$$

It still remains to be determined, what the multiplier Cos^nA becomes, when the arch A is continually diminished, and at last becomes equal to 0. For this purpose, it is necessary to observe, that $\frac{1}{\text{Cos}^2\text{A}} = \text{Sec}^2\text{A} = 1 + \text{Tan}^2\text{A}$; and that therefore $\text{Cos. A} = (1 + \text{Tan}^2\text{A})^{-\frac{1}{2}}$ Hence, $\text{Cos}^n\text{A} = (1 + \text{Tan}^2\text{A})^{-\frac{n}{2}} = 1 \quad \frac{n}{2}\text{Tan}^2\text{A} + \frac{n(n-2)}{2\cdot 4}\text{Tan}^4\text{A} -$ &c.

Substituting instead of n, its value $\frac{x}{\text{A}}$, we have,

$$\text{Cos}^n\text{A} = 1 - \frac{x}{2}\text{A}\cdot\frac{\text{Tan}^2\text{A}}{\text{A}^2} + \frac{x(x-2\text{A})}{2\cdot 4}\text{A}^2\cdot\frac{\text{Tan}^4\text{A}}{\text{A}^4} - \&c.$$

If we now suppose, that A is continually diminished, while the value of x remains the same, it is evident, the value of Cos^n A will approach continually to unity, and, if at last, we suppose $A = 0$, and consequently $\frac{\mathrm{Tan.}\ A}{A} = 1$, we shall have exactly $\mathrm{Cos}^n\ A = 1$

Hence we obtain these two formulæ,

$$\mathrm{Cos.}\ x = 1 - \frac{x^2}{1\cdot 2} + \frac{x^4}{1\cdot 2\cdot 3\cdot 4} - \frac{x^6}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6} + \&c$$

$$\mathrm{Sin.}\ x = x - \frac{x^3}{1\cdot 2\cdot 3} + \frac{x^5}{1\cdot 2\cdot 3\cdot 4\cdot 5} - \frac{x^7}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} + \&c$$

From these series, the sine and cosine of any arch may be calculated, the length of the arch being given in parts of the radius considered as unity.

6. The sine and cosine being determined, the tangent and cotangent may be found from the formulæ $\mathrm{Tan.}\ x = \frac{\mathrm{Sin}\ x}{\mathrm{Cos.}\ x}$, $\mathrm{Cotan.}\ x = \frac{\mathrm{Cos.}\ x}{\mathrm{Sin}\ x}$. It may, however, be convenient, to express the tangent and cotangent, in the following manner.

$$\mathrm{Tan.}\ x = \frac{x - \frac{x^3}{1\cdot 2\cdot 3} + \frac{x^5}{1\cdot 2\cdot 3\cdot 4\cdot 5} - \frac{x^7}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} + \&c}{1 - \frac{x^2}{1\cdot 2} + \frac{x^4}{1\cdot 2\cdot 3\cdot 4} - \frac{x^6}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6} + \&c}$$

$$\mathrm{Cotan.}\ x = \frac{1 - \frac{x^2}{1\cdot 2} + \frac{x^4}{1\cdot 2\cdot 3\cdot 4} - \frac{x^6}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6} + \&c}{x - \frac{x^3}{1\cdot 2\cdot 3} + \frac{x^5}{1\cdot 2\cdot 3\cdot 4\cdot 5} - \frac{x^7}{1\cdot 2\cdot 3\cdot 4\cdot 5\cdot 6\cdot 7} + \&c}$$

The secant and cosecant might be expressed in a similar manner, but these, as has already been observed, are most easily found from the tangents, simply by addition, by means of the formulæ,

$$\left.\begin{array}{l}\mathrm{Sec.}\ A = \mathrm{Tan}\ A + \mathrm{Tan.}\ (45^\circ - \tfrac{1}{2}A) \\ \mathrm{Cosec.}\ A = \mathrm{Cot.}\ A + \mathrm{Tan.}\ \tfrac{1}{2}A\end{array}\right\}\ (\mathrm{Prop\ VI}\ \S\ 3.)$$

7. The series of § 5, expressing the sine, cosine, and tangent of x, may be exhibited under a succinct form, by means of exponential

quantities. For this purpose, let e be the radical number of the hyperbolic system of logarithms, then, from a formula already investigated (Logar. § 23.), by putting $r = e$, A, the modulus, $= 1$, and $z =$ Hyp. Log. N, (N being any number,) we obtain,

$$e^z = N = 1 + \frac{z}{1} + \frac{z^2}{1\cdot 2} + \frac{z^3}{1\cdot 2\cdot 3} + \frac{z^4}{1\cdot 2\cdot 3\cdot 4} + \&c.$$

If, in this series, we put $z = x\sqrt{-1}$, it becomes,

$$e^{x\sqrt{-1}} = 1 + \frac{x\sqrt{-1}}{1} - \frac{x^2}{1\cdot 2} - \frac{x^3\sqrt{-1}}{1\cdot 2\cdot 3} + \frac{\quad}{1\cdot 2\cdot 3\cdot 4} + \frac{x^5\sqrt{-1}}{1\cdot 2\cdot 3\cdot 4\cdot 5} - \&c.$$

We have, in like manner, by changing the sign of $\sqrt{-1}$,

$$e^{-x\sqrt{-1}} = 1 - \frac{x\sqrt{-1}}{1} - \frac{x^2}{1\cdot 2} + \frac{x^3\sqrt{-1}}{1\cdot 2\cdot 3} + \frac{\quad}{1\cdot 2\cdot 3\cdot 4} \quad \frac{x^5\sqrt{-1}}{1\cdot 2\cdot 3\cdot 4\cdot 5} - \&c$$

Whence we derive, by adding and subtracting,

$$\frac{e^{x\sqrt{-1}} + e^{-x\sqrt{-1}}}{2} = 1 - \frac{x^2}{1\cdot 2} + \frac{x^4}{1\cdot 2\cdot 3\cdot 4} - \&c$$

$$\frac{e^{x\sqrt{-1}} - e^{-x\sqrt{-1}}}{2\sqrt{-1}} = x - \frac{x^3}{1\cdot 2\cdot 3} + \frac{x^5}{1\cdot 2\cdot 3\cdot 4\cdot 5} - \&c.$$

The second members of these equations, are the values which we have found for Cos. x and Sin. x. Hence we have,

$$\text{Cos. } x = \frac{e^{x\sqrt{-1}} + e^{-x\sqrt{-1}}}{2}, \text{ and}$$

$$\text{Sin } x = \frac{e^{x\sqrt{-1}} - e^{-x\sqrt{-1}}}{2\sqrt{-1}}$$

From these we derive,

$$\frac{1}{\sqrt{-1}} \times \frac{e^{x\sqrt{-1}} - e^{-x\sqrt{-1}}}{e^{x\sqrt{-1}} + e^{-x\sqrt{-1}}} = \frac{\text{Sin. } x}{\text{Cos. } x} = \text{Tan } x$$

8. The above formulas for Sin. x and Cos x, give us by addition and subtraction.

$$e^{x\sqrt{-1}} = \text{Cos.}\ x + \sqrt{-1}\ \text{Sin.}\ x, \text{ and}$$
$$e^{-x\sqrt{-1}} = \text{Cos.}\ x - \sqrt{-1}\ \text{Sin.}\ x.$$

Hence, by dividing the former of these equations by the latter, we obtain,

$$\frac{e^{x\sqrt{-1}}}{e^{-x\sqrt{-1}}}, \text{ or } e^{2x\sqrt{-1}} = \frac{\text{Cos.}\ x + \sqrt{-1}\ \text{Sin.}\ x}{\text{Cos.}\ x - \sqrt{-1}\ \text{Sin.}\ x} =$$
$$\frac{1 + \sqrt{-1}\ \text{Tan.}\ x}{1 - \sqrt{-1}\ \text{Tan.}\ x}.$$

Or by taking the hyperbolic logarithm of each member of this last equation,

$(\text{Log.}\ e) \times 2x\sqrt{-1}$, or, (since Hyp. Log. $e = 1$), $2x\sqrt{-1} =$ $\text{Log.}\left(\frac{1 + \sqrt{-1}\ \text{Tan.}\ x}{1 - \sqrt{-1}\ \text{Tan.}\ x}\right)$. But we know (Logar. § 12.) that $\text{Log.}\ \frac{1+y}{1-y} = 2y + \frac{2}{3}y^3 + \frac{2}{5}y^5 + \text{\&c.}$ Putting, therefore, $\sqrt{-1}\ \text{Tan.}\ x$ instead of y, and dividing both sides by $2\sqrt{-1}$, we find,

$$x = \text{Tan.}\ x - \tfrac{1}{3}\ \text{Tan}^3\ x + \tfrac{1}{5}\ \text{Tan}^5\ x - \tfrac{1}{7}\ \text{Tan}^7\ x + \text{\&c.*}$$

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* This expression for the arch, in terms of the tangent, was originally found by JAMES GREGORY, some time after the middle of the 17th century

The series for the arch, in terms of the sine, is likewise remarkable, and was first discovered by SIR ISAAC NEWTON Let S denote the sine of any arch whose length is represented by a, then radius being unity, we have,

$$a = S + \frac{1}{2}\frac{S^3}{3} + \frac{1 \cdot 3}{2 \cdot 4}\frac{S^5}{5} + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}\frac{S^7}{7} + \frac{1 \cdot 3 \cdot 5 \cdot 7}{2 \cdot 4 \cdot 6 \cdot 8}\frac{S^9}{9} + \text{\&c.}$$
$$\text{Or } a = S + \frac{S^3}{6} + \frac{3S^5}{40} + \frac{5S^7}{112} + \frac{35S^9}{1152} + \text{\&c}$$

This series, under its second form, is easily derived, by reverting the series for the sine, in terms of the arch.

This is a very simple and elegant formula, by means of which, an arch may be derived from its tangent, when the latter is less than unity.

9. We proceed now to shew, in what manner these formulæ are to be applied to the calculation of the trigonometrical tables.

It will be necessary first to determine the ratio of the circumference of a circle to the diameter, that is, to find the length of the circumference when the diameter is supposed equal to unity. For this purpose, we shall employ the above series, which expresses the arch in terms of the tangent. If, in this series, we put Tan. $x = 1$, we find the length of the arch of 45^0, or of one-eighth part of the circumference, equal to $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} -$ &c. The rate of convergency is here, however, by much too small to admit of the series being applied to any practical purpose.* But, by the following artifice, we shall be able to make it converge with considerable rapidity.

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* In the paper by Professor Wallace, in Vol VI of the Edin. Phil. Trans referred to in a preceding note, the following formulas for the rectification of an arch of a circle are investigated, which have the property of being equally applicable to every possible case of the problem They are farther remarkable on this account, that the terms of each approach continually to those of a geometrical series

Putting x for an arch of a circle, the first series investigated is,

$$\frac{1}{x} = \frac{1}{\text{Tan } x} + \frac{1}{2}\text{Tan }\frac{1}{2}x + \frac{1}{4}\text{Tan. }\frac{1}{4}x + \frac{1}{8}\text{Tan }\frac{1}{8}x + \frac{1}{16}\text{Tan. }\frac{1}{16}x + \&c.$$

In this expression for the reciprocal of an arch of a circle, the terms approach continually to those of a geometrical series, whose common ratio is $\frac{1}{4}$; so that the sum of all the terms following any assigned term approaches nearer to $\frac{1}{3}$ of that term, according as it is more advanced in the series. The expressions, Tan $\frac{1}{2}x$, Tan. $\frac{1}{4}x$, Tan $\frac{1}{8}x$, &c are easily deduced from Tan. x, and from one another by the formula,

$$\text{Tan }\frac{1}{2}\text{A} = \sqrt{\frac{1}{\text{Tan}^2\text{ A}} + 1} - \frac{1}{\text{Tan. A}}$$

The second formula is as follows.

$$\frac{1}{x^2} = \frac{1}{4}\,\frac{1 + \text{Cos. }x}{1 - \text{Cos. }x} + \frac{1}{6} - \left(\frac{1}{4^2}\,\frac{1 - \text{Cos }\frac{1}{2}x}{1 + \text{Cos. }\frac{1}{2}x} + \frac{1}{4^3}\,\frac{1 - \text{Cos. }\frac{1}{4}x}{1 + \text{Cos. }\frac{1}{4}x} + \frac{1}{4^4}\,\frac{1 - \text{Cos }\frac{1}{8}x}{1 + \text{Cos. }\frac{1}{8}x} + \&c.\right).$$

In this expression, the terms approach continually to to those of a geometrical

By Cor. Prop. III. § 3., and Cor. Prop. IV. of the same section, we have,

$$\text{Sin. }(A + B) = \text{Sin. } A \times \text{Cos. } B + \text{Cos. } A \times \text{Sin. } B.$$
$$\text{Cos. }(A + B) = \text{Cos. } A \times \text{Cos. } B - \text{Sin. } A \times \text{Sin. } B.$$

Hence, by dividing the former of these equations by the latter, we obtain,

$$\frac{\text{Sin. }(A + B)}{\text{Cos. }(A + B)} = \frac{\text{Sin. } A \times \text{Cos. } B + \text{Cos. } A \times \text{Sin. } B}{\text{Cos. } A \times \text{Cos. } B - \text{Sin. } A \times \text{Sin. } B}.$$

Or by dividing the numerator and denominator of the second member of this equation, by Cos $A \times$ Cos. B, and substituting the tangent instead of the sine divided by the cosine, it becomes,

$$\text{Tan. }(A + B) = \frac{\text{Tan. } A + \text{Tan } B}{1 - \text{Tan. } A \text{ Tan. } B}.$$

In like manner, from the formulæ for Sin. $(A - B)$ and Cos. $(A - B)$, we obtain,

$$\text{Tan. }(A - B) = \frac{\text{Tan. } A - \text{Tan. } B}{1 + \text{Tan } A \text{ Tan } B}.$$

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series, whose common ratio is $\frac{1}{16}$, so that the sum of all the terms which follow any assigned term, approaches the nearer to $\frac{1}{15}$ of that term, according as it is more advanced in the series. The expressions Cos $\frac{1}{2}x$, Cos. $\frac{1}{4}x$, Cos. $\frac{1}{8}x$, &c are to be found from Cos x and one another by the formula,

$$\text{Cos } \tfrac{1}{2}A = \sqrt{\frac{1 + \text{Cos } A}{2}}$$

Or, let a series of quantities, t, t', t'', t''', &c be found, such that

$$t = \frac{1 - \text{Cos. } x}{1 + \text{Cos. } x},\ t' = \frac{\sqrt{1+t} - 1}{\sqrt{1+t} + 1},\ t'' = \frac{\sqrt{1+t'} - 1}{\sqrt{1+t'} + 1},$$

$$t''' = \frac{\sqrt{1+t''} - 1}{\sqrt{1+t''} + 1}, \text{ \&c.}$$

Then will the above formula become

$$\frac{1}{x^2} = \frac{1}{4}\,\frac{1 + \text{Cos } x}{1 - \text{Cos. } x} + \frac{1}{6} - \left(\frac{1}{4^2}t' + \frac{1}{4^3}t'' + \frac{1}{4^4}t''' + \frac{1}{4^5}t'''' + \text{\&c.}\right)$$

If, in the above formula, for Tan. (A + B), we suppose A = B, it becomes,

$$\text{Tan. } 2A = \frac{2 \text{ Tan. } A}{1 - \text{Tan}^2 A}.$$

Now, in order to make the series for the arch in terms of the tangent, converge with sufficient rapidity when applied to the determination of the ratio of the circumference to the diameter, we

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Formulas expressing the third and fourth powers of the reciprocal of an arch are also investigated. But we will conclude this note by showing the application of the first formula to the computation of the length of the arch of 90°.

$$\frac{1}{\text{Tan } x} = \text{Cot } x = 0$$

$\text{Tan } \frac{1}{2} x = 1$	$\frac{1}{2} \text{ Tan } \frac{1}{2} x = 0.5000000000000$
$\text{Tan } \frac{1}{4} x = 0.4142135623731$	$\frac{1}{4} \text{ Tan } \frac{1}{4} x = 0.1035533905933$
$\text{Tan } \frac{1}{8} x = 0.1989123673796$	$\frac{1}{8} \text{ Tan } \frac{1}{8} x = 0.0248640450225$
$\text{Tan } \frac{1}{16} x = 0.0984914033571$	$\frac{1}{16} \text{ Tan } \frac{1}{16} x = 0.0061557127098$
$\text{Tan } \frac{1}{32} x = 0.0491268497694$	$\frac{1}{32} \text{ Tan } \frac{1}{32} x = 0.0015352140553$
$\text{Tan } \frac{1}{64} x = 0.0245486221089$	$\frac{1}{64} \text{ Tan } \frac{1}{64} x = 0.0003835792205$
$\text{Tan } \frac{1}{128} x = 0.012272462379$	$\frac{1}{128} \text{ Tan. } \frac{1}{128} x = 0.0000958786123$
$\text{Tan } \frac{1}{256} x = 0.006136000157$	$\frac{1}{256} \text{ Tan. } \frac{1}{256} x = 0.0000239687506$
$\text{Tan } \frac{1}{512} x = 0.003067971201$	$\frac{1}{512} \text{ Tan } \frac{1}{512} x = 0.0000059921319$
$\text{Tan } \frac{1}{1024} x = 0.00153398194$	$\frac{1}{1024} \text{ Tan } \frac{1}{1024} x = 0.0000014980293$
$\text{Tan } \frac{1}{2048} x = 0.00076699054$	$\frac{1}{2048} \text{ Tan } \frac{1}{2048} x = 0.0000003745071$

Sum of the remaining terms is nearly $\frac{1}{3}$ of the last term, i. e. nearly $= 0.0000001248357$

$$\frac{1}{x} = 0.6366197723677$$

Arch of 90°, or $x = 1.570796326795$

$\times\ 2$

3.14159265359.

= the circumference of a circle whose diameter is unity.

shall first determine some small arch, such that some multiple of it may be nearly equal to 45°. The tangent of the small arch, by which the multiple-arch thus found differs from 45°, may be obtained by means of the above formula, expressing the tangent of the difference of two arches; and hence, the differential arch itself may be determined.

Let $\frac{1}{5}$ or .2, therefore, be the tangent of some small arch, the tangent of the double arch will be found, by the formula,

$$\text{Tan. } 2A = \frac{2\text{Tan. } A}{1 - \text{Tan}^2 A}, \text{ to be equal to } \frac{.4}{.96} = \frac{.1}{.24} = \frac{1}{2.4}.$$

Again, from this last result, the tangent of the quadruple arch, is found by the same formula to be equal to $\frac{4.8}{4.76} = 1.00840336$. But since this tangent does not differ much from unity, it follows, that the quadruple of the arch, whose tangent is .2, does not differ considerably from 45°. To determine what the arch is, we have, in the formula which we have found for the arch in terms of the tangent, Tan. $x = .2$. Hence,

$T \quad x =$	0.200,000,000,00	$+ \quad T \quad x = +$	0.200,000,000,00
$T^3 \quad x =$	0.008,000,000,00	$- \frac{1}{3} T^3 \quad x = -$	0.002,666,666,67
$T^5 \quad x =$	0.000,320,000,00	$+ \frac{1}{5} T^5 \quad x = +$	0.000,064,000,00
$T^7 \quad x =$	0.000,012,800,00	$- \frac{1}{7} T^7 \quad x = -$	0.000,001,828,57
$T^9 \quad x =$	0.000,000,512,00	$+ \frac{1}{9} T^9 \quad x = +$	0.000,000,056,89
$T^{11} \quad x =$	0.000,000,020,48	$- \frac{1}{11} T^{11} \quad x = -$	0.000,000,001,86
$T^{13} \quad x =$	0.000,000,000,82	$+ \frac{1}{13} T^{13} \quad x = +$	0.000,000,000,06

\+ 0.200,064,056,95
— 0.002,668,497,10

Arch, whose tangent is .2 = 0.197,395,559,85
4

Arch, whose tangent is $\frac{4.8}{4.76} = 0.789,582,239,40$

Next, to find the small arch, by which the arch last found exceeds 45°. By the formula, $\text{Tan. }(A - B) = \frac{\text{Tan. } A - \text{Tan. } B}{1 + \text{Tan. } A \text{ Tan. } B}$, the tangent of the differential arch is found to be $\frac{1}{239}$, or 0.004,184,100,42, from which the arch itself is now to be found.

$$\begin{array}{rl} \text{Tan. } x = & 0.004,184,100,42 \\ -\tfrac{1}{3}\text{ Tan}^3\, x = & -0.000,000,024,42 \\ \hline \text{Differential arch} = & 0.004,184,076,00 \\ & 0.789,582,239,40 \\ \hline \text{The arch of } 45^0 = & 0.785,398,163,40 \\ & 4 \\ \hline & 3.141,592,653,6 \end{array}$$

3.141,592,653,6 = the circumference of the circle, the diameter being supposed equal to unity.

10. Having thus determined the ratio of the circumference to the diameter, we now proceed to shew the method of applying the series of § 5 and § 6, to the determination of the sine, cosine, tangent and cotangent of any arch, which is given in degrees and parts of a degree. For this purpose, it is necessary to have the length of the given arch expressed in parts of the radius, or which amounts to the same thing, it is necessary to have the ratio of this arch to the radius. Now, the radius being supposed equal to unity, the semicircumference, or the arch of 180°, is, as we have found, equal to 3.1415926536, or, carrying the computation to still greater accuracy, it is found equal to 3.14159265358979332. Let this number be represented by π, and let the ratio of m to n, express the ratio of the given arch to a quadrant, then shall the length of the given arch $\frac{m}{n}90^0$, be equal to $\frac{m}{n}.\tfrac{1}{2}\pi$. Hence, if we put in the series for the sine, cosine, tangent and cotangent, instead of π its value, and calculate the coefficients to a convenient number of decimal places, we shall have the following formulæ:

$\text{Sin. }\frac{m}{n}90^0 =$	$\text{Cos. }\frac{m}{n}90^0 =$
$1.570,796,326,794,897\frac{m}{n}$	$1.000,000,000,000,000$
$-0.645,964,097,506,246\frac{m^3}{n^3}$	$-1.233,700,550,136,170\frac{m^2}{n^2}$
$+0.079,692,626,246,167\frac{m^5}{n^5}$	$+0.253,669,507,901,048\frac{m^4}{n^4}$
$-0.004,681,754,135,319\frac{m^7}{n^7}$	$-0.020,863,480,763,353\frac{m^6}{n^6}$
$+0.000,160,441,184,787\frac{m^9}{n^9}$	$+0.000,919,260,274,839\frac{m^8}{n^8}$

$-0.000,003,598,843,235\frac{m^{11}}{n^{11}}$	$-0.000,025,202,042,373\frac{m^{10}}{n^{10}}$
$+0.000,000,056,921,729\frac{m^{13}}{n^{13}}$	$+0.000,000,471,087,478\frac{m^{12}}{n^{12}}$
$-0.000,000,000,668,804\frac{m^{15}}{n^{15}}$	$-0.000,000,006,386,603\frac{m^{14}}{n^{14}}$
$+0.000,000,000,006,067\frac{m^{17}}{n^{17}}$	$+0.000,000,000,065,660\frac{m^{16}}{n^{16}}$
$-0.000,000,000,000,044\frac{m^{19}}{n^{19}}$	$-0.000,000,000,000,529\frac{m^{18}}{n^{18}}$
	$+0.000,000,000,000,003\frac{m^{20}}{n^{20}}$

$\text{Tan.}\ \frac{m}{n}\ 90^\circ =$	$\text{Cotan.}\ \frac{m}{n}\ 90^\circ =$
$0.636,619,772,3675\frac{2mn}{n^2-m^2}$	$0.636,619,772,3675\frac{m}{n}$
$+0.297,556,782,0597\frac{m}{n}$	$-0.318,309,886,1837\frac{4mn}{4n^2-m^2}$
$+0.018,688,650,2773\frac{m^3}{n^3}$	$-0.205,288,889,4145\frac{m}{n}$
$+0.001,842,475,2034\frac{m^5}{n^5}$	$-0.006,551,074,7882\frac{m^3}{n^3}$
$+0.000,197,580,0714\frac{m^7}{n^7}$	$-0.000,345,029,2554\frac{m^5}{n^5}$
$+0.000,021,697,7245\frac{m^9}{n^9}$	$-0.000,020,279,1060\frac{m^7}{n^7}$
$+0.000,002,401,1370\frac{m^{11}}{n^{11}}$	$-0.000,001,236,6527\frac{m^9}{n^9}$
$+0.000,000,266,4132\frac{m^{13}}{n^{13}}$	$-0.000,000,076,4959\frac{m^{11}}{n^{11}}$
$+0.000,000,029,5864\frac{m^{15}}{n^{15}}$	$-0.000,000,004,7597\frac{m^{13}}{n^{13}}$
$+0.000,000,003,2867\frac{m^{17}}{n^{17}}$	$-0.000,000,000,2969\frac{m^{15}}{n^{15}}$
$+0.000,000,000,3651\frac{m^{19}}{n^{19}}$	$-0.000,000,000,0185\frac{m^{17}}{n^{17}}$
$+0.000,000,000,0405\frac{m^{21}}{n^{21}}$	$-0.000,000,000,0011\frac{m^{19}}{n^{19}}$
$+0.000,000,000,0045\frac{m^{23}}{n^{23}}$	
$+0.000,000,000,0005\ m^{25}$	

11. As has already been observed, the sines and cosines of the arches between 0^{0} and 45^{0}, comprehend the sines and cosines of the arches between 45^{0} and 90^{0}. Hence, it is evident, that in the above formulæ for Sin. $\frac{m}{n}90^{0}$, and Cos. $\frac{m}{n}90^{0}$, we may always suppose $\frac{m}{n} < \frac{1}{2}$; so that the series will in every case converge with such rapidity, as to render necessary the calculation of only a few terms, provided there be not required a great number of decimal places in the result.

12. As an example of the method of applying the formulæ above investigated, let it be required to calculate the sine, cosine, tangent, and cotangent, of the arch of 10^{0}.

Here, $\frac{m}{n} = \frac{1}{9}$. Hence, substituting this value of $\frac{m}{n}$, in the series for the sine, cosine, tangent, and cotangent of $\frac{m}{n}90^{0}$ successively, we find,

Sin. 10^{0}	$= 0.174,532,92$	Cos. 10^{0}	$= 1.000,000,00$
	$-0.000,886,10$		$-0.015,230,87$
	$+0.000,001,35$		$+0.000,038,66$
			$-0.000,000,04$
Sin. 10^{0}	$= 0.173,648,2$	Cos. 10^{0}	$= 0.984,807,8$
Tan. 10^{0}	$= 0.143,239,45$	Cot. 10^{0}	$= 5.729,577,94$
	$+0.033,061,86$		$-0.035,477,26$
	$+0.000,025,64$		$-0.022,809,87$
	$+0.000,000,03$		$-0.000,008,99$
Tan. 10^{0}	$= 0.176,327,0$	Cot. 10^{0}	$= 5.671,281,8$

13. After the same manner, may the sine, cosine, &c. of any other arch be determined. The facility with which the above results are obtained, is a proof of the excellence of this method. In the actual calculation of the tables, the above series are, however, to be employed in combination with the geometrical relations of the sines, cosines, &c. which have been laid down in § 3. If it be required to calculate the sine and cosine of every arch consisting of an exact number of minutes, from 1 minute to 5400 minutes, or 90^{0}, the first thing to be done, is to determine the sine and cosine of 1 minute, to a great degree of accuracy. Suppose, for ex-

ample, that the table of sines and cosines is to exhibit the sine and cosine true to the tenth decimal place, it will be requisite to find the values of the sine and cosine of 1 minute, true to the fifteenth decimal place. For, in constructing the table, we must calculate the sines and cosines to several more decimal places than we intend to retain, in order to be assured, that the errors which may accumulate in the course of 2700 operations, shall not affect the tenth decimal place of the last results. The calculation of the sine and cosine of 1′, to the requisite degree of accuracy, is easily accomplished by means of the formulæ for Sin. $\frac{m}{n}90^0$ and Cos. $\frac{m}{n}90^0$, if we suppose $\frac{m}{n}=\frac{1}{5400}$. In this manner, by taking the first two terms, we obtain,

$$\text{Sin. } 1' = 0.000,290,888,204,564$$
$$\text{Cos. } 1' = 0.999,999,957,692,025$$

The sine and cosine of 1 minute being determined, the sines and cosines of the multiple arches of 1′ are to be successively derived from each other by the formulæ already deduced, Prop. V. § 3, namely,

Sin. 2′=2Cos. 1′ Sin. 1′ — Sin. 0′	Cos. 2′=2Cos. 1′ Cos. 1′ — 1
Sin. 3′=2Cos. 1′ Sin. 2′ — Sin. 1′	Cos. 3′=2Cos. 1′ Cos. 2′ — Cos. 1′
Sin. 4′=2Cos. 1′ Sin. 3′ — Sin. 2′	Cos. 4′=2Cos. 1′ Cos. 3′ — Cos. 2′
Sin. 5′=2Cos. 1′ Sin. 4′ — Sin. 3′	Cos. 5′=2Cos. 1′ Cos. 4′ — Cos. 3′
&c. &c. &c.	&c. &c. &c.

14. In the above expressions for the sines and cosines of the multiple arches, Cos. 1′ occurs as a constant multiplier. But, we may remark, that this quantity being nearly equal to unity, there is a method of abbreviation, which it may be proper to point out.

Let $k = 2$ (1 — Cos. 1′) = 0.000,000,084,615,950, we shall have 2 Cos. 1′ = 2 — k. Hence, if in the formulæ

Sin. (A + B) = 2Cos. B. Sin. A — Sin. (A—B), (Cor. Prop. III.)
Cos. (A + B) = 2Cos. B. Cos. A — Cos. (A — B), (Cor. Prop. IV.)

we suppose B = 1′, and for 2Cos. B, insert 2 — k, we shall obtain,

Sin. (A + 1′) — Sin. A = Sin. A — Sin. (A — 1′) — k Sin. A
Cos. (A + 1′) — Cos. A = Cos. A — Cos. (A — 1′) — k Cos. A

Putting, therefore, A equal 1′, 2′, 3′, 4′, &c. successively, we obtain,

$$\begin{array}{l}
\text{Sin. } 2' - \text{Sin. } 1' = \text{Sin. } 1' - k \text{ Sin. } 1' \\
\text{Sin. } 3' - \text{Sin. } 2' = \text{Sin. } 2' - \text{Sin. } 1' - k \text{ Sin. } 2' \\
\text{Sin. } 4' - \text{Sin. } 3' = \text{Sin. } 3' - \text{Sin. } 2' - k \text{ Sin. } 3' \\
\text{Sin. } 5' - \text{Sin. } 4' = \text{Sin. } 4' - \text{Sin. } 3' - k \text{ Sin. } 4' \\
\quad \&c. \qquad \&c. \qquad \&c. \qquad \&c.
\end{array}$$

And

$$\begin{array}{l}
\text{Cos. } 2' - \text{Cos. } 1' = \text{Cos. } 1' - k \text{ Cos. } 1' \\
\text{Cos. } 3' - \text{Cos. } 2' = \text{Cos. } 2' - \text{Cos. } 1' - k \text{ Cos. } 2' \\
\text{Cos. } 4' - \text{Cos. } 3' = \text{Cos. } 3' - \text{Cos. } 2' - k \text{ Cos. } 3' \\
\text{Cos. } 5' - \text{Cos. } 4' = \text{Cos. } 4' - \text{Cos. } 3' - k \text{ Cos. } 4' \\
\quad \&c. \qquad \&c. \qquad \&c. \qquad \&c.
\end{array}$$

From these expressions, it appears, that in order to find the difference between the sine of 1′ and the sine of 2′, we have only to multiply the sine of 1′ by the small fraction k, and to subtract the product from the sine of 1′. Again, to find the difference of the sine of 3′ and the sine of 2′, it is only necessary to multiply the sine of 2′ by k, and subtract the product from the difference of the sines of 2′ and 1′. In like manner, to find the difference of the sines of 3′ and 4′, it is only necessary to multiply the sine of 3′ by k, and subtract the product from the difference of Sin. 2′ and Sin. 3′, and so on. These differences being calculated, it is evident, that Sin. 2′, Sin. 3′, Sin. 4′, may be successively deduced with great facility. But, in calculating the differences, the one from the other, the only operation, somewhat tedious, which is to be performed, is the multiplication by the fraction k. Now, it is to be observed, *first*, That it is only necessary that we find the product true to the fifteenth decimal place, which will require but a short process of calculation, and, *secondly*, That these multiplications may be much abridged by finding previously the product of the constant number 84615950, by each of the nine digits; for, by this means, we shall have immediately the partial products which result from the different figures of the multiplier, and it will only remain to add together these products, observing to extend each to the fifteenth decimal place.

The same method is to be pursued in the calculation of the cosines, and, when we have calculated as far as the sine and cosine of 45°, the table will be completed.

15. The sines, such as they result from the calculations now pointed out, are expressed in parts of the radius, and are called *Natural Sines:* but it has been found in practice, that much ad-

vantage is derived from employing the logarithms of the sines, instead of the sines themselves. The same may be said of the tangents and secants. Hence, in the trigonometrical tables, besides the natural sines, tangents and secants, their logarithms are likewise inserted; or frequently the former are altogether omitted. The natural sines being calculated, their logarithms are easily obtained from the logarithmic tables. But, as the supposition of radius being equal to unity, would render negative all the logarithms of the sines, it is usual to assume radius equal to 10,000,000,000; which amounts to the same thing as to multiply by 10,000,000,000 all the sines found upon the supposition of radius being equal to 1. Upon this supposition, the radius or Sin. 90°, which frequently occurs in calculation, has for its logarithm 10 units, and those angles which have the logarithms of their sines negative, must necessarily be much smaller than any which are met with in practice.

The logarithms of the sines being found, we can easily deduce from them the logarithms of the tangents and secants; for since

$$\text{Tan. A} = \frac{\text{R} \times \text{Sin. A}}{\text{Cos. A}}, \text{ and Sec. A} = \frac{\text{R}^2}{\text{Cos. A}},$$

it follows, that

$$\text{Log. Tan. A} = 10 + \text{Log. Sin. A} - \text{Log. Cos. A},$$

and $$\text{Log. Sec. A} = 20 - \text{Log. Cos. A}.$$

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16. From Prop. III. and IV. a great number of trigonometrical formulæ may be deduced. We shall here deduce those which are of most frequent use.

Let A and B be any two arches of a circle whose radius is denoted by R, and let A be the greater arch: Then, from Prop. III. and IV. with their corollaries, we have,

I. $$\text{Sin. A} + \text{Sin. B} = \frac{2}{\text{R}} \text{Sin.} \tfrac{1}{2}(\text{A} + \text{B}) \text{Cos.} \tfrac{1}{2}(\text{A} - \text{B}).$$

II. $$\text{Sin. A} - \text{Sin. B} = \frac{2}{\text{R}} \text{Sin.} \tfrac{1}{2}(\text{A} - \text{B}) \text{Cos.} \tfrac{1}{2}(\text{A} + \text{B}).$$

III. $$\text{Cos. A} + \text{Cos. B} = \frac{2}{\text{R}} \text{Cos.} \tfrac{1}{2}(\text{A} + \text{B}) \text{Cos.} \tfrac{1}{2}(\text{A} - \text{B}).$$

IV. $$\text{Cos. B} - \text{Cos. A} = \frac{2}{\text{R}} \text{Sin.} \tfrac{1}{2}(\text{A} + \text{B}) \text{Sin.} \tfrac{1}{2}(\text{A} - \text{B}).$$

V. $\text{Sin. A Cos. B} = \frac{1}{2}\text{ R Sin. }(A + B) + \frac{1}{2}\text{ R Sin. }(A - B).$

VI. $\text{Sin. B Cos. A} = \frac{1}{2}\text{ R Sin. }(A + B) - \frac{1}{2}\text{ R Sin. }(A - B).$

VII. $\text{Cos. A Cos. B} = \frac{1}{2}\text{ R Cos. }(A - B) + \frac{1}{2}\text{ R Cos. }(A + B).$

VIII. $\text{Sin. A Sin. B} = \frac{1}{2}\text{ R Cos. }(A - B) - \frac{1}{2}\text{ R Cos. }(A + B).$

IX. $$\text{Sin. }(A + B) = \frac{\text{Sin. A Cos. B} + \text{Cos. A Sin. B}}{R}.$$

X. $$\text{Sin. }(A - B) = \frac{\text{Sin. A Cos. B} - \text{Cos. A Sin. B}}{R}.$$

XI. $$\text{Cos. }(A + B) = \frac{\text{Cos. A Cos. B} - \text{Sin. A Sin. B}}{R}.$$

XII. $$\text{Cos. }(A - B) = \frac{\text{Cos. A Cos. B} + \text{Sin. A Sin. B}}{R}.$$

If, in formulas IX. and XI. we suppose the arches A and B equal, we obtain,

XIII. $$\text{Sin. } 2A = \frac{2\text{ Sin. A Cos. A}}{R}.$$

XIV. $$\text{Cos. } 2A = \frac{\text{Cos}^2\text{ A} - \text{Sin}^2\text{ A}}{R}.$$

If, in this last formula, the expression $R^2 - \text{Cos}^2\text{ A}$ be put instead of its equal $\text{Sin}^2\text{ A}$, we find

XV. $$\text{Cos. } 2A = \frac{2\text{ Cos}^2\text{ A} - R^2}{R},$$

and if, in the same formula, we put $R^2 - \text{Sin}^2\text{ A}$ instead of $\text{Cos}^2\text{ A}$, it becomes

XVI. $$\text{Cos. } 2A = \frac{R^2 - 2\text{ Sin}^2\text{ A}}{R}.$$

From these two last, by multiplying by R, then transposing, and extracting the square root, we obtain

XVII. $$\text{Cos. A} = \sqrt{\frac{R^2 + R\text{ Cos. } 2A}{2}},$$

$$\text{or Cos. } \tfrac{1}{2}\text{ A} = \sqrt{\frac{R^2 + R\text{ Cos. A}}{2}}.$$

XVIII. $\text{Sin. } A = \sqrt{\frac{R^2 - R \text{ Cos. } 2A}{2}}$,

or $\text{Sin. } \frac{1}{2} A = \sqrt{\frac{R^2 - R \text{ Cos. } A}{2}}$.

Dividing by each other formulas, I, II, III, IV, and observing that $\frac{\text{Sin. } A}{\text{Cos. } A} = \frac{\text{Tan. } A}{R} = \frac{R}{\text{Cot. } A}$, we find

XIX. $\frac{\text{Sin. } A + \text{Sin. } B}{\text{Sin. } A - \text{Sin. } B} = \frac{\text{Sin. } \frac{1}{2}(A+B) \text{ Cos. } \frac{1}{2}(A-B)}{\text{Cos. } \frac{1}{2}(A+B) \text{ Sin. } \frac{1}{2}(A-B)} = \frac{\text{Tan. } \frac{1}{2}(A+B)}{\text{Tan. } \frac{1}{2}(A-B)}$.

XX. $\frac{\text{Sin. } A + \text{Sin. } B}{\text{Cos. } A + \text{Cos. } B} = \frac{\text{Sin. } \frac{1}{2}(A+B)}{\text{Cos. } \frac{1}{2}(A+B)} = \frac{\text{Tan. } (A+B)}{R}$.

XXI. $\frac{\text{Sin. } A + \text{Sin. } B}{\text{Cos. } B - \text{Cos. } A} = \frac{\text{Cos. } \frac{1}{2}(A-B)}{\text{Sin. } \frac{1}{2}(A-B)} = \frac{\text{Cot. } \frac{1}{2}(A-B)}{R}$.

XXII. $\frac{\text{Sin. } A - \text{Sin. } B}{\text{Cos. } A + \text{Cos. } B} = \frac{\text{Sin. } \frac{1}{2}(A-B)}{\text{Cos. } \frac{1}{2}(A-B)} = \frac{\text{Tan. } \frac{1}{2}(A-B)}{R}$.

XXIII. $\frac{\text{Sin. } A - \text{Sin. } B}{\text{Cos. } B - \text{Cos. } A} = \frac{\text{Cos. } \frac{1}{2}(A+B)}{\text{Sin. } \frac{1}{2}(A+B)} = \frac{\text{Cot. } \frac{1}{2}(A+B)}{R}$.

XXIV. $\frac{\text{Cos. } A + \text{Cos. } B}{\text{Cos. } B - \text{Cos. } A} = \frac{\text{Cos. } \frac{1}{2}(A+B) \text{ Cos. } \frac{1}{2}(A-B)}{\text{Sin. } \frac{1}{2}(A+B) \text{ Sin. } \frac{1}{2}(A-B)} = \frac{\text{Cot. } \frac{1}{2}(A+B)}{\text{Tan. } \frac{1}{2}(A-B)}$.

If we divide formula IX. by formula XI., and observe that $\frac{\text{Sin. } (A+B)}{\text{Cos. } (A+B)} = \frac{\text{Tan. } (A+B)}{R}$, we have

$$\frac{\text{Tan. } (A+B)}{R} = \frac{\text{Sin. } A \text{ Cos. } B + \text{Cos. } A \text{ Sin. } B}{\text{Cos. } A \text{ Cos. } B - \text{Sin. } A \text{ Sin. } B};$$

or, multiplying by R, both sides of the equation; then dividing numerator and denominator of the second member by Cos. A Cos. B, and substituting $\frac{\text{Tan. } A}{R}$ and $\frac{\text{Tan. } B}{R}$ instead of $\frac{\text{Sin. } A}{\text{Cos. } A}$ and $\frac{\text{Sin. } B}{\text{Cos. } B}$; and, lastly, multiplying numerator and denominator of the result by R^2, we find

XXV. $\text{Tan.}\,(A+B) = \dfrac{R^2\,(\text{Tan. }A + \text{Tan. }B)}{R^2 - \text{Tan. }A\ \text{Tan. }B}$.

By proceeding in the same manner with formulas X. and XII., we obtain

XXVI. $\text{Tan.}\,(A-B) = \dfrac{R^2\,(\text{Tan. }A - \text{Tan. }B)}{R^2 + \text{Tan. }A\ \text{Tan. }B}$.

If, in formula XXV., we suppose $A = B$ and $2A = B$ successively, we get

XXVII. $\text{Tan. }2A = \dfrac{2R^2\ \text{Tan. }A}{R^2 - \text{Tan}^2\ A}$.

XXVIII. $\text{Tan. }3A = \dfrac{R\,(\text{Tan. }A + \text{Tan. }2A)}{R^2 - \text{Tan. }A\ \text{Tan. }2A} = \dfrac{3R^2\ \text{Tan. }A - \text{Tan}^3\ A}{R^2 - 3\text{Tan}^2\ A}$.

If we multiply both sides of formula IX. by R^2, divide by Cos. A Cos. B; and substitute in the result Tan. A for $\dfrac{R\ \text{Sin. }A}{\text{Cos. }A}$, and Tan. B for $\dfrac{R\ \text{Sin. }B}{\text{Cos. }B}$, we find

XXIX. $\text{Tan. }A + \text{Tan. }B = \dfrac{R^2\ \text{Sin.}\,(A+B)}{\text{Cos. }A\ \text{Cos. }B}$.

In like manner we obtain

XXX. $\text{Tan. }A - \text{Tan. }B = \dfrac{R^2\ \text{Sin.}\,(A-B)}{\text{Cos. }A\ \text{Cos. }B}$.

XXXI. $\text{Cot. }B + \text{Cot. }A = \dfrac{R^2\ \text{Sin.}\,(A+B)}{\text{Sin. }A\ \text{Sin. }B}$.

XXXII. $\text{Cot. }B - \text{Cot. }A = \dfrac{R^2\ \text{Sin.}\,(A-B)}{\text{Sin. }A\ \text{Sin. }B}$.

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17. Having explained the nature and calculation of sines,

tangents, &c. we now proceed to explain the arrangement and use of the trigonometrical tables.

EXPLANATION OF THE TABLES

OF

LOGARITHMIC SINES, TANGENTS, AND SECANTS.

1. *To find the Logarithmic Sine, Tangent, or Secant of any Number of Degrees and Minutes.*

If the number of degrees be less than 45°, seek them at the top of the page, then in a line with the given number of minutes in the left-hand marginal column, under the word sine, tangent, or secant, you have the logarithmic sine, tangent, or secant of the proposed number of degrees and minutes.

If the number of degrees be above 45° and less than 90°, seek them at the bottom of the page, then against the minutes in the right-hand marginal column, and above the word sine, tangent, or secant, respectively, you have the logarithm sought.

When the degrees exceed 90°, take the supplement of the arch, that is, subtract the given degrees and minutes from 180°, and look out the sine, tangent, or secant of the remainder as above.

EXAMPLES.

	Arches.		*Sines.*	*Tangents.*	*Secants.*
	18°	15′	9.495772	9.518185	10.022414
	33	45	9.744739	9.824893	10.080154
	47	9	9.865185	10.032624	10.167439
	64	56	9.957040	10.330009	10.372970
	135	30	9.845662	9.992420	10.146758
Supplt.	44	30			

Note.—The logarithmic cosine, cotangent, or cosecant of any number of degrees and minutes, may be found in the same manner as above, in the columns marked cosine, cotangent, cosecant, respectively.—Thus, the cosine of 42° 51′ is 9.865185, which is also the sine of its complement 47° 9′. The cotangent of 71° 45′ is 9.518185 = to the tangent of 18° 15′, and the cosecant of 56° 15′ is 10.080154 = to the secant of 33° 45′.

2. *To find the Sine, Tangent, or Secant of any Number of Degrees, Minutes and Seconds.*

Find the sine, tangent, or secant corresponding to the given number of degrees and minutes, as before, and also the tabular difference from the column marked D, multiply this difference by the given number of seconds, cut off two decimal places from the right of the product, and the remaining figures are the part to be added for the seconds.

EXAMPLES.

Required the logarithmic sine of 19° 24′ 36″?

Sine 19° 24′ =	9.521349	Diff.	598
	215		36
Sine 19° 24′ 36″ =	9.521564		3588
			1794
			215|28

Arches.	*Sines.*	*Tangents.*	*Secants.*
44° 25′ 37″	9.845098	9.991312	10.146214
57° 7′ 52″	9.924237	10.189664	10.265427
26° 12′ 43″	9.645120	9.692347	10.047127
39° 42′ 50″	9.805469	9.919405	10.113935
71° 56′ 19″	9.978055	10.486643	10.508588

Note.—The cosine, cotangent, and cosecant of any number of degrees, minutes, and seconds, are to be found in the same manner; except that the proportional part for the seconds is to be subtracted.

EXAMPLES.

Arches.	*Cosines.*	*Cotangents.*	*Cosecants.*
18° 17′ 24″	9.977486	10.480797	10.503310
32° 5′ 35″	9.927979	10.202642	10.274663
58° 49′ 56″	9.713949	9.781650	10.067701
83° 12′ 15″	9.073102	9.076164	10.003062
21° 46′ 52″	9 967833	10.398387	10.430554

3. *To find the number of Degrees and Minutes corresponding to any given Logarithmic Sine, Tangent, or Secant; Cosine, Cotangent, or Cosecant.*

Seek, in its respective column, the sine, tangent, &c. nearest to that given; if you find the given sine, &c., or the next less, in a column titled at the top, you have the number of degrees at the top of the page, and number of minutes in the left hand marginal column. But if you find the sine, &c., or the next less, in a column titled at the bottom, you have the number of degrees at the bottom of the page, and the minutes in the right-hand marginal column.

EXAMPLES.

Sines.	*Arches.*	*Tangents.*	*Arches.*
9.724632	32° 2′	9.876454	36° 57
9.953298	63 54	10.109765	52 9

Secants.	*Arches.*
10.043624	25° 15′
10.423631	67 51

4. *To find the Seconds corresponding to the remainder of any Logarithmic Sine, Tangent, &c. after the Degrees and Minutes have been found.*

If the given sine, tangent, &c. be found exactly in the table, there will be no remainder for seconds. But, if there be a remainder, the corresponding seconds are to be found by annexing

two cyphers to the difference between the tabular sine, tangent or secant next less, and the given one, and dividing by the tabular difference; the quotient is the number of seconds to be added to the degrees and minutes.

The seconds corresponding to the remainder of any logarithmic cosine, cotangent, or cosecant, are to be found in the same manner: but the seconds must be subtracted from the degrees and minutes in order to find the true arch.

EXAMPLES.

1. To find the degrees, minutes, and seconds corresponding to the logarithmic sine 9.880054.

$$\left.\begin{array}{r}\text{Sine in the table, next}\\ \text{less than given sine,}\end{array}\right\} 9.879963 = \text{Sin. } 49^\circ\ 20'$$

$$\text{Given sine} = 9.880054$$

$$\text{Tab. Diff.} = 181)9100(50''$$

Hence, to the given sine, 9.880054, corresponds the arch $49^\circ\ 20'\ 50''$.

2. To find the number of degrees, minutes, and seconds corresponding to the logarithmic cotangent 10.008688.

$$\left.\begin{array}{r}\text{Cotangent in the table, next less}\\ \text{than the given cotangent,}\end{array}\right\} 10.008591 = \text{Cot. } 44^\circ\ 26'$$

$$\text{Given cotangent} = 10.008688$$

$$\text{Tab. Diff.} = 421)9700(23''$$

Hence $44^\circ\ 26' - 23'' = 44^\circ\ 25'\ 37''$ is the arch to which the given cotangent 10.008688 corresponds.

3. Required the degrees, minutes, and seconds answering to the logarithmic tangent 10.199471?—Ans. $57^\circ\ 43'\ 6''$.

4. Required the degrees, minutes, and seconds corresponding to the cosine 9.924237?—Ans. $32^\circ\ 52'\ 8''$.

Note.—The same rules are applicable to obtuse angles if we take the supplement of the angle instead of the angle itself. For the sine, tangent, and secant of an arch, is likewise the sine, tangent and secant of the supplement of that arch.

5. *To find the Natural Sine, Tangent, or Secant of any Angle, or number of Degrees and Minutes.*

Find the logarithmic sine, tangent, or secant, by the table; cancel its index, and find the nearest logarithm to it, in the table of logarithms of numbers, the natural number corresponding to this logarithm, is the natural sine, tangent, or secant respectively. If the index of the logarithmic sine, tangent, &c. be under 10, prefix to the natural sine, &c. as many cyphers as make the complement of its index to 9. But, if the index be 10, 11, 12, or 13, then one, two, three, or four figures respectively, are to be pointed off for integers; the rest are decimals.

The natural sine agreeing to any number of degrees and minutes, may also be found more readily, at once from the table of natural sines, the arrangement and use of which, are sufficiently obvious, from the explanation already given of the table of logarithmic sines; and the natural sine and cosine being known, the natural tangent, &c. are easily calculated (§ 3. Prop. VI).

EXAMPLES.

Arches.	*N. Sines.*	*N. Tangents.*	*N. Secants.*
23° 20′	.396080	.431358	1.089068
— 30′	.008727	.008727	1.000038
87° 15′	.998848	20.81883	20.84283
89° 30′	.999962	114.5887	114.5930

6. *To find the Logarithmic and Natural Versed-sine of any Angle.*

To twice the logarithmic sine of half the given angle, add the constant logarithm 0.301030, and from the index of the sum, subtract the logarithm of the radius, or 10; then shall the result be the logarithmic versed-sine of the given angle. Seek this logarithm in the table of the logarithms of numbers, the natural number corresponding to it will be the natural versed-sine, the position of the decimal point being ascertained precisely in the same manner as in the natural sine.

Or, the natural versed-sine may be found by subtracting the natural cosine of the angle from the radius, (unity,) if the angle be less than 90°, or by adding, if greater.

Ex To find the logarithmic, and natural versed-sine of 76° 48′.

$$\begin{array}{rr} \text{Log. Sin. } \tfrac{1}{2}\,(76^\circ\ 48') = \text{Log. Sin. } 38^\circ\ 24' = & 9.793195 \\ & 2 \\ \hline & 19.586390 \\ \text{Log. } 2 = & 0.301030 \\ \hline & 19.887420 \\ & 10.000000 \\ \hline \text{Logarithmic versed-sine } 76^\circ\ 48' = & 9.887420 \end{array}$$

Now, corresponding to the decimal part of the logarithmic versed-sine, we obtain the natural versed-sine of 76° 48′ = 77165.

PLANE TRIGONOMETRY.

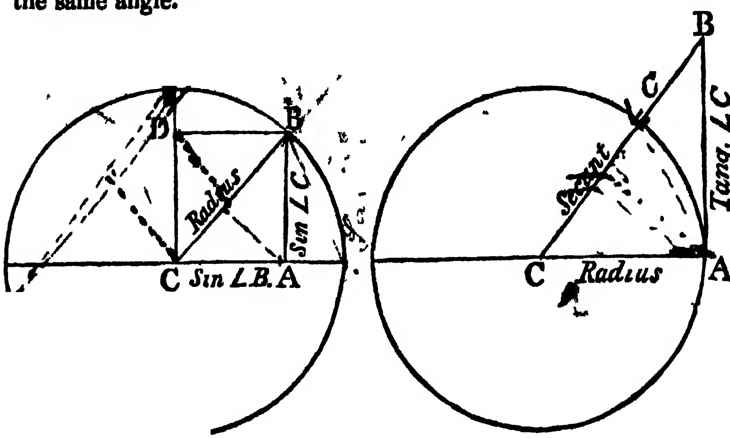
I. Plane Trigonometry is the method of determining the measures of the unknown parts of plane triangles, from certain parts being given. For the conveniency of calculation, it is usual to divide the general problem which trigonometry proposes to resolve, into two, according as the triangle has or has not a right angle.

Solution of Right-Angled Plane Triangles.

2. A right-angled triangle consists of five parts, namely, the three sides and two acute angles; the right angle, being a constant quantity, is not reckoned. Of these, any two being given, and one of these two being a side, the other parts of the triangle may be found.

3. In right-angled triangles, the side opposite to, or subtending the right-angle, is called the *Hypotenuse*: the other sides, which contain the right-angle, are sometimes called *Legs*: Or, the one is denominated the *Base*, the other, the *Perpendicular*.

4. If the hypotenuse be assumed equal to the radius, then will the sides be the sines of the angles opposite to them; and if either side be considered as radius, the other side will be the tangent of the angle opposite to it; and the hypotenuse will be the secant of the same angle.



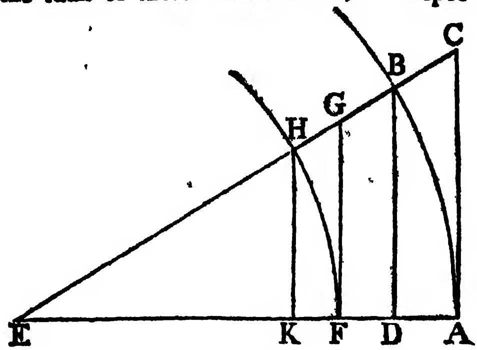
This appears sufficiently evident, by comparing the figures with the definitions already given of the sine, tangent, and secant.

In the calculation of right-angled plane triangles, any side, whether given or required, may be made radius to find a side, but a given side must always be made radius to find an angle.

THEOREM.

The sine, versed-sine, tangent, and secant of an arch, which is the measure of any given angle, is to the sine, versed-sine, tangent, and secant of any other arch, which is the measure of the same angle as the radius of the first arch is to the radius of the second.

Let AB and HF be two arches, which measure the same angle AEB, and let the radii of these arches be EA, EF respectively. Let BD be the sine, AC the tangent, and EC the secant of the arch AB; also let HK be the sine, FG the tangent, and EG the secant of the arch FH. Since AC, BD, FG, and HK are parallel, being each perpendicular to



EA, we have BD : HK, or Sin. AB : Sin. FH : : Rad. EB : Rad. EH: Again, we have ED : : EB or EA : : EK : EH or EF, therefore, by division, DA . EA : : KF : EF, and alternately, DA : KF : : EA : EF, that is, Ver. Sin. AB : Ver. Sin. FH : : Rad. EA : Rad. EF. Farther, we have AC : FG, or Tan. AB : Tan. FH : : Rad. EA : Rad. EF; and EC : EH, or Sec. AB : Sec. FH : : Rad. EA : Rad. EF.

5. From this theorem, it appears, that as the trigonometrical tables exhibit in numbers the sines, tangents, secants, &c. of certain angles to a given radius, they also exhibit the ratio of the sines, tangents, &c. of the same angles to any radius whatever. Upon this principle the solution of the different cases in right-angled plane triangles depends; and from the theorem, we may deduce the following general rules.

RULES.

Write the word *radius* upon one side of the triangle, and mark the names on the other sides accordingly (§ 4.); then,

To find a Side.

As the term or name on the given side
Is to that on the required side,
So is the given side .
To the required side.

And to find an Angle.

As the side made radius
Is to the other given side,
So is radius
To the term or name upon that side.

Note.—From this property of a plane triangle, that the three angles are together equal to two right-angles, or 180°, the following very useful corollaries arise.

1st, When two angles of a triangle are given, the third is also given; for it is the supplement of the sum of the other two, and may be found by subtracting their sum from 180°.

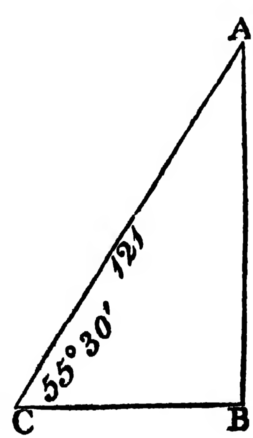
2d, When one angle of a triangle is given, the sum of the other two may be found, by subtracting the given angle from two right-angles, or 180°.

3d, If one angle of a triangle be right, the other two are acute, and together make another right-angle; and, if one of the acute angles be given, the other is also given, being the complement of the other given one, or what it wants of 90°.

PROBLEM I.

Given the angles and hypotenuse of a right-angled plane triangle, to find the base and perpendicular.

Ex. 1. In the triangle ABC, right-angled at B, suppose the angle C 55° 30′, and the hypotenuse AC 121 yards, required the sides AB and BC?



Geometrically.

Draw the indefinite line BC; at the point C, with the chord of 60°, describe an arch, and upon it lay off the quantity of the angle C, 55° 30′; then measure the hypotenuse, 121 equal parts, from C to A, and from A let fall a perpendicular upon CB; ABC is the triangle proposed. Measure the sides AB and BC on the scale from which AC was taken.

By Calculation.

The hypotenuse AC being radius, then AB is the sine of the angle C, and BC the sine of angle A, or cosine angle C. Hence,

To find AB.		To find BC.	
As radius	10.000000	As radius	10.000000
To sine of C, 55° 30′	9.915994	To Cosine of C, 55° 30′	9.753128
So is AC, 121	2.082785	So is AC, 121	2.082785
To AB, 99.719	1.998779	To BC, 68.535	1.835913

The base BC being radius, then AB is the tangent, and AC the secant of the angle C. Hence,

To find AB.		To find BC.	
As secant of C, 55° 30′	10.246872	As secant of C, 55° 30′	10.246872
To tangent of C, 55° 30′	10.162866	To radius	10.000000
So is AC, 121	2.082785	So is AC, 121	2.082785
To AB, 99.719	1.998779	To BC, 68.535	1.835913

The perpendicular AB being radius, then BC becomes the tangent of the angle A, or the cotangent of the angle C, and AC becomes the secant of angle A, or cosecant of angle C. Hence,

To find AB.		To find BC.	
As cosec. of C, 55° 30′	10.084006	As cosec. of C, 55° 30′	10.084006
To radius	10.000000	To cotan. of C, 55° 30′	9.837134
So is AC, 121	2.082785	So is AC, 121	2.082785
To AB, 99.719	1.998779	To BC, 68.535	1.835913

General Rule for Gunter's Scale.

Extend the compasses from the first term to the second, that extent will reach from the third to the fourth term; observing to take the line marked *Num.* for feet, yards, miles, &c. the line mark-

ed S, for sines of angles, and that marked T, for tangents. The radius is 90° of sines, and 45° of tangents.

Ex. 2. In the triangle ABC, right-angled at B, let the hypotenuse AC be 1045 feet, and the angle A 35° 56′; what is the length of the base and perpendicular?—*Ans.* Length of the base, AB = 846.135 feet. Length of the perpendicular, BC = 613.25 feet.

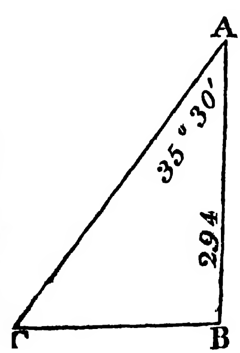
3. A ship, from latitude 20° 30′ north, sailed N. W. by N. 235 miles, what is her departure from the meridian, and what her difference of latitude, and the latitude come to?—*Ans.* Dep. from meridian 130.5 miles. Diff of Lat. = 195.4 miles. Hence the latitude come to, is 23° 45′ N.

4. Suppose the one end of a rope 350½ feet long, fixed at the top of an eminence, and the other end brought down to the plane below, so that its direction make with the plane an angle of 50° 40′, required the perpendicular height of the eminence, and the space of the level covered by the rope?—*Ans.* Height of the eminence = 271.102 feet. Space of the level = 222.158 feet.

PROBLEM II.

Given the angles and one side, to find the hypotenuse and other side.

Ex. 1. In the right-angled triangle ABC, right-angled at B, let the angle at A be 35° 30′, and the side AB 294 feet; required the base BC, and the hypotenuse AC?



Geometrically.

Make AB = 294, from a scale of equal parts, and at the point A make an angle of 35° 30′, from the line of chords, then from B draw the perpendicular BC, and ABC is the triangle proposed in the example. Measure BC and AC severally, by taking them in the compasses, and applying them to the scale from which AB was taken.

By Calculation.

The Hypotenuse AC being radius,

To find BC.		To find AC.	
As cosine of A, 35° 30′	9.910686	As cosine of A, 35° 30′	9.910686
To sine of A, 35° 30′	9.763954	To radius	10.000000
So is AB, 294	2.468347	So is AB, 294	2.468347
To BC, 209.7	2.321615	To AC, 361.13	2.557661

The base BC being radius,—

To find BC.		To find AC.	
As cotang. of A, 35° 30′	10.146732	As cotang. of A, 35° 30′	10.146732
To radius	10.000000	To cosec. of A, 35° 30′	10.236046
So is AB, 294	2.468347	So is AB, 294	2.468347
To BC, 209.7	2.321615	To AC, 361.13	2.557661

The perpendicular AB being radius,—

To find BC.		To find AC.	
As radius	10.000000	As radius	10.000000
To tangent of A, 35° 30′	9.853268	To secant of A, 35° 30′	10.089314
So is AB, 294	2.468347	So is AB, 294	2.468347
To BC, 209.7	2.321615	To AC, 361.13	2.557661

2. In the triangle ABC, right-angled at B, suppose the base BC 374½ yards, and the angle A 52° 8′, required the other side AB, and the hypotenuse AC?—*Ans.* Side AB = 291.19 yards. Hypotenuse AC = 474.386 yards.

3. Suppose a ship to sail S. W. by W. until she has made 140 miles of southing, required the distance sailed, and also how far she is west from the meridian of the place sailed from?—*Ans.* Distance sailed 252 miles. Dep. from meridian 209.5 miles west.

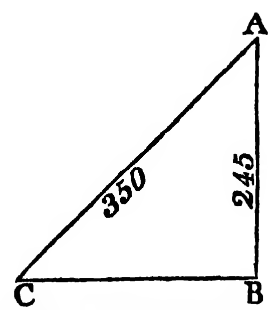
4. Observing the sun's altitude to be 30° 45′, and the length of the shadow of a tree at the same time to be 70 feet 3 inches on the horizontal plane; what is the height of the tree, and what will be the length of a rope which will reach from the extremity of the shadow to the top of the tree?—*Ans.* Height of the tree = 41.794 feet. Length of the rope = 81.742 feet.

PROBLEM III.

Given the hypotenuse and one side, to find the angles and the other side.

Ex. 1. In the right-angled triangle ABC, right-angled at B, let the hypotenuse AC be 350 feet, and the perpendicular AB 245 feet; required the angles A and C, and the base BC?

Geometrically.



Draw the line BC terminated towards B, but unlimited towards C, and at the point B draw AB perpendicular to BC, and make AB = 245, from a scale of equal parts; from the same scale take AC = 350; place the one foot of the compasses in A, and with the other, describe an arch, cutting BC in the point C, join AC, and ABC is the triangle required. The angles are to be measured on the line of chords.

By Calculation.

The hypotenuse AC being radius,—

To find angle C.		To find BC.	
As AC, 350	2.544068	As radius	10.000000
To BA, 245	2.389166	To cos. of C, 44° 25′ 37″	9.853786
So is radius	10.000000	So is AC, 350	2.544068
To sine of C, 44° 25′ 37″	9.845098	To BC, 249.95	2.397854

The perpendicular AB being radius,—

To find angle A.		To find BC.	
As AB, 245	2.389166	As radius	10.000000
To AC, 350	2.544068	To tang. of A, 45° 34′ 23″	10.008688
So is radius	10.000000	So A B, 245	2.389166
To secant of A, 45° 34′ 23″	10.154902	To BC, 249.95	2.397854

The base BC being radius, to find itself,—

As tang. of C, 44° 25′ 37″	9.991312	As secant of C, 44° 25′ 37″	10.146214
To radius	10.000000	To radius	10.000000
So is AB, 245	2.389166	So is AC, 350	2.544068
To BC, 249.95	2.397854	To BC, 249.95	2.397854

The side BC may also be found, independently of the angles, by means of the known property of a right-angled triangle; that the square of the hypotenuse is equal to the sum of the squares of the two sides. For, since $AC^2 = AB^2 + BC^2$, it follows, that $BC^2 = AC^2 - AB^2 = (AC + AB) \cdot (AC - AB)$; and therefore $BC = \sqrt{(AC + AB) \cdot (AC - AB)}$. Or, $\text{Log. } BC = \dfrac{\text{Log. }(AC + AB) + \text{Log. }(AC - AB)}{2}$.

From which BC is easily determined. Thus,

$AC = 350$	
$AB = 245$	
	Logarithms.
$AC + AB = 595$	2.774517
$AC - AB = 105$	2.021189
	2)4.795706
$BC = 249.95$,	2.397853

2. Suppose the hypotenuse of a right-angled triangle to be 274.5 yards, and its base 196.25; what are the two acute angles, and the

perpendicular?—*Ans.* Angle opposite the base = 45° 38′ 17″; angle adjacent to the base 44° 21′ 43″. Length of the perpendicular = 191.927 yards.

3. Suppose a ship to have sailed between south and east 204 miles, and thereby made her difference of latitude, or southing, 126 miles; upon what course did she sail?—*Ans.* Course, S. 51° 51′ 20″ E., or S. E. ½E. nearly.

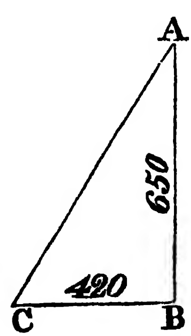
4. A ship sailed from latitude 49° 30′ north, between the south and west 135 leagues, till, by a good observation, she is found in latitude 45° 15′, required the course on which she sailed, and her departure from the meridian?—*Ans.* Course, S. 50° 58′ 38″ W., or S. W. ½W. nearly. Dep. from merid. = 104.9 leagues.

PROBLEM IV.

Given the base and perpendicular, to find the angles and hypotenuse.

Ex. 1. In the right-angled triangle ABC, right-angled at B, let the perpendicular AB be 650 feet, and the base BC 420 feet; required the acute angles A and C, and the hypotenuse AC?

Geometrically.



Make BC = 420, taken from a scale of equal parts, and from B raise the perpendicular AB, and make it = 650 from the same scale; join AC, and ABC is the triangle required. With the chord of 60° taken from the line of chords, describe arches upon the angular points A and C as centres, for the measure of the angles; the chords of these arches applied to the same line of chords, will give the quantity of each angle.

By Calculation.

The base BC being radius,—

To find angle C.		To find AC.	
As BC, 420	2.623249	As radius	10.000000
To AB, 650	2.812913	To sec. of C, 57° 7′ 52″	10.265427
So is radius	10.000000	So is BC, 420	2.623249
To tang. of C, 57° 7′ 52″	10.189664	To AC, 773.88	2.888676

The perpendicular AB being radius,—

To find angle A.		To find AC.	
As AB, 650	2.812913	As radius	10.000000
To BC, 420	2.623249	To sec. of A, 32° 52′ 8″	10.075763
So is radius	10.000000	So is AB, 650	2.822913
To tang. of A, 32° 52′ 8″	9.810336	To AC, 773.88	2.888676

The hypotenuse AC being radius, to find itself.

As sine of C, 57° 7′ 52″	9.924237	As sine of A, 32° 52′ 8″	9.734573
To radius	10.000000	To radius	10.000000
So is AB, 650	2.812913	So is BC, 420	2.623249
To AC, 773.88	2.888676	To AC, 773.88	2.888676

The hypotenuse may also be found independently of the angles; for, by *Euc. Elem.* B. I. Prop. XLVII., we have $AC = \sqrt{AB^2 + BC^2} = \sqrt{AB\left(AB + \frac{BC^2}{AB}\right)}$. This last form of the expression for AC, is by much the most convenient for logarithmic calculation.

Logarithms.

$$BC = 420 \ldots\ldots 2.623249$$
$$2$$
$$\text{Log. } BC^2 = 5.246498$$
$$AB = 650 \ldots\ldots 2.812913 \quad 2.812913$$
$$\frac{BC^2}{AB} = 271.384 \ldots\ldots 2.433585$$
$$AB + \frac{BC^2}{AB} = 921.384 \ldots\ldots 2.964441$$
$$\text{Log. } AB\left(AB + \frac{BC^2}{AB}\right) = 5.777354$$
$$AC = 773.88, \quad 2.888677$$

2. In the triangle ABC, right-angled at B, suppose the side AB 495.45 yards, and the side BC 560.5 yards; what are the acute angles A and C, and the hypotenuse AC?—*Ans.* Angle A = 48° 31′ 31″. Angle C = 41° 28′ 29″. Hypotenuse AC = 748.086 yards.

3. Suppose three towns, A, B, C, to be so situated, that A lies 35¾ miles south from B, and C lies 50¼ miles west from B; the bearings of A from C, and of C from A, are required?—*Ans.* The bearing of A from C, is S. 54° 34′ 13″ E., or S. E. ¾E., nearly. The bearing of C from A, is N. W. ¾W., nearly.

4. When the sun shines, if a steeple, 196 feet high, project a shadow 237 feet 9 inches, on the horizontal plane, what is the sun's altitude at that time?—*Ans.* Sun's altitude = 39° 30′ 7″.

Solution of Oblique-Angled Triangles.

6. In an oblique-angled triangle, six parts are concerned, viz. the three sides and three angles. Of these, one side and other two parts being given, the other parts may be found.

7. The solution of oblique-angled triangles, depends upon the following theorems.

THEOREM I.

. The sides of a plane triangle are to one another, as the sines of the angles opposite to them.

Let ABC be a triangle; AB: AC : : Sin. C : Sin. B.

From the vertext A draw AD perpendicular to BC the opposite side. Then in the right-angled triangle ACD we have,

AC : AD : : Rad · Sin. C,

and, by inversion,

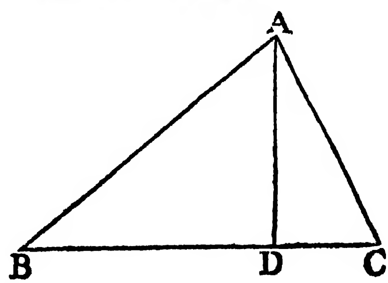
AD : AC : : Sin. C : Rad.

Now, in the right-angled triangle ABD,

AB : AD : : Rad. : Sin. B.

Hence, *ex æquo*, inversely,

AB : AC : : Sin. C : Sin. B.



In the same manner, it may be proved, that AB : BC : : Sin. C : Sin. A, and that AC : CB : : Sin. B : Sin. A.

THEOREM II.

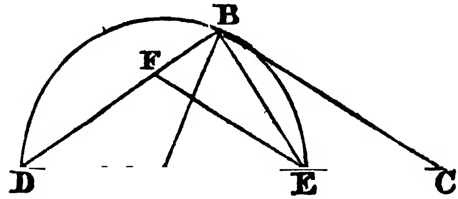
In any plane triangle, as the sum of any two sides is to their difference, so is the tangent of half the sum of the opposite angles to the tangent of half their difference.

Let ABC be a triangle, of which the side AC is greater than AB, and consequently the angle ABC greater than ACB,

AC+AB : AC—AB : : Tan. $\frac{1}{2}$(ABC+BCA) : Tan. $\frac{1}{2}$(ABC—BCA).

About the centre A, at the distance AB, let the semicircle DBE be described, meeting CA produced in D. Join DB, BE; and through E draw EF parallel to BC. Then, because the angle DAB is the exterior angle of the triangle ABC, it is equal to the sum of the two interior and opposite angles ABC, ACB. But the angle DEB is equal to half the angle DAB, therefore the angle DEB is equal to half the sum

of the angles ABC, ACB. Again, since AB is equal to AE, the angle ABE is equal to AEB. But the angle AEB is equal to the two angles EBC, BCE, therefore, also the angle ABE is equal to the sum of the angles EBC, BCE. To each



of these, add the angle EBC, then the whole angle ABC, is equal to twice the angle EBC, together with the angle BCE; whence, it is evident, that the angle EBC, or the alternate angle BEF, is equal to half the difference of the angles ABC, BCA. Now, DBE being an angle in a semicircle, is a right angle. Therefore, to the same radius EB, DB will be the tangent of the angle DEB, and FB the tangent of BEF. So that BD : BF : : Tan. DEB : Tan. BEF . : Tan. $\frac{1}{2}$(ABC + ACB) : Tan. $\frac{1}{2}$(ABC — ACB). Also, since AD and AE are each equal to AB, it is evident, that DC is the sum of the sides AB and AC; and CE is their difference. But, because EF is parallel to BC, we have DC : CE : : DB : BF, that is,

AC+AB : AC—AB . . Tan. $\frac{1}{2}$(ABC+BCA) : Tan. $\frac{1}{2}$(ABC—BCA),

Scholium.

When two sides, and the included angle of a triangle are given, half the difference of the two remaining angles can be found by the above proposition.

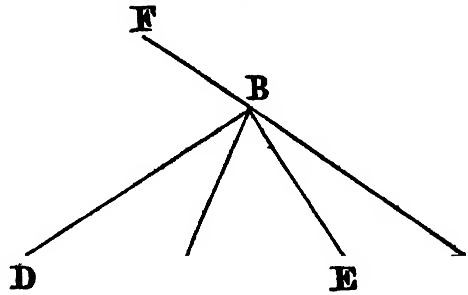
Then, half the difference being added to half the sum, gives the greater angle, and half the difference being taken from half the sum, gives the less.

THEOREM III.

In any plane triangle, the cosine of half the difference of any two of its angles is to the cosine of half their sum, as the sum of the sides opposite to these angles is to the remaining side of the triangle. Also, the sine of half the difference of the angles is to the sign of half their sum, as the difference of the opposite sides is to the remaining side.

Let ABC be a triangle, of which the side AC is greater than AB, and the angle ABC than ACB;

Cos. $\frac{1}{2}$(ABC—ACB):Cos. $\frac{1}{2}$(ABC+ACB): : AC+AB : BC, and Sin. $\frac{1}{2}$(ABC—ACB) : Sin. $\frac{1}{2}$(ABC+ACB) : AC—AB : BC.



In CA and CA produced take AE and AD each equal to AB: join BD, BE and produce CB to F: Then the angle DBE is a right angle; the angle AEB is half the sum, and the angle EBC half the difference of the angles ABC, ACB: Also DC is the sum and EC the difference of the sides AC and AB. Now in the triangle DBC we have

Sin. DBC or FBD : Sin. BDC : : DC : BC.

But FBD is the complement of the angle EBC, and BDC is the complement of the angle AEB; therefore

Sin. FBD = Cos. EBC = Cos. $\frac{1}{2}$(ABC — ACB),
Sin. BDC = Cos. AEB = Cos. $\frac{1}{2}$(ABC + ACB):

Hence we have

Cos. $\frac{1}{2}$(ABC — ACB) : Cos. $\frac{1}{2}$(ABC + ACB) : : AB + AC : BC

Again, in the triangle EBC we have

Sin. EBC : Sin. BEC or AEB : : EC : BC.

But Sin. EBC = Sin. $\frac{1}{2}$(ABC — ACB), and Sin. AEB = Sin. $\frac{1}{2}$(ABC + ACB),

wherefore

Sin. $\frac{1}{2}$(ABC — ACB) : Sin. $\frac{1}{2}$(ABC + ACB) : : AC — AB : BC.

THEOREM IV.

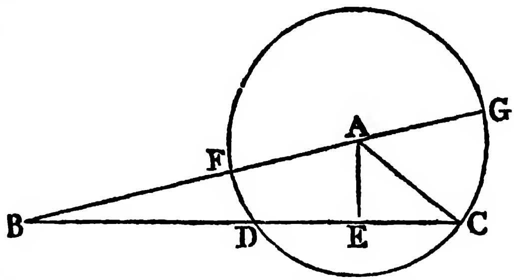
In any plane triangle, as the base, or longest side, is to the sum of the other two sides, so is the difference of these sides, to the dif-

ference of the segments of the base, made by a perpendicular let fall upon it from the oposite angle.

In the oblique-angled triangle ABC, let a perpendicular AE be drawn from the vertex A to the base BC;

$$BC : AB + AC :: AB - AC : BE - EC.$$

About A as a centre at the distance AC, the shortest side, describe a circle FDC; produce BA to meet the circle in G; then is BG the sum of the sides BA and AC, and BF is their difference; also BD is the difference of BE, and EC the segments of the base. Now, from a known property of the circle, the rectangle contained by BG and BF, is equal to the rectangle contained by BC and BD; hence, it follows, that BC : BG : . BF . BD, that is,



$$BC : AB + AC :: AB - AC : BE - EC.$$

SCHOLIUM.

When the three sides of a triangle are given, the difference of the segments of the base may be found by the above proposition. Then, half the difference added to half the sum, gives the greater segment, and half the difference taken from half the sum, gives the less. In each of the right-angled triangles ABE, ACE, into which the given triangle ABC is divided by the perpendicular, there are given, therefore, the hypotenuse and base, from which the angles may be found by Prob. III. of the solution of right-angled triangles. Hence the angles of the triangle ABC become known.

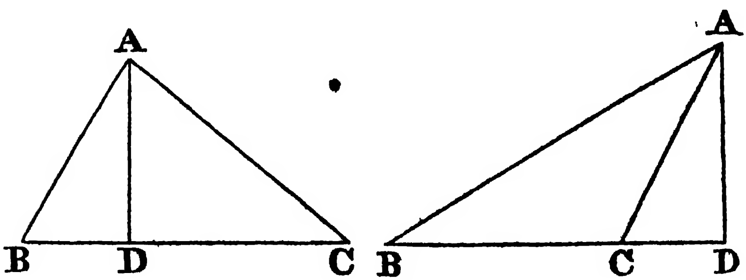
THEOREM V.

In any triangle, twice the rectangle contained by any two of the sides is to the difference between the sum of the squares of those sides and the square of the base, as the radius is to the cosine of the angle included by the two sides.

Let ABC be any triangle, twice the rectangle contained by AB and BC is to the difference between the sum of the squares of AB and BC and the square of AC, as the radius is to the cosine of angle B: that is,

$$2AB \times BC : AB^2 + BC^2 - AC^2 :: \text{Rad.} : \text{Cos. } B.$$

From A draw AD perpendicular to BC, then $2BC \times BD = AB^2 + BC^2 - AC^2$. But



$$BC \times BA : BC \times BD :: BA : BD :: \text{Rad.} : \text{Cos. } B;$$

therefore also $2BC \times BA : 2BC \times BD :: \text{Rad.} : \text{Cos. } B.$

Now $2BC \times BD = AB^2 + BC^2 - AC^2$; wherefore

$$2AB \times BC : AB^2 + BC^2 - AC^2 :: \text{Rad.} : \text{Cos. } B.$$

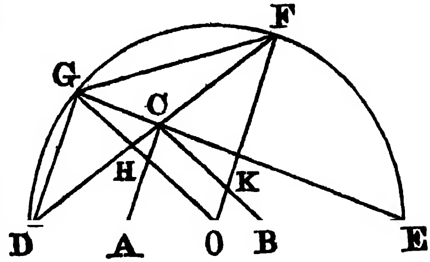
THEOREM VI.

In any triangle, the rectangle contained by two sides is to the rectangle contained by the excesses of half the perimeter above those sides, as the square of the radius is to the square of the sine of half the angle included between them.

Let ABC be a triangle of which AB is the base; in AB produced both ways take AD = AC, and BE = BC, and bisect DE in O: then DO is half the perimeter, and AO, BO are its excesses above AC, BC, the sides of the triangle: it is to be proved that

$$AC \times BC \text{ or } AD \times BE : AO \times BO :: \text{Rad}^2 : \text{Sin}^2 \tfrac{1}{2}ACB.$$

Join DC, EC; draw OF parallel to AC, meeting DC in F, and OG parallel to BC, meeting EC in G: and because the triangles DAC, DOF are similar and DA = AC, therefore DO = OF: In like manner, because the triangles EBC, EOG are similar, and EB = BC, therefore EO = OG; hence OG and OF are each equal to half of DE.



From O as a centre, with OD or OE as a radius, describe a semicircle, which will pass through G and F, also join DG, GF: And because OG is parallel to CB, and OF to CA, these lines form a parallelogram HCKO, of which the opposite angles ACB, GOF are equal, but the angle GDF at the circumference is half the angle GOF at the centre, therefore GDF or GDC is half the angle ACB. Again, because AC is parallel to OF, and BC parallel to OG,

$$AD : OA :: DC : CF,$$

$$\text{and } BE : OB :: EC : CG,$$

therefore $AD \times BE . AO \times OB :: DC \times EC : CF \times CG.$

Now the triangles DCE, GCF being evidently equiangular,

$$EC : DC :: CF : CG;$$

$$\text{hence } DC \times EC : DC^2 :: CF \times CG : CG^2,$$

and by altern. $DC \times EC : CF \times CG :: DC^2 . CG^2$:

Therefore $AD \times BE : AO \times BO :: DC^2 : CG^2$.

But in the triangle DCG, which has the angle at G a right angle,

$$DC : CG :: \text{Rad.} : \text{Sin. } CDG \text{ or Sin. } \tfrac{1}{2}ACB,$$

$$\text{and } DC^2 : CG^2 :: \text{Rad}^2 : \text{Sin}^2 \tfrac{1}{2}ACB:$$

Wherefore $AD \times BE : AO \times BO :: \text{Rad}^2 : \text{Sin}^2 \tfrac{1}{2}ACB$.

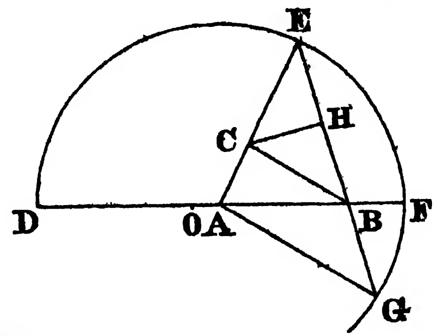
THEOREM VII.

In any triangle the rectangle contained by two sides is to the rectangle contained by half the perimeter and its excess above the base as the square of the radius is to the square of the cosine of half the angle included between them.

Let ABC be a triangle of which AB is the base; in BA produced take AD equal to the sum of AC, CB, and bisect BD in O; then DO is half the perimeter, and AO is its excess above the base: It is to be proved that

$$AC \times BC : DO \times OA :: R^2 : \text{Cos}^2 \tfrac{1}{2}ACB.$$

In AC produced take CE = CB; join EB; draw AG parallel to CB, meeting EB in G; and draw CH to bisect the angle ECB; then EH will be equal to HB, and the angles at H will be right-angles: and because the angles ECB, ACB are together equal to two right-angles, HCE and half of ACB will be together equal to a right-angle; therefore HCE is the complement of half the angle ACB.



The triangles ECB and EAG being similar and EC = CB; therefore EA = AG, but EA = AD by construction; therefore a circle described from A as a centre, with AD as a radius, will pass through E and G: let it meet AB produced in F; and since DF = 2DA and DB = 2DO, it follows that BF = 2AO.

Again, because BC is parallel to AG,

$$AC : CE :: GB : BE,$$

$$\text{therefore } AC \times CE : CE^2 :: GB \times BE : BE^2;$$

But $GB \times BE = DB \times BF = 4DO \times AO$; and $BE^2 = 4EH^2$

$$\text{therefore } AC \times CE : CE^2 :: 4DO \times AO : 4EH^2$$

$$:: DO \times AO : EH^2,$$

and by alternation $AC \times CE : DO \times AO :: CE^2 . EH^2.$

Now the angle ECH being the complement of half ACB,

$$CE : EH :: \text{Rad.} : \text{Cos.} \tfrac{1}{2}ACB,$$

$$\text{and } CE^2 : EH^2 :: \text{Rad}^2 : \text{Cos}^2 \tfrac{1}{2}ACB:$$

Therefore $AC \times CE : DO \times AO :: R^2 \text{Cos}^2 \tfrac{1}{2}ACB$.

THEOREM VIII.

In any triangle, the rectangle contained by half the perimeter and its excess above the base is to the rectangle contained by its excesses above the sides as the square of the radius is to the square of the tangent of half the included angle.

Let c denote the base, and a and b the sides of the triangle, and C the angle opposite to c: Put $p = \frac{1}{2}(a + b + c)$: Then, it is to be proved that

$$p\,(p - c) : (p - a)\,(p - b) :: \text{Rad}^2 : \text{Tan}^2 \tfrac{1}{2}C.$$

For, since by Theor. VII., $p\,(p - c) : ab :: \text{Cos}^2 \tfrac{1}{2}C : \text{Rad}^2$,

and by Theor. VI. $ab : (p - a)\,(p - b) :: \text{Rad}^2 : \text{Sin}^2 \tfrac{1}{2}C$;

therefore, *ex æquali*,

$$p\,(p - c) : (p - a)\,(p - b) :: \text{Cos}^2 \tfrac{1}{2}C : \text{Sin}^2 \tfrac{1}{2}C;$$

$$\text{but } \text{Cos}^2 \tfrac{1}{2}C : \text{Sin}^2 \tfrac{1}{2}C :: \text{Rad}^2 : \text{Tan}^2 \tfrac{1}{2}C,$$

$$\text{therefore } p\,(p - c) : (p - a)\,(p - b) :: R^2 : \text{Tan}^2 \tfrac{1}{2}C.$$

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8. The application of the preceding theorems to the solution of the cases of oblique-angled triangles, is contained in the following problems.

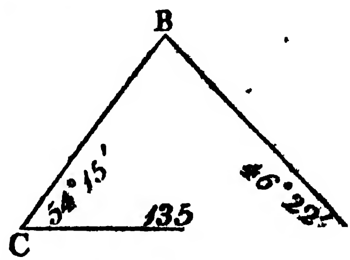
PROBLEM I.

Given the angles, and one side of an oblique-angled triangle, to find the other sides.

Ex. 1. In the triangle ABC, suppose angle A 46° 22′, angle B 79° 23′, and, consequently, the angle C 54° 15′, also the side AC 135 feet; required the sides AB and BC?

Geometrically.

Having measured the side AC = 135 from a scale of equal parts, at the point C make the angle ACB = 54° 15′ from a line of chords; also at the point A, the other extremity of the side AC, make the angle CAB = 46° 22′; the sides BC and AB will fall in their proper position, and are to be measured on the same scale from which AC was taken.



By Calculation.

To find AB.		To find BC.	
As sine B, 79° 23′	9.992501	As sine B, 79° 23′	9.992501
Is to sine C, 54° 15′	9.909328	To sine A, 46° 22′	9.859601
So is AC, 135	2.130334	So is AC, 135	2.130334
To AB, 111.47	2.047161	To BC, 99.41	1.997434

2. In the oblique-angled triangle ABC, let there be given the side BC 5304 yards, the angle B 40° 34′, and the angle C 36°; required the sides AB and AC?—*Ans.* AB = 3205.31 yards. AC = 3546.38 yards.

3. Coasting along the shore, I saw a cape bearing from me directly west; I steered away W. N. W. 60 miles, and then the same cape bore from me S. W. by W.; required the distance from each station to the cape?—*Ans.* Distance of the cape from first station 89.796 miles. Distance from second station 41.829 miles.

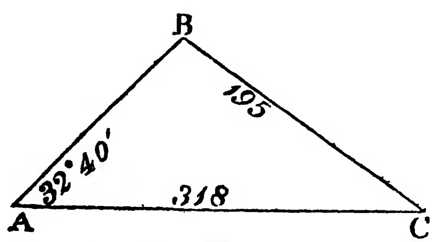
4. In the triangular field ABC, let the side AB be 3045 links, the angle at A 34° 45′, and the angle at B 50° 10′; required the length of the other sides AC and BC?—*Ans.* AC = 2347.52 links. BC = 1742.49 links.

PROBLEM II.

Given two sides, and an angle opposite to one of them, to find the other angles and the other side.

Ex. 1. In the oblique-angled triangle ABC, obtuse at B, let the side AC be 318 yards, the side BC 195 yards, and the angle A 32° 40′; required the angles B and C, and the side AB?

Geometrically.



Draw AC = 318, from a scale of equal parts, and at the point A make an angle of 32° 40; then, with 195 equal parts in the compasses, set one foot in C, and describe an arch intersecting AB in B, join CB, and ABC is the triangle required. Measure the angles B and C on the line of chords, and the side AB on the same scale from which the other sides were taken.

Note.—When the side opposite to the given angle is greater than the other given side, the angle opposite to this latter side is necessarily acute. When the side opposite to the given angle is less than the other given side, but greater than the perpendicular drawn to the unknown side from the opposite angle, the same *data* will give two different triangles, and the angle opposite the latter given side will be either acute or obtuse. For the arch described from the centre C, at the distance CB, will then cut AB in two points on the same side of AC. When the side opposite the given angle is

equal to the perpendicular, the angle opposite the other given side is a right-angle. When the side opposite the given angle is less than the perpendicular, the proposed triangle is impossible.

By Calculation.

To find the angle B.		To find the side AB.	
As the side BC, 195	2.290035	As sine of A, 32° 40′	9.732193
Is to side AC, 318	2.502427	To sine of C, 29°	9.685571
So is sine A, 32° 40′	9.732193	So is BC, 195	2.290035
To Sine of 61° 40′	9.944585	To AB, 175.15	2.243413
Hence angle B, 118° 20′			

It is to be observed, that angle B being an obtuse angle, the arch 61° 40′, which is found directly from the tables, is not the measure of angle B, but the measure of its supplement; and that, therefore, it is necessary to subtract 61° 40′ from 180°, in order to find the true value of the angle B. To determine angle C, we have only to take the sum of angles A and B from 180; which gives angle C = 29°

Ex. 2. In the oblique-angled triangle ABC, suppose the side AB 4101 feet, the side BC 2900 feet, and the angle A 30°; required the angles B and C, and the side AC?—*Ans.* In this example, the angle C may be either obtuse or acute. If angle C is acute, then angle C = 44° 59′ 49″, angle ABC = 105° 0′ 11″, and AC = 5602.29 feet. But, if angle C be obtuse, then angle ACB = 135° 0′ 11″, angle ABC = 14° 59′ 49″, and AC = 1500 85 feet.

3. In the triangular field BCD, let the side CD be 465 links, the side DB 543 links, and the angle C, opposite to the side DB, 70° 35′, required the angles B and D, with the length of the side BC?—*Ans.* Angle B = 57° 23′ 35″. Angle D = 52° 1′ 25″. And BC = 453.84 links.

4. A headland was observed to bear N. W. by W; and having steered N. N. E. 54 miles, we then came to an anchor 63 miles from the same headland; required the distance of the headland from the first place of observation, and its bearing from the place where we anchored?—*Ans.* Distance of headland from first place of observation 44.652 miles. Bearing from second station S 66° 32′ 20″. W. or W. S. W. nearly.

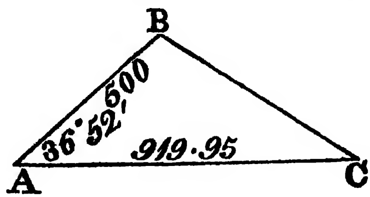
PROBLEM III.

Given two sides and the included angle, to find the other angles and the third side.

Ex. 1. In the triangle ABC, suppose the side AC 919.95 feet, the side AB 500 feet, and the contained angle A 36° 52′, required the angles B and C and the side BC?

Geometrically.

Draw the side AC, making it equal to 920 nearly, from a scale of equal parts. At the point A, in the straight line AC, make the angle CAB = 36° 52′ from a scale of chords, and from the same scale of equal parts, from which AC was taken, measure the side AB = 500.



Join BC; then ABC is the triangle required. The side BC is to be measured upon the scale of equal parts, and the angles B and C upon the line of chords.

By Calculation.

To find the angles B and C.

As AC + AB, 1419.95	3.152273
To AC — AB, 419.95	2.623198
So is Tan. ½(B + C), 71° 34′	10.477162
To Tan. ½(B — C), 41° 35′	9.948087

Angle B = 113° 9′

C = 29° 59′

To find BC.

As sine C, 29° 59′	9.698751
To sine A, 36° 52′	9.778119
So is AB, 500	2.698970
To BC, 600.26	2.778338

Or thus, (Theor. III.)

As Cos. $\frac{1}{2}$(B — C), 41° 35′	9.873896	As Sin. $\frac{1}{2}$(B — C), 41° 35′	9.821977
To Cos. $\frac{1}{2}$(B + C), 71° 34′	9.499963	To Sin. $\frac{1}{2}$(B + C), 71° 34′	
So is AC+AB, 1419.95,	3.152273	So is AC—AB, 419.95,	2.623198
To BC, 600.26	2.778340	To BC, 600.27	2.778346

2. In the obtuse-angled triangle ABC, let the side AB be 290 yards, the side BC, 410 yards, and the contained angle B, 105°; required the angles A and C, and the side AC?—*Ans.* Angle A = 44° 59′ 37″. Angle C = 30° 0′ 23″. Side AC = 560.13.

3. Suppose BCD a triangular field; the side BC measures 10 chains 30 links, the side CD 12 chains 60 links, and the contained angle C, measured with an angular instrument, is found to be 56° 30′, required the other angles B and D, and the side BD?—*Ans.* Angle B = 72° 20′ 16″. Angle D = 51° 9′ 44″. Side BD = 11.027 chains.

4. There are three cities, A, B, and C, in a triangular situation to each other; A lies 360$\frac{1}{2}$ miles due west from B, and C lies 230$\frac{1}{4}$ miles S. W. by W. from B; what is the distance between A and C, and what their bearings from each other?—*Ans.* Distance of A from C, is 211.996 miles. Bearing of C from A, is S. 52° 53′ 9″ E., or S. E. $\frac{3}{4}$E. nearly; and, consequently, the bearing of A from C, is nearly N. W. $\frac{3}{4}$W.

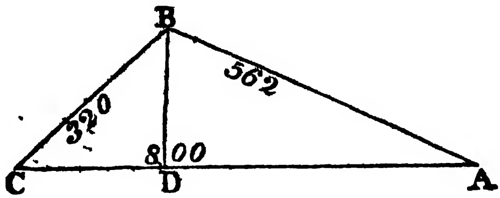
PROBLEM IV.

Given the three sides of a triangle to find the angles.

Ex. 1. In the triangle ABC, let AB = 562, AC = 800, and BC = 320; required the angles?

Geometrically.

Draw the side AC, making it equal to 800, from a scale of equal parts; then, from the same scale, take AB = 562; set one foot of the compasses in A, and with the other describe an arch; next take BC = 320 as a radius, and with one foot in C, intersect the former arch in the point B; join AB and BC, and ABC is the triangle required.



The angles are to be measured on the line of chords.

By Calculation.

To find CD and AD, the segments of the base.

As AC, 800	2.903090
To AB + BC, 882	2.945469
So is AB — BC, 242	2.383815
To AD — DC, 266.805	2.426194

Now, $\frac{1}{2}(AD + DC) = 400$
$\frac{1}{2}(AD - DC) = 133.403$

AD = 533.403

DC = 266.597

To find angle BCD.		To find angle BAD.	
As CD, 266.597	2.425855	As AD, 533.403	2.727055
To BC, 320	2.505150	To AB, 562	2.749736
So is radius	10.000000	So is radius	10.000000
To secant BCD 33° 34′ 47″	10.079295	To secant B 18° 21′ 23″	10.022681

Hence, we obtain angle ABC = 128° 3′ 50″.

Or, without finding the segments of the base we may determine the angles by Theor. V., VI., VII., or VIII. But it is to be observed, that a small angle may be more correctly determined from its sine than from its cosine; because in this case a small error in the cosine produces a considerable error in the angle: On the contrary, an angle nearly equal to a right angle, may be more correctly determined from its cosine than from its sine, because a small error in the sine produces a considerable error in the angle. Hence when the angle sought is acute, the solution by Theor. VI. is preferable to that by Theor. VII.; but when the angle is obtuse, the solution by Theor. VII. is to be preferred. It may always be known whether an angle is acute or obtuse by considering whether the square of the side opposite to the angle is less or greater than the sum of the squares of the sides containing the angle.

To find angle ABC. (Theor. VII.)

As AB × BC	562 × 320 Arith. Comp. Log.	7.250264 7.494850
To $\frac{1}{2}$ Perimeter	841	2.924796
× ($\frac{1}{2}$ Perim. — AC)	× 41	1.612784
So is Rad^2		20.000000
To Cos^2 $\frac{1}{2}$ABC	2)	19.282694
Cos. $\frac{1}{2}$ABC, 64° 1′ 55″		9.641347
	2	

Angle ABC $= 128^{\circ}\ 3'\ 50''$

One of the angles being thus found, another may be found upon the principle that the sides are to each other as the sines of the opposite angles: and then the third angle becomes known.

2. In the oblique-angle triangle ABC, suppose the side AC 105 yards, the side AB 85 yards, and the side BC 50 yards, it is required to find the three angles A, B, and C?—*Ans.* Angle A $= 28^{\circ}\ 4'\ 21''$. B $= 98^{\circ}\ 47'\ 50''$. C $= 53^{\circ}\ 7'\ 49''$.

3. Suppose A, B, and C three towns; let A be distant from B 76 miles, and from C 37 miles; also let B's distance from C be 53 miles; required the bearings of A from the towns B and C, supposing C to be situated N. W. by W. from B, and from A somewhere between North and East?—*Ans.* The bearing of A from B,

is N. 82° 30′ W., or W. by N. $\frac{1}{4}$ W, nearly. And the bearing of A from C, is S. 57° 48′ W., or S. W. by W. nearly.

4. Let there be two ports in the same parallel of latitude, whose distance is 48 leagues; and suppose a ship to sail from the one port 30 leagues between North and East, and another ship to sail from the other port 42 leagues between North and West, where she meets with the first ship; required the course of each?—*Ans.* The course of the first ship, is N. 30° E., or N. N. E. $\frac{3}{4}$E. nearly. The course of the second, is N. 51° 47′ 12″ W., or N. W. $\frac{1}{2}$W. nearly.

MENSURATION

OF

HEIGHTS AND DISTANCES.

1. When a line which joins two points in space is accessible throughout its whole extent, it may in general be measured by the successive application of some line of a known length; but when it is inaccessible, or cannot be directly measured, it may then be considered as a side of a triangle, of which as many parts can be found as are sufficient to determine all the others, and its length may be calculated by the rules of trigonometry. In this manner the mensuration of inaccessible lines is reduced to that of accessible lines and angles.

2. The instruments commonly used for measuring heights and distances, are a chain, a square, a quadrant, a theodolite, and a sextant.

A chain is used for measuring those distances or lines which are to be given sides of triangles. The English chain is 4 poles in length, and consists of 100 links; therefore each link should be 7.92 inches long. The Scotch chain is 74 feet in length, and therefore each of the links is 8.88 inches.

A square is used for finding the ratio of the sides of a right-angled triangle. Two of its sides are divided each into 100 equal parts; and it is furnished with a plummet suspended from the opposite angle, and with sights fixed on one of the undivided sides.

A quadrant is used for determining verticle angles. It is divided into degrees, &c. and is furnished with a plummet suspended from the centre, and with sights fixed on one of the radii.

A theodolite is used for measuring both horizontal and vertical angles. It consists of a circle and a circular segment at right angles to each other, and each divided into degrees, &c. When the instrument is used, the circle is placed in a horizontal and the segment in a vertical plane by means of a level. To the segment are attached sights, or a telescope, moveable about the centre both of the circle and of the segment. On the circle, horizontal angles are measured; and vertical angles, whether of elevation or depression, are measured on the segment.

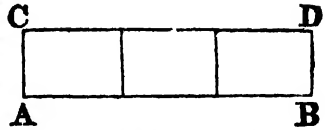
A sextant is employed to measure angles contained by lines situated in any plane whatever. When constructed on a small scale the sextant is more portable than the theodolite; but it is less suited to surveying, because the angles determined by it, when out of the plane of the horizon, must be reduced to that plane by calculation.

3. The operation of measuring an angle being much more easy than that of measuring a side, it is usual to measure only one side which is denominated the *Base*.

Method of Measuring a Base Line.

4. Let it be proposed to measure on the ground the distance AB. Begin by fixing in a vertical position, by means of a plumb-line, two straight poles AC, BD at the extreme points A, B. These poles (which are likewise called pickets or station-staves) enable the observer to fix the other intermediate pickets, which may be found necessary for marking out the line AB. All the pickets must be placed perpendicularly, that they may all be in the vertical plane of the visual ray CD. When great accuracy is required, a small trench may be drawn from A to B, applying from time to time the eye to the pickets, and in their vertical plane, in

order to direct the trench as nearly as possible in a straight line: or we may connect the pickets with each other by cords. Having thus accurately marked out the line AB, it is next to be

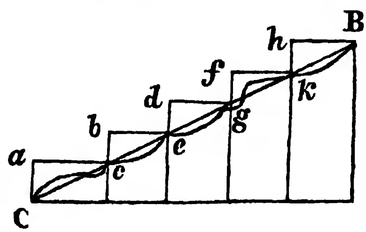


measured by the chain, or by a straight rod of any convenient length.

When this degree of minuteness is not required, the direction of the line may be ascertained by the person at one end of the chain, directing the person at the other end to place himself in such a position, as to be seen exactly in a line with the picket or mark towards which they are measuring *.

This method of measuring a base supposes the ground between the points A and B to be level, and in this case the distance measured is called the *horizontal distance.*

5. When the ground is inclined or uneven we may proceed as follows. After having marked, by pickets, the direction of the base line BC, we may employ for measuring the distance two or more straight and inflexible rods, of any convenient length, which are to be brought into a horizontal position, *ac*, *be*, &c. by means of a spirit level. The perpendiculars C*a*, *bc*, *de*, &c. represent the position of the plumb-line, which should be suspended from the extremity of each rod, to ascertain the exact point at which the extremity of the succeeding rod is to be placed. The sum of all the distances *ac*, *be*, *dg*, *fk*, hB is equal to AC the horizontal distance of the points B, C, which therefore becomes known.



6. If we wish to find AB, the perpendicular altitude of the point B above C, it is only necessary to measure the heights *a*C, *bc*, *de*, *fg*, *kh*. then adding or substracting these heights according as the elevations or depressions of the ground direct, the result is the

* The termination of each chain length is marked by a small arrow or rod, stuck into the ground by the person who leads, and taken up by the person who follows. There are ten arrows which accompany the chain; so that by attending to the number of times that the arrows have been transferred from the follower to the leader of the chain, the whole number of chains or links measured are easily determined.

height AB required. When AB and AC are known, the sloping distance BC is easily found: for $BC = \sqrt{AC^2 + AB^2}$.

7. When the height of the point B above the horizontal line AC is all that is required, the instruments commonly employed to determine the partial heights aC, bc, &c. are a spirit-level and a pair of staves, each composed of two pieces that slide out into a rod of ten feet in length, every foot being divided centesimally. The intervals between the staves should not exceed 400 yards. When the objects are very remote a good theodolite is the instrument to be used. and allowance must be made for refraction and the earth's curvature. This operation is denominated *Leveling*, and is a delicate and important branch of general surveying.

Of the Measurement of Angles.

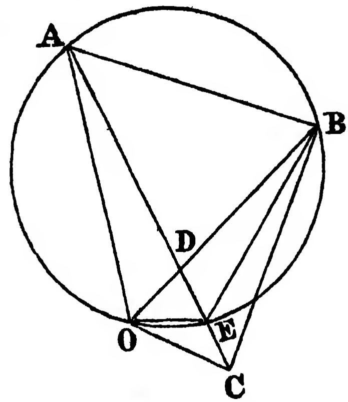
8. Though when two angles of a triangle are known the third may be found; yet, where accuracy is wanted, it is proper to measure, if it is in our power, all the three angles of every triangle whose sides are to be determined from the base and the angles. If the sum of the three angles, as found by observation, be very nearly 180°, we are sure that our observations have been made with precision and the difference between the sum and 180° is to be divided equally among the three angles, unless we have reason to doubt the accuracy of one observation more than that of another. Thus, suppose the sum of the three observed angles to be 180° 0′ 30″, we should subtract 10″ from each of the angles before we proceed to find the unknown sides of the triangle. To diminish the probability of errors; and to enable us to estimate nearly the certainty or uncertainty of the results, it is proper, in choosing the position of the triangles, to observe the following rules as far as local circumstances will permit.

Rule I. When only one side of a triangle is to be determined, make the base as nearly as possible equal to the side required.

Rule II. When two sides of a triangle are to be determined, make the triangle as nearly as possible equilateral.

Rule III. When the base cannot be made nearly equal to the side, or to each of the sides, sought, make the base as long as possible, and the angles at the base as nearly as possible equal to each other.

9. It often happens, that the theodolite cannot be placed at the centre of the object which has been observed at the other points of station. Suppose, for example, that the vane of a spire has been observed, it may be impossible to place the theodolite at the point of the base immediately under the vane. Hence the angles must be taken at a point as near as possible, and afterwards reduced by calculation to the centre of the station.



Let O be the centre of a permanent station, where the angle AOB subtended by two remote objects A and B is to be determined: let C be a given point at a little distance, where the instrument is placed, and the angle ACB actually measured. Suppose the distance CO given, also the angles ACO, BCO; and the distances AO, BO, or at least their values nearly. Let D be the point of intersection of AC, BO: and because the angle ADB is the sum of the angles CAO, AOB, also the sum of the angles CBO, ACB, therefore

$$\text{CAO} + \text{AOB} = \text{CBO} + \text{ACB},$$
$$\text{and AOB} - \text{ACB} = \text{CBO} - \text{CAO}.$$

Now, in the triangle COA, COB,

$$\text{BO} : \text{OC} :: \text{Sin. BCO} : \text{Sin. CBO}$$
$$\text{and AO} : \text{OC} :: \text{Sin. ACO} : \text{Sin. CAO}.$$

From these proportions the angles CBO, CAO may be found; and their difference, which is also the difference of the angles AOB, ACB, will be known.

It is useful in practice to have a formula that expresses the difference of the angles AOB, ACB in minutes of a degree. For this purpose put the angles AOB $=$ O, ACB $=$ C, ACO $= v$, then OCB $=$ C $+ v$: also, put the lines AO $= m$, BO $= n$, OC $= d$, from the above proportions we have

$$n : d :: \text{Sin.}\ (\text{C} + v) : \text{Sin. CBO} = \frac{d}{n}\ \text{Sin.}\ (\text{C} + v)$$

$$m : d :: \text{Sin.}\ v \quad \text{Sin. CAO} = \frac{d}{m}\ \text{Sin.}\ v.$$

But small angles being nearly proportional to their sines, it follows that the number of minutes in the angle CBO will be $\frac{\text{Sin CBO}}{\text{Sin. } 1'}$ nearly; and in like manner the number of minutes in the angle CAO will be $\frac{\text{Sin. CAO}}{\text{Sin. } 1'}$ nearly; therefore $\text{CBO} = \frac{d \text{ Sin. } (C + v)}{n. \text{ Sin. } 1'}$, $\text{CAO} = \frac{d \text{ Sin. } v}{m \text{ Sin.} 1'}$: hence since CBO — CAO = O — C we have

$$O - C = \frac{d \text{ Sin. } (C + v)}{n \text{ Sin. } 1'} - \frac{d \text{ Sin. } v}{m \text{ Sin.} 1'}.$$

This is the correction of the angle C expressed in minutes of a degree. *

10. A different and more simple expression for the correction of the angle C may be found as follows:

Let a circle be described about the triangle ABO, meeting AC in E and join OE, BE: let the angles AOB, ACB, ACO, and the lines AO, BO, OC, be denoted by the same letters as before; and in addition put the angle ABO = B. Then

$$O - C = \text{CBO} - \text{CAO or EBO} = \text{EBC}.$$

In the triangle OEC

$$\text{Sin CEO or AEO} : \text{Sin. EOC} :: \text{OC} : \text{CE},$$

and in the triangle BEC

$$\text{BE} : \text{EC} :: \text{Sin. BCE} : \text{Sin. EBC} :$$

Observing now that AEO = ABO = B, and that EOC = AEO — OCE = B — v; also that EB and BO = n are nearly equal, these proportions may be expressed thus:

$$\text{Sin. B} : \text{Sin. } (B - v) :: d : \text{CE}$$

$$n : \text{CE} :: \text{Sin. C} :: \text{Sin. EBC}$$

$$\text{hence CE} = \frac{d \text{ Sin. } (B - v)}{\text{Sin. B.}}; \text{ and}$$

* In the application of this formula we must consider whether Sin. $(C + v)$ and Sin. v are positive or negative. The sine of any arch between 0° and 180° is positive, and between 180° and 360° negative. The cosine between 0° and 90° is positive, between 90° and 270° it is negative, and between 270° and 360° it is again positive.

$$\text{Sin. EBC} = \frac{\text{CE Sin. C}}{n} = \frac{d \text{ Sin. } (B - v) \text{ Sin. C}}{n \text{ Sin. B.}}$$

But, on account of the smallness of the angle EBC, the number of minutes it contains will be $\frac{\text{Sin. EBC}}{\text{Sin. } 1'}$ nearly; therefore, because $O - C = EBC$, we have in minutes of a degree

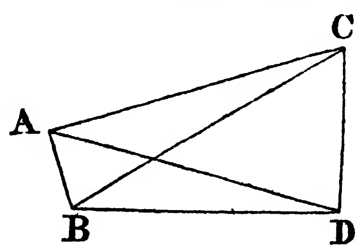
$$O - C = \frac{d \text{ Sin. } (B - v) \text{ Sin. C}}{n \text{ Sin. B Sin. } 1'}$$

This expression for the difference of the angles O and C is not quite so accurate as the former, yet in practice it is near enough the truth. It requires that approximate values of the distance BO and of the angle ABO be known, but the distance CO should be accurately determined.

It is evident that if the instrument be placed at E, in the circumference of a circle passing through the points A, B, O, the observed angle AEB will be equal to the angle AOB at the station: this may be done by moving the instrument along CA until the angle OEA is found to be equal to OBA; and then no correction is wanted.

11. When a theodolite is used in surveying, the angles are taken at once in a horizontal plane, but when a sextant is used, the angles are measured in the planes of the objects, and if they are oblique, the corresponding horizontal angles are found by calculation.

Let AC, BC be the straight lines which contain the given oblique angle ACB: in CD a perpendicular to the horizon take any point D, and let a horizontal plane pass through D and meet CA, CB in A and B; join AB, AD, BD: Then ADB is the horizontal angle corresponding to the given oblique angle ACB, and CAD, CBD, are the inclinations of the lines AC, BC to the horizontal plane. Put the angles CAD $= a$, CBD $= b$, the given oblique angle ACB $=$ C, and its corresponding horizontal angle ADB $=$ D.



In the triangle ABC we have, (Pl. Trig. §. 7. Theor. V.)

$$2AC \times BC : AC^2 + BC^2 - AB^2 :: \text{Rad.} : \text{Cos. ACB};$$

hence, $R(AC^2 + BC^2 - AB^2) = 2AC \times CB \text{ Cos. C}.$

In like manner in the triangle ABD,

$$R\,(AD^2 + BD^2 - AB^2) = 2AD \times DB \text{ Cos. } D$$

Substracting the latter of these equations from the former, and observing that,

$$AC^2 - AD^2 \text{ and } BC^2 - BD^2 = CD^2, \text{ we obtain}$$

$$2R + CD^2 = 2AC \times CB \text{ Cos. C} - 2AD \times DB \text{ Cos. D.}$$

Dividing by $2AC \times CB$, this equation becomes,

$$R. \times \frac{CD}{AC} \times \frac{CD}{BC} = \text{Cos. } C \cdot \frac{AD}{AC} \times \frac{DB}{BC} \text{ Cos. D.}$$

But $\frac{CD}{AC} = \frac{\text{Sin. } a}{R}$, $\frac{CD}{BC} = \frac{\text{Sin } b}{R}$, $\frac{AD}{AC} = \frac{\text{Cos. } a}{R}$, and $\frac{BD}{BC} = \frac{\text{Cos. } b}{R}$, therefore, by substituting, we obtain

$$\frac{\text{Sin. } a \text{ Sin. } b}{R} = \text{Cos. C} \quad \frac{\text{Cos. } a \text{ Cos. } b}{R^2} \text{ Cos. D};$$

hence we have

$$\text{Cos. D} = \frac{R^2 \text{ Cos. C} - R \text{ Sin. } a \text{ Sin. } b}{\text{Cos. } a \text{ Cos. } b}.$$

From this formula the reduced angle D may easily be determined. The formula may, however, be put under a form more convenient for logarithmic calculation. By multiplying both sides of the equation by R, and then subtracting the results from R^2, and reducing, we find

$$R^2 - R \text{ Cos. D} = \frac{R^2 (\text{Cos. } a \text{ Cos. } b + \text{Sin. } a \text{ Sin. } b) - R^3 \text{ Cos. C.}}{\text{Cos. } a \text{ Cos. } b}$$

But $R^2 - R \text{ Cos. D} = 2\text{Sin.}^2 \tfrac{1}{2}D$, (page 56 formula XVIII.)

and $\text{Cos. } a \text{ Cos. } b + \text{Sin. } a \text{ Sin. } b = R \text{ Cos. } (a - b)$; (page 55 formula XII.

therefore we have

$$2\text{Sin.}^2 \tfrac{1}{2}D = \frac{R^3 (\text{Cos. } (a - b) - \text{Cos. C.})}{\text{Cos. } a \text{ Cos. } b.}$$

Now in formula IV. page 54, namely,

$$R\,(\text{Cos. } B - \text{Cos. } A) = 2\text{Sin. } \tfrac{1}{2}(A + B)\ \text{Sin. } \tfrac{1}{2}(A - B),$$

assume $B = a - b$, $A = C$, and we have

$$R^3\,(\text{Cos. } (a - b) - \text{Cos. } C) = 2R^2\ \text{Sin. } \tfrac{1}{2}(C + a - b) \times \text{Sin. } \tfrac{1}{2}(C - a + b).$$

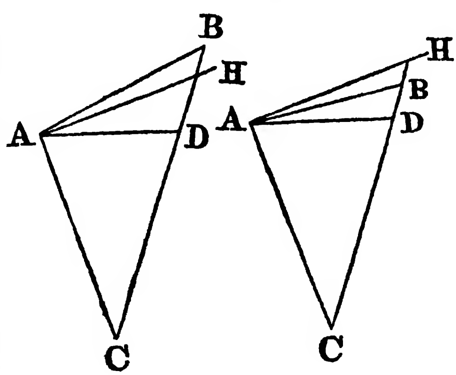
If we put $\frac{1}{2}(C + a + b) = s$; then $\frac{1}{2}(C + a - b) = s - b$, and $\frac{1}{2}(C - a + b) = s - a$. Hence, by substituting and extracting the square root, we obtain

$$\text{Sin. } \tfrac{1}{2}D = R\,\sqrt{\frac{\text{Sin. } (s - a)\ \text{Sin. } (s - b)}{\text{Cos. } a\ \text{Cos. } b}}.*$$

We have supposed the angles a, b to be both elevations. If one be a depression the formula will still hold true, provided the arch of depression be regarded as negative.

12. When the height of one station above another is to be measured, it is necessary to apply to the observed elevation or depression, a correction, on account of the earth's curvature, in order to find the true verticle angle, from which the height is to be determined.

Let A and B be the two stations, and let the height of the one above the other be required. Let C be the centre of the earth. Draw the horizontal line AH in the plane of the triangle ABC; then BAH will be the apparent angle of elevation or depression of B, according as it is above or below the horizon of A.



* When the lines which contain the given oblique angle have their inclinations to the horizontal plane less than 2° or 3°, the general solution does not conveniently apply. In this case it is better, instead of seeking the horizontal angle directly, to find its difference from the oblique angle. This difference, which we shall denote by d, may be determined with sufficient accuracy from the following approximate expression.

Let a and b denote the inclinations in minutes of a degree; we have also in minutes

$$d = \left\{ \left(\frac{b + a}{2}\right)^2 \frac{\text{Tan } \frac{1}{2}C}{R} - \left(\frac{b - a}{2}\right)^2 \frac{\text{Cot. } \frac{1}{2}C}{R} \right\} \frac{1}{3438}.$$

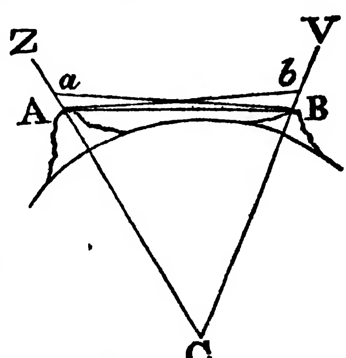
Take CD = CA, and join AD; the line BD is the difference between the heights of stations A and B: and to determine BD, the verticle angle BAD must be known. This angle differs from the apparent elevation or depression HAB by the angle HAD; therefore HAD is the correction of the verticle angle, depending on the earth's curvature.

• In the isosceles triangle CAD, the sum of the angles, that is, $2CAD + C$, is equal to two right-angles, therefore $CAD + \frac{1}{2}C =$ one right-angle: But the angle CAH ($= CAD + DAH$) is a right-angle: wherefore $CAD + DAH = CAD + \frac{1}{2}C$, and taking away the common angle CAD, there remains $DAH = \frac{1}{2}C$. Hence, the correction on account of the earth's curvature is *half the arch intercepted between the stations.*

The angle ADB being nearly a right-angle, when the vertical angle DAB and the distance AD are known, DB, the height of the one station above the other is easily found.

13. In observing the elevation or depression of a remote object, or of a heavenly body, a correction must be applied for the effect of refraction.

Let A and B be two stations remote from each other, and C the earth's centre, draw the lines CA, CB, and produce them towards Z and V the zeniths of the stations, join AB, then the true zenith distance of B, as seen from A, is the angle ZAB; and the true zenith distance of A, as seen from B, is the angle VBA: these, however, cannot be directly measured on account of refraction which increases the elevation of distant objects. This effect is produced by the rays of light being gradually bent from their original rectilineal direction, in passing through the atmosphere into a curve that is concave towards the earth. The change which this produces in the position of objects is always in the verticle plane. Let a, b be the positions to which the points A and B appear to be elevated when seen from each other. The errors produced by refraction are the angles bAB, aBA; and these will be nearly equal if the angles be observed at the same instant, which may be done by setting two watches to the same time, or by making a signal at one station so as to be seen from the other.



Put the greater apparent zenith distance ZAb = d, the lesser

VBa = d', the refraction bAB = aBA = r, and the angle C at the centre of the earth = C. Then

$$ZAB = ABC + C, \text{ and } VBA = BAC + C,$$

$$\text{therefore } ZAB + VBA = ABC + BAC + C + C:$$

$$\text{But } ABC + BAC + C = 180^\circ;$$

$$\text{Therefore } ZAB + VBA = 180^\circ + C.$$

$$\text{Again } ZAB = ZAb + bAB = d + r,$$

$$\text{and } VBA = VBa \times aBA = d' + r,$$

$$\text{therefore } ZAB + VBA = d + d' + 2r.$$

Putting the two values of ZAB + VBA equal to each other, we have

$$d + d' + 2r = 180^\circ + C.$$

Hence we find

$$r = 90^\circ + \tfrac{1}{2}C - \tfrac{1}{2}(d + d').$$

14. Such are the *principal* circumstances to be attended to, and corrections to be made in finding the requisite *data* for determining the relative position and elevation of points on the surface of the earth. When this operation is carried to a great extent, as in surveying a kingdom, in addition to some of the more profound theories of pure mathematics, the aid of astronomy and other branches of natural philosophy is required. The following examples will serve to illustrate the principles already laid down.

EXAMPLES.

1. At the distance of 130 feet from the bottom of a tower AB, and on the same horizontal plane with it, I directed the sights of a square to its summit A, and observed that the plummet cut 76 equal parts on the side adjacent to the sights; required the height of the tower, supposing the square $5\frac{1}{2}$ feet above the ground?—*Ans.* 176.553 feet.

2. Let BC be a horizontal plane, on which stands the perpendicular object AB; let the observer be placed 196 yards from the bottom of the object, at C, and let the height of his eye be 5 feet; suppose the plummet cuts 45 equal parts, upon the side of the square opposite to the sights, what is the height of the object?—*Ans.* 269.6 feet.

Note.—If the plummet cut the opposite angle, the distance of the object from the place of the observer, is the same with its height above his eye; therefore, if to the distance we add the height of the observer's eye, this will give the whole height of the object.

3. Standing upon the top of a tower AB, whose height is 120 feet, I placed a square so as, through the sights, to observe a house on the plane below, and found the plummet cut $80\frac{1}{2}$ equal parts on the side opposite to the sights, required the distance of the house C from the bottom of the tower?—*Ans.* 149.068 feet.

4. From the top of a hill 210 yards high, I observed a tree at the bottom, whose distance from the centre of the hill I wished to know, placing the sights of a square towards the tree, the plummet cut $86\frac{1}{4}$ equal parts on the side adjacent to the sights: what is the distance of the tree?—*Ans.* 181.125 yards.

5. From a station A, upon a horizontal plane, I observed a tree F, and going off at right-angles to AF, 350 yards, to another station B, and placing a square so as through the sights to see the former station A, then directing the sights of the index to the tree F, the index cut 56 on the edge opposite to the sights of the square, required the distance of the tree from the first station?—*Ans.* 625 yards.

6. Suppose a mirror C to be placed horizontally 140 feet distant from the bottom of a tower AB, and an observer, whose eye is 5 feet 9 inches from the ground, at D, $8\frac{1}{2}$ feet distant from the mirror, to see the image of the summit of the tower in the mirror at C, required the height of the tower AB?—*Ans.* 94.706 feet.

7. Suppose a staff DE, 9 feet long, to be fixed perpendicularly 290 feet from the bottom of an object AB, whose height is sought, and another staff FG, 5 feet long, 20 feet farther from the object, so that the summits of all the three are in the same straight line; what is the height of the object?—*Ans.* 67 feet.

8. Let AB be an inaccessible spire, whose height is required by the help of two staves, the one 9 feet, the other $5\frac{1}{4}$ feet long, at

the first station the staves are placed perpendicular, so that the summit of the spire may be seen over their tops, the distance between the staves is then found to be 8 feet; and at a second station, 120 feet from the shortest staff in the first station, the same staff is placed, and the longest betwixt it and the object, so that the summits of both and of the spire may be in the same straight line; here the distance between the staves is 14¾ feet; required the height of the spire*?—*Ans.* 71.917 feet.

9. Wanting to know my distance from an inaccessible object, suppose a tree, I took two stations, A and B, at the distance of 150 feet from each other, and in the same straight line with the tree; then from A, the station nearest the object, in a line perpendicular to AB, I measured AC, 160 feet, and set up a mark at the extremity C; then from B, the other station, in a line also perpendicular to AB, I measured the distance BD, 275½ feet, where I observed the tree and the mark C to be in the same straight line with the point D; required the distance of the tree from the station A?—*Ans.* 207.79 feet.

10. At the distance of 310 feet from a wall, the angle of elevation is observed to be 15° 40′; required the height of the wall?—*Ans.* 86.9424 feet.

11. Required the height of a tower, and also the distance of its summit from the place of observation, 120 yards from the bottom of the tower, supposing the angle of elevation at the same place to be 30° 15′?—*Ans.* Height of the tower 69.98 yards. Distance of the summit, 138.915 yards.

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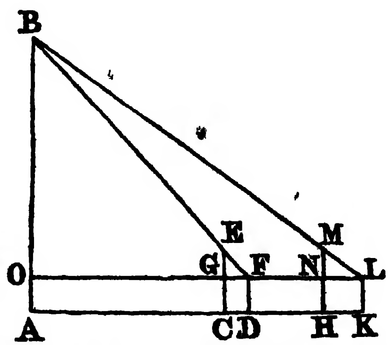
* *Solution.* From similar triangles, we have,
LO . OB LN NM, and
FO : OB : FG : GE or NM,
hence LO : FO . LN FG,
therefore, by division,
LF FO : : LN—FG : FG,
but FO : OB : · FG : MN;
therefore,
LN—FG : LF MN : OB.
Hence, since the three first terms of this proportion are given, the fourth, OB, may be found; to which the length of the shortest staff LK being added, we obtain the whole height AB.



Instead of two staves, the geometrical square may be used for finding the ratio of LO to OB, and of FO to OB; in which case, the solution is similar to the above.

12. What is the perpendicular height of a hill, whose angle of elevation near the bottom is 33° 26′, and 125 yards farther off, 24° 30′, and what the distance of the perpendicular from the first station?—*Ans.* Height of the hill, 183.92 yards. Distance of the perpendicular, 278.58 yards.

13. To find the height of an object on the top of a hill, there are given the elevation of the hill 40°, and the elevation of the object at the same station 51°. Also the elevation of the object at another station, 100 yards distant, in a direct line from the former, 33° 45′?—*Ans.* 46.666 yards.

14. From the top of a tower, whose height was 120 feet, I took the angles of depression of two objects, which lay in a direct line from, and upon the same horizontal plane with, the bottom of the tower. The depression of the nearer object, was found to be 57°, and that of the farther, 25° 30′; what is the distance between the two objects, and what is the distance of each from the bottom of the tower?—*Ans.* The distance of the nearer object, 77.93 feet. Distance of farther object, 251.59 feet. Distance between the objects, 173.66 feet.

15. Being on the side of a river, and wanting to know the distance of a house on the other side, I measured 268 yards in a right line, by the side of the river, and found that the two angles, one at each end of this line, subtended by the other end and the house, were 38° 40′, and 92° 46′; what was the distance between each station and the house?—*Ans.* Distance from the one station, 354.38 yards. Distance from the other station, 221.67 yards.

16. Wanting to know the breadth of a river, I measured a base of 250 yards in a straight line close by one side of it; and at each end of this line, I found the angles subtended by the other end, and a tree close to the bank on the other side of the river, to be 53°, and 79° 12′; what was the perpendicular breadth of the river?—*Ans.* Breadth = 264.74 yards.

17. To find the distance between two inaccessible objects A and B; at the two stations C and D, 300 yards distant, the angles ACD 95° 20′, and ACB 37°, at the one station, and the angles BDC 98° 45′, and BDA 45° 15′ at the other station are given?—*Ans.* 479.79 yards.

18. Required the distance between two parish churches on the farther side of a river, supposing the straight line between them to subtend angles at two stations 60° 20′, and 67°, the distance between these stations being 430 yards; also the other angles at the stations 42° and 45° 15′ respectively?—*Ans.* 804.4 yards.

19. The ship Speedwell sailing along the coast, the master observed a cape which bore from him N. W. by N., and another headland bearing N.N.E.; then standing away E.N.E.½E. 21 miles, he found the first bore from him W.N.W., and the second N. by W. ½W., required the distance of the cape from the headland?—*Ans.* 28.085 miles.

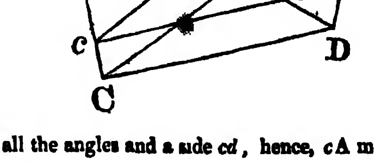
20. To find the height of an object on the top of a hill; there are given the elevation of the hill 26° 30′, the elevation of the object at the same station 47°, and the elevation of a second station in the same vertical plane, but farther from the object, 13° 20′; also the distance between the stations 130 yards, and the angle at the second station contained by a straight line to the first, and another to the summit of the object 38° 40′?—*Ans.* 86.0878 yards.

21. From two stations on the side of a hill, distant from each other 160 yards, the angles of depression of the bottom of a tower on an opposite hill are observed to be 28° 30′ and 35°, also, at the upper station, the depression of the summit of the tower is 7° 20′, and its elevation at the under station 13° 30′, and all the angles are taken in the same vertical plane, required the height of the tower * ?—*Ans.* 224.88 yards

* The problem may be stated in general terms, thus

Given the distance between two stations A and B, with the angles observed at the objects C and D, to find the distance between the objects C and D

Solution Draw the line *cd* of any convenient length, and at the points *c*, *d*, make the angles *dcb*, *bc*A equal to the angles observed at C, and the angles *cd*A, A*db* equal to the angles observed at D, join A*b*, and in A*b*, produced if necessary, take AB of the given length through the point B draw BD parallel to *bd*, and meeting A*d*, or A*d* produced, in D also through D draw DC parallel to *dc*, meeting A*c* or A*c* produced in C, and CD is the distance required. Join BC, and let us assume *cd* equal to any convenient number, suppose 1000. Then,



1. In the triangle *cd*A are given all the angles and a side *cd*, hence, *c*A may be found

2. In the triangle *cdb* are given all the angles and the side *cd*, whence *cb* may be found.

3. In the triangle A*bc* are given the sides A*c* and *cb*, and the contained angle A*cb*, whence A*b* may be found.

Lastly, The figures A*bdc*, ABDC, are similar; therefore, by construction, A*b* AB . . *cd* . CD, and, consequently, CD may be determined.

22. Wanting to know the height of, and my distance from, an object on the other side of a river, which seemed to be on a level with the place where I stood, close by the side of the river, and not having room to measure backward on the same plane, because of the immediate rise of the bank, I placed a mark where I stood, and measured in a direction from the object, up the ascending ground, to the distance of 264 feet, where it was evident that I was above the level of the top of the object; there the angles of depression were found to be, viz. of the mark left at the river's side, 42°; of the bottom of the object, 27°; and of its top, 19°; required, then, the height of the object, and the distance of the mark from its bottom?—*Ans.* Height, 57.27 feet. Distance, 150.5 feet.

23. If the height of the mountain called the Peak of Teneriffe be $2\frac{1}{2}$ miles, and the angle taken at the top of it, as formed by a plumb-line, and a line conceived to touch the earth in the horizon, or farthest visible point, be 87° 58′, it is required from hence to determine the diameter of the earth, and the utmost distance that can be seen on its surface from the top of the mountain, supposing the form of the earth to be perfectly spherical?—*Ans.* Distance, 140.876 miles. Diameter, 7936 miles.

24. From an eminence 360 feet in height, the angle of depression of the top of a steeple on the same horizontal plane was found to be 41°, and of the bottom 54°, required the height of the steeple?—*Ans.* 132.63 feet.

25. From the summit of a hill, the angles of depression of the top and bottom of a tower on the plane below, whose height is 120 feet, were observed to be 30° and 33° respectively; the height of the hill is required?—*Ans.* 1081.5 feet.

26. Required the height of a spire, supposing its elevation at the first station to be 15° 40′, and the horizontal angle at the first station, between the spire and the second station 76°, and the horizontal angle at the second station 63° 40′; also the distance between the two stations 550 links of a Scotch chain?—*Ans.* 213.595 links, or 158.06 feet.

27. On the top of a monument whose height is 60 feet, stands a statue 12 feet high; at what distance from the monument may the statue be viewed under an angle of 3°, and what is the greatest

angle under which it can be viewed* ?—*Ans.* Distance from the bottom of the monument, at which the object will be seen under an angle of 3°, 208.23 feet, or 20.75 feet; and the greatest angle under which, it can be seen from a point in the horizontal line, is 5° 12′ 57″.

28. The elevation of a tower at one station is 20° 45′, and at another, 60 yards from the former, but not in the same straight line with it and the foot of the tower, 19° 40′; and at a third sta-

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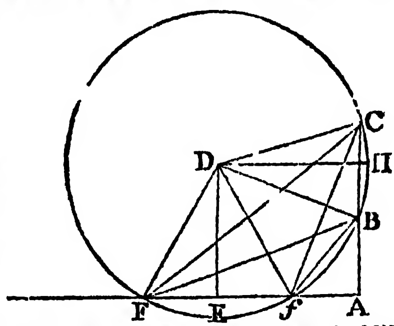
* *Solution* Let AB be the height of the monument, and BC the height of the object upon its top. Upon BC describe an arch of a circle that will contain an angle equal to the angle under which the object is supposed to be seen. If the problem be possible, this circle will either touch the horizontal line AF, or will cut it in two points.

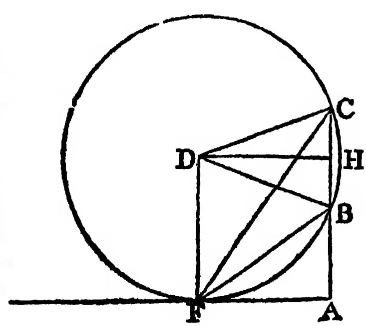


First let it cut it in F and f, then if FB, FC, also fB, fC, be joined, it is evident that the line BC subtends, at each of the points F and f, an angle equal to the angle proposed From D, the centre of the circle, draw DE, DH perpendicular to Ff, BC, join DB, DC

Then it is evident that the angle HDC, being equal to the given angle CFB or CfB, is also given. But BH is also given, therefore DH or AE may be determined.

Now, $AE^2 = FA \cdot Af + EF^2 = CA \cdot AB + EF^2$. From this equation, we find,

Ef or $EF = \sqrt{AE^2 - CA \cdot AB}$. Hence EF or E$f$, and consequently AF or Af may be determined.



Next, to find the greatest angle under which BC can be seen Join DF or Df, when BFC or BDH is the greatest possible, it is evident that the angle DBH will be the least possible; and the line DH, as also DB or DF, the radius of the circle, will be the least possible But DF will be the least possible when it is perpendicular to AF, in which case AF will be a tangent to the circle at the point F, therefore, when the angle BDH or BFC is the greatest possible, BD = DF or AH, and is therefore given. Hence, it is evident, that the angle BFC = BDH may be determined.

tion, 30 yards distant from each of the former, 21° 48′; required the height of the tower* ?—*Ans.* 28.013 yards.

29. At three stations in the same straight line with the foot of a tower, the angles of elevation are such, that the first is double, and the second the complement of the third; also, the distance of the first and second stations is 27, and the distance of the second and third 100 yards; required the height of the tower† ?—*Ans.* 116.74 yords.

30. Let A, B, and C be three towns, whose distance from the spire O is sought, supposing A 31.4 miles distant from B, and 28

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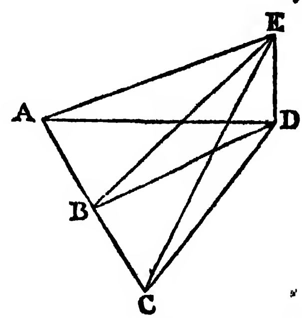
* *Solution* Let A, B, C, be the three stations which are in the same straight line, and such that AB = BC Let DE be the object whose height is to be measured Join AD, BD, and CD, then, DE being perpendicular to the plane of the triangle ADC, the angles ADE, BDE, and CDE are right-angles Join also AE, BE, and CE, then, Rad : tang EAD :: AD : DE; hence,



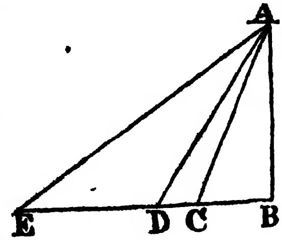
$$DE = AD \times \text{tang } EAD$$

In like manner, BD = ED × tang. BED = tang EAD × cotan EBD × AD, and CD = ED × tang CED = tang EAD × cotan ECD × AD But $AD^2 + DC^2 = 2AB^2 + 2BD^2$ Hence we have $AD^2 + \text{tang}^2 EAD \times \text{cotan}^2 ECD \times AD^2 = 2AB^2 + 2\,\text{tang.}^2 EAD \times \text{cotan}^2 EBD \times AD^2$; from which equation, we find,

$$AD = \sqrt{\frac{2AB^2}{1 + \tan.^2 A \times \cot.^2 C - 2\tan.^2 A \times \cot^2 B}}.$$

From this expression, AD is easily determined Then, in the triangle ADE, the angle EAD, and the side AD being known, the side DE may be found.

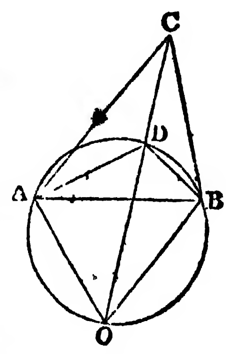
† *Solution.* Since the angle ACB is double AEB, and also equal to AEB + EAC, the angle AEB is equal to the angle EAC, hence AC = CE Now, ADB being the complement of AEB, is equal to EAB, hence the triangles AEB, ADB are similar, and EB : AB :: AB : BD; hence $AB^2 = EB \times BD$ But $AB^2 = AC^2 - CB^2 = (AC + CB) \times (AC - CB) = EB \times (EC - CB)$ Therefore EB × (EC—CB) = EB × BD, wherefore EC — CB = BD = CB + CD, and 2CB = CE — CD = ED; so that BC is given; and, therefore, AB being equal to $\sqrt{EB \times BD}$, may be determined.



miles from C, and B 32.6 miles from C; also the angles at the spire, AOB 48° 58′, and BOC 23° 6′; what is the distance between the spire and each of the three towns* ?—*Ans.* AO = 41.34 miles; BO = 30.81 miles; CO = 58.62 miles.

31. Suppose the distances between three trees to be respectively 267, 346, and 209 yards, and the angles, at a point within the triangle, subtended by those distances, 128° 40′, 140°, and 91° 20′ respectively, required the distance from this point to each of the trees† ?—*Ans.* 104.05, 189.33, and 178.86 yards respectively.

32. A gentleman wanted to know the contents of a square field, but had forgotten the dimensions, only he remembered that the distance between a large oak which grew within the field, and three



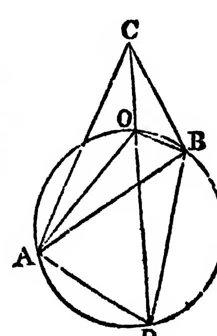
* *Solution* Let A, B, and C be the positions of the three towns, make the angle ABD = AOC, and BAD = BOC Having thus determined the point D, through the points A, B, D, describe a circle, join CD, let DC be produced till it meet the circle AOBD in the point O, and O will be the position of the spire Then,

1 In the triangle ABC, the three sides being given, the angle BAC may be found

2 In the triangle ABD, the angles DAB, DBA, and the side AB being given, the side AD may be found.

3. In the triangle ACD, the sides AC, AD, and the included angle CAD, are given, hence the angle ACD may be found

Lastly, In the triangle AOC, the angles and the side AC are given, hence OA and OC may be found, and in the triangle AOB, the sides AB, AO, and the angle AOB are given, hence OB may be found.



† *Solution.* Let A, B, and C be the positions of the three trees. At the extremities of the line AB, one of the sides of the triangle, make the angles BAD, ABD, equal to the supplements to the angles BOC, AOC, respectively Through the points A, B, D, describe the circle AOBD; join DC, and DC will intersect the circle in O, the place of observation

The method of computation, is sufficiently obvious, from that of the preceding example.

of its corners in a successive order, were 116, 156, 166 yards; required the contents of the field?*—*Ans.* 8.136 acres.

* *Solution.* The problem may be stated in more general terms, thus

Given the distances of three of the angles of a square, from a given point, to construct the square and determine its area.

Construction. Let P be the given point, and let *ap*, *bp*, *cp*, be the given distances of the three angles of the square, from the point P, and let *ap* be the distance of that angle which lies between the other two.

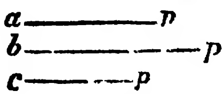


Fig 1

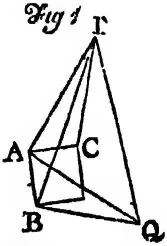
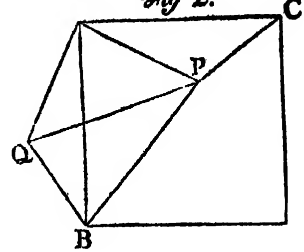


Fig 2.



From P draw a straight line PA, equal to *pa*, and draw AQ perpendicular to AP, and equal to AP On P and Q as centres, with radii equal to *bp* and *cp*, describe arcs intersecting each other in B. Join AB, and AB shall be a side of the square required.

Draw AC perpendicular to AB, and in such a direction that the angle PAC may be of the same kind as the angle QAB, (that is, so that the angles PAC, QAB, may be either both acute or both obtuse), take AC equal to AB, and A, B, C shall be the three angles of the square, whose distances from P are equal to *ap*, *bp*, *cp*, respectively, as required.

Demonstration. Join QB, PB, PC. It is evident from the construction, that the distances of the points A and B from P are equal to *ap* and *bp* respectively; therefore, it only remains to prove, that the distance of P from C is equal to the remaining line *cp*.

Because the angle QAC in fig. 1., or the angle PAB in fig 2, is common to each of the right angles PAQ, CAB, therefore, taking it from both, the remaining angles PAC, QAB, are equal Now, by construction, PA is equal to QA, and CA to BA, therefore, the triangles PAC, QAB, are equal, and their remaining sides PC, QB are equal. But, by construction, QB is equal to *cp*; therefore, PC is equal to *pc*, as was to be demonstrated.

Calculation of AB, *the side of the square.*

1 Join PQ; then, in the right-angled triangle QAP, because the equal sides QA, PA are given, the hypotenuse PQ is given

2 In the triangle PBQ, the three sides PB, BQ, and PQ being given, the angle BPQ may be found

3. The angle APQ is given (for it is an angle of 45°), therefore, the angle APB (= APQ − BPQ, fig. 1., or = APQ + BPQ, fig 2.), is given.

33. From the top of a tower 130 feet in height the angle subtended by a line joining two objects A and B on the plane below was found by the sextant to be $64^{0}\ 10'$; the depression of the object A was $6^{0}\ 20'$, and that of B, $8^{0}\ 46'$; required the distance of each object from the bottom of the tower; also the bearing of the object B from it, supposing A to lie S.E. by S. $\frac{1}{2}$E., and B to be situated between south and west?—*Ans.* Distance of A, 1171.28 feet. Distance of B, 842.99 feet. Bearing of B from the tower S. $25^{0}\ 22'\ 54''$ W., or S.S.W.$\frac{1}{4}$W. nearly.

34. From an elevated station A the depression of a distant object B was observed to be $1^{0}\ 6'$, and the elevation of another object C, $1^{0}\ 30'$, also the angle subtended by the straight line BC, $97^{0}\ 36'$, required the corresponding horizontal angle?—*Ans.* $97^{0}\ 34'\ 30''$.

35. The distance between two stations B and C, on a declivity, is 220 yards. At B the oblique angle ABC between C and an object A on the top of the hill was found by a sexant to be $77^{0}\ 8'$, and at C the angle ACB between B and A $62^{0}\ 18'$, also at C the elevation of the station B was found to be $8^{0}\ 32'$, and that of the object A $32^{0}\ 12'$; required the horizontal distances of the object from the stations, and its height above each of them?—*Ans.* Distance from C, 279.1 yards. Height above it, 175.7 yards. Distance from B, 263.1 yards. Height above it, 143.1 yards.

36. From a station on the surface of the earth the apparent elevation of the summit of a mountain was observed to be $2^{0}\ 7'$, and, again, from the summit of the mountain the apparent depression of the station was found to be $2^{0}\ 24'\ 10''$, also the arch intercepted on a great circle of the earth, between the station and the point immediately under the summit, had previously been computed at 22.064 miles; required the true height of the mountain, supposing the circumferance of the earth to be 24856 miles?—*Ans.* 4597 feet.

Lastly, In the triangle APB, the sides AP and PB being given, and the included angle APB known, the remaining side AB (that is, the side of the square) may be found.

Note.—It appears from the construction, that the same *data* will afford two solutions to the problem; for the triangle PBQ may be on either side of the line PQ. It also appears, that $bp + cp$ must not be less than $\sqrt{2} \times ap$; for BP + BQ cannot be less than PQ, that is, less than $\sqrt{2AP^2}$, or $\sqrt{2} \times AP$.

APPENDIX

TO THE

MENSURATION OF HEIGHTS AND DISTANCES,

CONTAINING

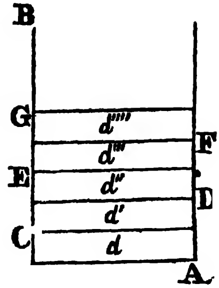
The application of Logarithms to the Mensuration of Heights by the Barometer.

1. It is found, by experiment, that the atmosphere or the air which surrounds this earth, is capable of being compressed into a much smaller space than that which it naturally occupies, that it is condensed, in proportion to the force by which it is compressed; that, like all other bodies with which we are acquainted, it gravitates or has weight; and that it is of an elastic or springing nature, the force of its elasticity being equal and opposite to the compressing force.

2. From these properties, it follows, that, the weight or pressure of the superincumbent air being diminished as we ascend in the atmosphere, the density of the air will also be diminished. Let us inquire by what law this diminution of the density is regulated.

Let AB be a column of air perpendicular to the surface of the earth, and reaching to the farthest part of the atmosphere. It is evident, that whatever is proved with respect to this portion of the atmosphere, will hold with respect to every other contiguous portion, and consequently with respect to the whole atmosphere.

Suppose, now, that this column is divided by planes parallel to the horizon, into a vast number of strata, AC, CD, DE, EF, &c. of equal thickness: it is evident, that if we conceive the number



of these strata to be indefinitely great, we may, without any sensible error, suppose the air in any one of them, to be of an uniform density throughout. Let d, d', d'', d''', d'''', &c. represent the densities of the different strata AC, CD, DE, EF, &c. respectively, from the surface upwards.

Then, since the density of the air is always as the force by which it is compressed, and, since the force by which the air in every part of the atmosphere is compressed, must be proportional to the density (or quantity) of the superincumbent air, it follows that

$$d : d' :: d' + d'' + d''' + d'''' \&c : d'' + d''' + d'''' + \&c.$$

therefore, $d - d' : d' :: d' : d'' + d''' + d'''' + \&c$

In like manner, $d' : d'' :: d'' + d''' + d'''' + \&c. : d''' + d'''' + \&c.$

hence $d' - d'' : d'' :: d'' : d''' + d'''' + \&c.$

But $d' : d'' :: d'' + d''' + d'''' + \&c. : d''' + d'''' + \&c.$

and alternately $d' : d'' + d''' + d'''' + \&c. :: d'' : d''' + d'''' + \&c.$

Hence it follows, that $d - d' : d' :: d' - d'' : d''$

and therefore we have $d : d' :: d' : d''$.

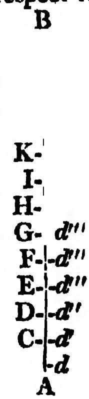
In the same manner, it may be proved, that $d' : d'' :: d'' : d'''$, and so on; that is, the densities or quantities of air, in the equal strata AC, CD, DE, &c. are in geometrical progression. It is also to be observed, that the heights of these strata above the surface of the earth, are in arithmetical progression. And, although we have supposed the parts into which the column of air is divided, to be indefinitely small, their number having been supposed indefinitely great, yet it holds, in general, that, if the altitudes above the surface of the earth be taken in arithmetical progression, the densities of the air at those altitudes will be in geometrical progression decreasing, for, in any geometrical series, it is evident, that any of its terms, equally distant from each other, are also in geometrical progression, since ratios compounded of the same number of equal ratios are equal to one another.

3. If any arithmetical series, whose first term is 0, be applied to a geometrical series, whose first term is unity, in such a manner that the first term of the one may correspond to the first term of the other, and each of the succeeding terms of the former, to each of the succeeding terms of the latter in their order, as in the following example,

0, 1, 2, 3, 4, 5, 6, 7, &c.
1, 3, 9, 27, 81, 243, 729, 2187, &c.

it appears, from the nature of logarithms, which has already been explained, that the terms in the arithmetical series will be the logarithms of the corresponding terms in the geometrical series, according to some particular system. For, in the equation $a = r^x$, where a represents any number, x the logarithm of that number, and r the radical number of the system, it is evident, that, in order that the values which may be given to a may constitute a geometrical series, the corresponding value of x must constitute an arithmetical series; and that, since r is susceptible of any affirmative value whatever, except of unity, the successive values of a and x may become equal to the terms of any geometrical and arithmetical series whatever.

4. To apply this to what has been demonstrated with respect to the atmosphere: let AB be a straight line reaching to the farthest part of the atmosphere, and perpendicular to the surface of the earth. In AB take AC, CD, DE, &c. equal to each other, and let d, d', d'', d''', d'''', &c. represent the densities of the air at the points A, C, D, &c.



Suppose now, that d''''' is the unit with which we compare all the other densities, then, it is evident, from what has been just shewn, with respect to geometrical and arithmetical series, that, if GF, GE, GD, GC, GA, and d'''', d''', d'', d', d be expressed in numbers, the former will be the logarithms of the latter, according to some particular system. It is likewise evident, that the logarithm of the density at any point above G will be negative.

5. From what has now been shewn we may infer, that

$$AC = (GA - GC) = \text{Log. } d - \text{Log. } d'.$$

Hence, if we can determine the densities at the points A and C, and likewise the *modulus* of the system of logarithms which is adapted to the atmosphere, we shall be enabled to find AC the perpendicular height of the point C, above the surface A.

6. The barometer enables us to determine, at any time and place, the density of the air. For, the density being always as the pressure, and the height of the mercury in the barometer being also as the pressure, it is evident, that the height of the column of mercury will always be proportional to the density, and of course will serve as a measure of it. Suppose B, therefore, to represent

the height of the barometer at the point A, and b that at the point C, and we shall have,

$$AC = \text{Log. B} - \text{Log. } b$$

7. With regard to the modulus of the system of logarithms to be employed, it can be determined only by experiment.

It may be determined as follows Suppose the height of the point C above A, the surface of the earth to have been ascertained geometrically to be equal to 96 61 fathoms. Let the mercury in the barometer placed at A, the surface of the earth be supposed to stand at the height of 30 inches, then, if the barometer be placed at C, the mercury in the tube will be found to sink down to 29 34 inches, the temperature of the air being supposed to be uniformly 32°.

From this, it appears, that, according to the system of logarithms applicable to the atmosphere, Log. $\frac{30}{29.34} = 96.61$. But, according to the common system of logarithms, Log $\frac{30}{29\ 34} = 0\ 009661$, and it has been shewn, in treating of the nature of logarithms, that the logarithms of the same number, according to different systems, are to each other as the moduli of these systems, therefore, since the modulus of the common system of logarithms has been found to be equal to .4342945, we have,

0.009661 : 96.61 . . .4342945 . 4342 945 = the modulus of the system of logarithms, which is applicable to the atmosphere.

From this, it appears, that the temperature of the air being supposed to be uniformly 32°, if we employ the English fathom as the measuring unit, the modulus of the system of logarithms, which is applicable to the atmosphere, is equal to the modulus of the common system, multiplied by 10,000, from which we may infer, that

$$AC = 10{,}000\ (\text{Com. Log. B} - \text{Com. Log. } b).$$

8. The air, however, is not all of the same temperature as we ascend in the atmosphere, nor is it at all times at the temperature of 32°, as we have here supposed. It, therefore, becomes necessary to apply a correction to the elevation, as found by the above formula, except when the medium temperature of the air happens to be 32°. A correction is also required for the temperature of the mercury itself, for that being seldom the same in the two positions of the barometer, a reduction is necessary to bring the mercury to the same temperature at both stations.

9. In the first place, it is found by experiment, that quicksilver expands nearly $\frac{1}{10,000}$ part of its whole bulk for every degree of Fahrenheit's thermometer. If, therefore, we put m and n to represent the temperatures of the mercury in the barometer, at the points A and C, as indicated by the thermometer which is attached to the barometer, respectively, it is evident, that $b + \frac{m - n}{10,000}\,b$, will express the barometrical altitude at C, when reduced to what it would be, if the temperature of the mercury were, at both stations the same.

10. Again, it is known that air expands nearly .00244 of its whole bulk, for every degree of Fahrenheit's thermometer. Let us suppose, that the temperature of the air is increased so as to exceed 32°, by p degrees, and let C be the point whose perpendicular altitude above A, the surface of the earth, is required. It is evident, that, if the temperature be supposed to be reduced to 32°, the thickness of the stratum of air which lies between the points A and C will be diminished by a quantity equal to .00244 of the whole thickness of the stratum, multiplied by the number of degrees, by which the mean temperature of the air exceeds 32°. Hence the portion of air which formerly occupied the space between A and C will now occupy a less space, suppose that between A and c; and, therefore, a portion of the air which was formerly situated above the point C will be allowed to descend below C, so that the pressure upon the barometer at C being thus diminished, the column of mercury in the tube will sink somewhat, but if the barometer be brought down to the point c, it is evident, that the mercury will again ascend to its original height. The height of the barometer at A will continue the same, as the pressure is not changed, so long as the quantity of superincumbent air remains the same.

From this it appears, that the height of the mercury will be the same when the barometer is placed at c, the temperature being supposed 32°, as when it is placed at C, the temperature being $(32 + p)°$.* Hence it follows, that, since in determining the modulus of the system of logarithms, which is applicable to the atmosphere, we supposed the temperature of the air to be uniformly 32°, if the temperature be increased, the formula AC = 10,000 (Com. Log. B — Com. Log. b) will give, not the real distance between the points A and C, but only the distance

between A and c; to which, if we add Cc, we shall have AC, the altitude required. It is evident, that, when the temperature of the air is below 32°, the correction for the true altitude must be subtracted from, instead of being added to, the approximate altitude found by the above formula.

Let t represent the temperature of the air at the lower station, and t' that at the higher station; then will $\frac{t+t'}{2}$ be the mean temperature, and may be taken for what the temperature would be, were it uniformly the same throughout the whole. Thus, we have, upon the whole, the true elevation expressed by this formula:

$$AC = 10{,}000\left(\text{Com. Log. } B - \text{Com. Log. } \left(b + \frac{m-n}{10{,}000}b\right)\right) \times \left(1 + .00244\left(\frac{t+t'}{2} - 32^\circ\right)\right).$$

This formula is applicable, whether the temperature be above or below 32 degrees. If the centigrade thermometer is used, because the beginning of the scale agrees with the temperature of 32° of Fahrenheit's thermometer, the formula becomes more simple, and if the expansion for air and mercury be both adapted to the degrees of this scale, the height is expressed as follows,

$$AC = 10{,}000\left(\text{Com. Log. } B - \text{Com. Log. } (b + .00018\,(m-n)\,b)\right) \times \left(1 + .00441\left(\frac{t+t'}{2}\right)\right).$$

11. In practice it is not necessary that the two situations of the barometer should be vertical to each other; for, though their horizontal distance be considerable, it does not produce any material alteration.

EXAMPLES.

1. It is required to determine the perpendicular height of a hill, from the following observations?

	Altitude of the Barometer.	*Temperature of the Mercury.*	*Temperature of the Air.*
At the foot of the hill,	29.56 inches.	63°	56°
At the summit of the hill,	28.27 inches.	54°	48°

First, $b + \frac{m}{10,000} b = 28.27 + 0.025 = 28.295 =$ Reduced barometrical altitude at upper station.

Com. Log. 29.56 is 1.470704
Com. Log. 28.295 is 1.451710

$$\begin{array}{r} 0.018994 \\ 10000 \\ \hline 189.94 \text{ fathoms.} \\ 6 \end{array}$$

Approximate height is 1139.64 feet.

Again, $\frac{t + t'}{2} - 32^0 = 52^0 \quad 32^0 = 20^0.$

Hence $(1 + 0.00244 \times 20) \times 1139.64 = 1139.64 + 55.61 =$ 1195 25 feet $=$ corrected height of the hill.

2. It is required to determine the perpendicular distance between two situations, where the following observations were made?

	Barom altit	*Attached therm.*	*Detached therm*
Lower place,	29.88 inches.	28^0	26^0
Upper place,	29.03 inches.	26^0	24^0

—*Ans*. 733.89 Feet.

3. Let the height of the barometer, at two places, be 28.65 and 29.9 inches, also let the temperature of the mercury, and of the air at both places, be 32^0; required the perpendicular distance between those two places?—*Ans*. 1112.76 feet.

4. Required the height of a mountain, at the bottom of which, the height of the mercury in the barometer was 30.5 inches, the temperature of the air and of the mercury being 17^0, and at the top the height of the mercury was 28.12 inches, the temperature being $11^0.4$, supposing the temperatures estimated according to the centigrade scale?—*Ans*. 2222.13 feet.

12. The following method enables us to determine altitudes by means of barometrical observations, without the assistance of logarithmic tables.

We have already found that, B being taken to represent the barometrical altitude at the lower station A, and b to represent that at the upper station C, the perpendicular altitude AC is equal to Log. B — Log b = Log. $\frac{B}{b}$. (§ 6).

It is evident, that the quantity $\frac{B}{b}$ will be but a little greater than unity for those heights which we may most frequently have occasion to determine. Let $\frac{B}{b}$ be put equal to $1 + y$, supposing y a fraction, then, we shall have, (Logar § 7.)

$$\text{Log } \frac{B}{b} = \text{Log } (1 + y) = A\left(y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \text{\&c.}\right)$$

Now, if from $1 + y$ we subtract its reciprocal $\frac{1}{1+y}$, which, by division, is found to be equal to $1 - y + y^2 - y^3 + \text{\&c.}$, we obtain,

$$1 + y - \frac{1}{1+y} = 2y - y^2 + y^3 - \text{\&c.} = 2\left(y - \tfrac{1}{2}y^2 + \tfrac{1}{2}y^3 - \text{\&c.}\right)$$

And, by multiplying both sides of this equation by $\frac{1}{2}$A, we find,

$$\tfrac{1}{2}A\left(1 + y - \frac{1}{1+y}\right) = A\left(y - \tfrac{1}{2}y^2 + \tfrac{1}{2}y^3 - \text{\&c.}\right)$$

But, it is evident, that the one side of this last equation coincides with the series expressing the logarithm of $1 + y$ in the first and second terms, and that the third term of the one differs but little from the third term of the other Hence, when y is a small fraction, the result of the one will be nearly equal to the result of the other, from which it follows, that y being a small fraction, we will have,

$$\text{Log } (1 + y) = \tfrac{1}{2}A\left(1 + y - \frac{1}{1+y}\right)$$

But the altitude AC is equal to Log. $\frac{B}{b}$, or Log. $(1 + y)$.

Therefore, substituting $\frac{B}{b}$ for $1 + y$, we have,

$$AC = \tfrac{1}{2}A\left(\frac{B}{b} - \frac{b}{B}\right).$$

It is to be observed, that $\tfrac{1}{2}A$ has already been determined (§ 7) to be equal to $\frac{4342.9}{2} = 2171.4$ fathoms.

13. As an example of the application of this formula, let us resume the last example in § 11

The barometrical altitudes being reduced, as before, we have,

$$\tfrac{1}{2}A\frac{B}{b} = \frac{2171.4 \times 29.56}{28.295} = 2268.5, \text{ and}$$

$$\tfrac{1}{2}A\frac{b}{B} = \frac{2171.4 \times 28.295}{29.56} = 2078.5.$$

Hence AC $= 2268.5 - 2078.5 = 190$ fathoms, the approximate altitude, nearly the same as before, which, being properly corrected for the expansion of the air, will give the true altitude required.

14. The method of measuring altitudes by means of barometrical and thermometrical observations, notwithstanding the attention that has been paid to the subject, has not yet attained such a degree of perfection as to supersede geometrical or trigonometrical measurements But the facility and expedition with which it is performed, renders it extremely useful when no very great degree of accuracy is required

SPHERICAL TRIGONOMETRY.

§ I. DEFINITIONS

I. Any circle of a sphere, whose plane passes through the centre, is called a great circle of the sphere.

Cor. All great circles of a sphere are equal, and any two of them bisect one another.

II. The poles of a great circle of a sphere are the two points in which the straight line drawn through the centre, perpendicular to the plane of the circle, meets the surface of the sphere.

Cor. The arch of a great circle, between either pole and the circumference of another great circle, is a quadrant.

III. A spherical angle is an angle on the surface of the sphere, contained by the arches of two great circles which intersect one another, and is the same with the inclination of the planes of these great circles.

Cor. The measure of a spherical angle is the intercepted arch of a great circle whose pole is the angular point.

IV. A spherical triangle is a figure upon the surface of a sphere, comprehended by three arches of three great circles, each of which is less than a semicircle.

2. Any triangle, whether spherical or plane, consists of six parts, namely, the three sides and the three angles.* The object of

* In a right-angled triangle, the right angle being constant, five parts only are considered. A spherical triangle may have three right angles; and then each of its sides is a quadrant; or it may have two right angles, then each of the sides opposite to these angles is a quadrant, and the remaining angle and its opposite side are both measured by the same number of degrees. It is only necessary to consider such triangles as have but one right angle

Spherical Trigonometry is to resolve the following problem. Having given any three of the six parts of a spherical triangle, to determine the other three parts. In the solution of plane triangles, it was found to be a necessary condition that one of the given parts should be a side, because otherwise the triangle could have no determinate magnitude. But in spherical triangles this condition is not required, because we consider, not the absolute magnitude of the sides, but the ratio to a quadrant, or what is the same thing, the number of degrees, &c., which they contain.

3. In spherical, as in plane trigonometry, the general problem is, for convenience of calculation, usually divided into two; according as the triangle has or has not a right angle

Solution of Right-angled Spherical Triangles.

4. The solution of right-angled spherical triangles depends on the following theorems.

THEOREM I.

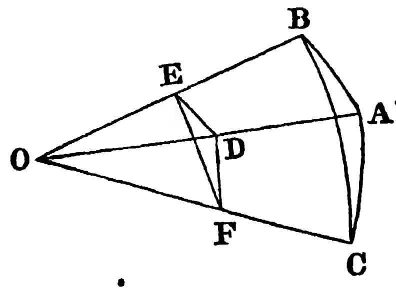
In every right-angled spherical triangle, the radius is to the sine of the hypotenuse, as the sine of either of the oblique angles is to the sine of the opposite side.

Let ABC be a spherical triangle, right-angled at A, then,

Rad. : Sin. BC :: Sin. B : Sin. AC.

From O the centre of the sphere draw the radii OA, OB, OC, in OC take OF equal to the radius in the tables, and from F draw FD perpendicular to OA: the line FD will also be perpendicular to the plane OAB, because by hypothesis the angle at A is a right angle, and the two planes OAB, OAC are therefore perpendicular to each other. (Def. III.) From the point D draw DE perpendicular to OB, and join EF: the line EF will also be perpendicular to OB; so that the angle DEF will be the mea-

sure of the inclination of the two planes OBA, OBC, and will therefore be equal to the angle B of the spherical triangle ABC. (Def. III.) Now, in the triangle DEF, right-angled at D, we have Rad. . Sin. DEF : : EF : DF, but the angle DEF = B, and since OF = Rad., we have EF = Sin. EOF = Sin. BC, and DF = Sin. AC. Wherefore Rad. Sin. B . . Sin. BC . Sin. AC, and by alternation,



Rad. Sin. BC Sin. B Sin. AC.

In the same manner it may be demonstrated that Rad . Sin. BC . . Sin. C : Sin. AB.

THEOREM II

In every right-angled spherical triangle radius is to the cosine of either of the oblique angles, as the tangent of the hypotenuse to the tangent of the side adjacent to that angle.

Let ABC be a spherical triangle, right-angled at A , (See figure of the preceding Theor), then,

Rad. : Cos. B : Tan. BC : Tan. AB.

For, making the same construction as in the last theorem, in the right-angled triangle DEF we have Rad. Cos. DEF . . EF . DE. But the angle DEF = B, EF = Sin. BC, OE = Cos. BC . also in the triangle OED right-angled at E, we have DE =

$$\text{OE} \times \frac{\text{Tan. DOE}}{\text{Rad.}} = \frac{\text{Cos. BC} \times \text{Tan. AB}}{\text{Rad.}}$$, therefore Rad. . Cos. B

$$. \quad \text{Sin. BC} \ . \ \frac{\text{Cos. BC} \times \text{Tan. AB}}{\text{R}} \quad \frac{\text{R} \times \text{Sin BC}}{\text{Cos. BC}} : \text{Tan. AB},$$

that is,

Rad. . Cos. B : : Tan. BC . Tan. AB.

In the same manner it may be demonstrated that Rad Cos. C . Tan. BC : Tan. AC

Cor Since Rad. . Cos. B : Tan. BC . Tan. AB; and because Cot. BC . Rad. . : Rad. : Tan. BC, by equality Cot. BC : Cos. B : : Rad. . Tan. AB. Also, because Cot. AB : Rad. : : Rad. . Tan. AB, therefore Cot. BC : Cos. B : : Cot. AB : Rad.

THEOREM III.

In every right-angled spherical triangle, radius is to the cosine of either of the sides, as the cosine of the other side is to the cosine of the hypotenuse.

Let ABC be a spherical triangle right-angled at A, (See figure of Theor I.), then,

Rad. . Cos. AB : Cos. AC . Cos. BC.

For, the same construction remaining as in the two preceding theorems, in the triangle ODF right-angled at D, and in which the hypotenuse OF = Rad we have OD = Cos. DOF = Cos. AC Again, in the triangle ODE right-angled at E, we have $OE = \frac{OD \times Cos\ DOE}{Rad.} = \frac{Cos.\ AC \times Cos.\ AB}{R}$ But in the triangle OEF we have OE = Cos. BC, therefore Cos. BC = $\frac{Cos.\ AC \times Cos\ AB}{R}$. Wherefore

Rad. . Cos. AB : Cos. AC Cos. BC.

THEOREM IV.

In every right-angled spherical triangle the cosine of either of the sides is to the radius as the cosine of the oblique angle opposite to that side is to the sine of the other oblique angle.*

Let A, B, C, denote the three angles of any right-angled spherical triangle, A being the right angle; and let a, b, c, denote the opposite sides respectively, then,

Cos. c : Rad. . Cos. C Sin. B.

* This theorem and the two following might be demonstrated directly, each by a particular construction, but it is preferable to derive them by the way of analysis, from the three preceding theorems.

From Theor. I. and II. we obtain $\text{Sin. B} = \frac{\text{R} \times \text{Sin. } b}{\text{Sin. } a}$, $\text{Cos. C} = \frac{\text{R} \times \text{Tan. } b}{\text{Tan. } a}$: hence, by dividing the latter of these equations by the former, we find

$$\frac{\text{Cos. C}}{\text{Sin. B}} = \frac{\text{Tan. } b}{\text{Sin. } b} \times \frac{\text{Sin. } a}{\text{Tan. } a} = \frac{\text{Cos. } a}{\text{Cos. } b}.$$

But by Theor. III. $\frac{\text{Cos. } a}{\text{Cos. } b} = \frac{\text{Cos. } c}{\text{R}}$, consequently

$$\frac{\text{Cos C}}{\text{Sin. B}} = \frac{\text{Cos. } c}{\text{R}}; \text{ wherefore}$$

$$\text{Cos, } c : \text{Rad.} . : \text{Cos. C} . \text{Sin. B}.$$

In the same manner it may be demonstrated that Cos. b : Rad. : . Cos. B : Sin. C.

THEOREM V.

In every right-angled spherical triangle the tangent of either of the oblique angles is to the radius, as the tangent of the side opposite to that angle is to the sine of the other side.

Let A, B, C, a, b, c, denote the angles and sides of a right-angled spherical triangle, as in the last theorem, then,

$$\text{Tan. B} : \text{Rad.} : : \text{Tan. } b : \text{Sin. } c.$$

From Theor. I. and II we have $\text{Sin. B} = \frac{\text{R} \times \text{Sin. } b}{\text{Sin. } a}$,

$$\text{Cos. B} = \frac{\text{R} \times \text{Tan. } c}{\text{Tan. } a}, \text{ hence we obtain}$$

$$\frac{\text{Sin. B}}{\text{Cos. B}} \text{ or } \frac{\text{Tan. B}}{\text{R}} = \frac{\text{Sin. } b \times \text{Tan. } a}{\text{Tan. } c \times \text{Sin. } a} = \frac{\text{R} \times \text{Sin. } b}{\text{Cos. } a \times \text{Tan. } c}.$$

But by Theor. III., we have $\text{Cos. } a = \frac{\text{Cos. } b \times \text{Cos. } c}{\text{R}}$; therefore we obtain

$$\frac{\text{Tan. B}}{\text{R}} = \frac{\text{R}^2 \times \text{Sin. } b}{\text{Cos. } b \times \text{Cos. } c \times \text{Tan. } c} = \frac{\text{Tan. } b}{\text{Sin. } c}:$$

Wherefore

Tan. B : Rad. : : Tan. b : Sin. c.

In the same manner it may be demonstrated that Tan. C : Rad. : : Tan. c : Sin. b.

Cor. Since Tan. B : Rad. : : Tan. b : Sin. c; and because Tan. B : Rad. : : Rad. : Cot. B, therefore Rad. : Cot. B : : Tan. b : Sin. c. Also, because Cot. b : Rad. : : Rad. : Tan. b; therefore Cot. b : Cot. B : : Rad. : Sin. c.

THEOREM VI.

In every right-angled spherical triangle, radius is to the cosine of the hypotenuse as the tangent of either of the oblique angles is to the cotangent of the other oblique angle.

Let A, B, C; a, b, c, denote the angles and sides of a right-angled spherical triangle, as before, then,

Rad. : Cos. a : : Tan. B : Cot. C : : Tan. C : Cot. B.

From Theor. V. we obtain

$$\frac{\text{Tan. B} \times \text{Tan. C}}{\text{R}^2} = \frac{\text{Tan. } b \times \text{Tan. } c}{\text{Sin. } b \times \text{Sin. } c} = \frac{\text{R}^2}{\text{Cos. } b \times \text{Cos. } c}.$$

But $\frac{\text{Tan. B}}{\text{R}^2} = \frac{1}{\text{Cot. B}}$, or $\frac{\text{Tan. C}}{\text{R}^2} = \frac{1}{\text{Cot. C}}$; and by Theor. III. we have Cos. b × Cos. c = Rad. × Cos. a; therefore

$$\frac{\text{Tan. B}}{\text{Cot. C}} \text{ or } \frac{\text{Tan. C}}{\text{Cot. B}} = \frac{\text{R}}{\text{Cos. } a}.$$ Wherefore

Rad. : Cos. a : : Tan. B : Cot. C : : Tan. C : Cot. B.

Cor. Because Rad. : Cos. a : : Tan. B . Cot. C, and Cot. B : Rad. . : Rad. : Tan. B, therefore Cot. B : Cos. a : : Rad. : Cot. C.

5. By a particular arrangement and classification of the parts of a spherical triangle, all the theorems employed in the solution of right-angled spherical triangles, are reduced to two, and included in one enunciation.*

* This is what is called the *Rule of the Circular parts.* It was invented by Napier, and is of great use in Spherical Trigonometry.

Definitions.

I. If, in any spherical triangle, we set aside the right angle, and consider only the five remaining parts of the triangle, viz. the three sides, and the two oblique angles, then, the two sides, which contain the right angle, and the complements of the other three, namely, of the two angles, and of the hypotenuse, are called the *Circular Parts.*

II. When, of the five circular parts, any one is taken for the *Middle Part*, then, of the remaining four, the two which are immediately adjacent to it on the right and left, are called *Adjacent Parts*, and the other two, each of which is separated from the middle by an adjacent part, are called *Opposite Parts*.

Proposition.

In any right-angled spherical triangle, the rectangle under the radius, and the sine of the middle part, is equal to the rectangle under the tangents of the adjacent parts, or to the rectangle under the cosines of the opposite parts.

6. It is to be remarked, that when an unknown part of a spherical triangle is determined by its sine only, there are two values of that part, and consequently two triangles which satisfy the conditions of the question. For the same sine which corresponds to an arch or angle, corresponds also to the supplement of that arch or angle, without any change in the direction of the sine to distinguish the arch or angle from its supplement. This is not the case, however, when an unknown part is determined by its cosine, tangent, or cotangent. For though the same *numerical value* of the cosine, tangent and cotangent corresponds to an arch, or angle, and its supplement, there is a difference of direction, marked by the positive or negative sign of the numerical value, which distinguishes the arch, or angle, and its supplement. If the cosine, tangent, or cotangent, by which an unknown part is determined, be positive, that part is less than 90°; but if the cosine, tangent, or cotangent,

be negative, the part determined by it is greater than 90°. The following general principles make it easy to determine from the given parts, whether the unknown parts are greater or less than 90°.

7. Let A, B, C, denote the angles of any right-angled spherical triangle, and a, b, c, the opposite sides respectively; then from Theor. IV., we have Cos. c : Rad. :: Cos. C : Sin. B, therefore

Cos. C $= \frac{\text{Sin. B}}{\text{R}} \times$ Cos. c. Now, since the multiplier $\frac{\text{Sin. B}}{\text{R}}$ is always positive, it follows from this equation that Cos. C will be positive or negative according as Cos. c is positive or negative. Hence we infer that,

In any right-angled spherical triangle, according as the sides are greater or less than quadrants, the opposite angles will be greater or less than right angles: and conversely.

8. From Theor. III. we have Rad : Cos. b :: Cos. c : Cos. a, therefore Cos. $a = \frac{\text{Cos. } b \times \text{Cos. } c}{\text{R}}$. From this equation, it is evident, that if Cos. b and Cos. c be both positive, or both negative, Cos. a will be positive, but if Cos. b and Cos. c have opposite signs, then Cos. a will be negative. Hence,

In any right-angled spherical triangle, if the sides be greater or less than quadrants, the hypotenuse will be less than a quadrant; but if one of the sides be greater, and the other less than a quadrant, the hypotenuse will be greater than a quadrant: and conversely.

9. From Theor. VI. we have Rad. : Cos. a :: Tan. B : Cot. C, therefore Cos. $a = \frac{\text{Cot. C}}{\text{Tan. B}} \times$ Rad. From this equation, it is evident that Cos. a will be positive or negative according as Tan. B and Cot. C are affected by the same or by opposite signs. Hence we conclude that,

In any right-angled spherical triangle, if the oblique angles be greater or less than right angles, the hypotenuse will be less than a quadrant; but if one of the oblique angles be greater and the other less than a right angle, the hypotenuse will be greater than a quadrant: and conversely.

10. From Theor. II. we have Rad. : Cos. B : : Tan. a : Tan. c; therefore Tan. $a = \frac{\text{Tan. } c}{\text{Cos. B}} \times \text{R}$. From this equation, we in like manner infer that,

In any right-angled spherical triangle, if an oblique angle and its adjacent side be each greater or each less than 90°, the hypotenuse is less than a quadrant; but if one of them be greater and the other less than 90°, the hypotenuse will be greater than a quadrant: and conversely.

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11. We proceed now to apply the principles laid down to the solution of the cases of right-angled spherical triangles.

PROBLEM I.

Given the hypotenuse and one of the angles, to find the sides and the remaining angle.

Ex. 1. In the spherical triangle ABC, right-angled at A, let the hypotenuse BC be 68° 36′, and the angle B, 35° 48′, required the remaining parts of the triangle?

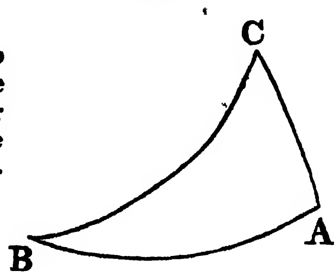
Solution.

To find AB.

Making complement angle B the middle part, then AB and complement BC are the adjacent parts. Hence,

$$\text{Rad.} \times \text{Cos. B} = \text{Tan. AB} \times \text{Cot. BC}.$$

Converting this expression into an analogy, and arranging the terms so as to have Tan. AB for the last term; observing, at the same time, that Cot. BC : R . . R : Tan. BC, we obtain,



As radius	10.000000	
To Tan. BC, 68° 36′	10.406829	
So is Cos. B, 35° 48′	9.909055	(Theor. II.)
To Tan. AB, 64° 12′ 39″	10.315884	

Since the hypotenuse BC is less than 90°, the angle B and its adjacent side AB are of the same species; (§. 10.) hence AB is less than 90°.

To find AC.

Making AC the middle part, then complement angle B and complement BC are opposite parts. Hence,

$$\text{Rad.} \times \text{Sin. AC} = \text{Sin. B} \times \text{Sin. BC}.$$

As radius	10.000000	
To Sin B, 35° 48′	9.767124	
So is Sin. BC, 68° 36′	9.968976	(Theor. I.)
To Sin. AC, 33°	9.736100	

The side AC is of the same species with its opposite angle B, (§. 7.) and is therefore less than 90°.

To find angle C.

Making complement BC the middle part, then complement angle B and complement angle C are the adjacent parts. Hence,

$$\text{Rad.} \times \text{Cos. BC} = \text{Cot. B} \times \text{Cot. C}.$$

As radius	10.000000	
To Tan. B, 35° 48′	9.858069	
So is Cos. BC, 68° 36′	9.562146	(Theor. VI.)
To Cot. C, 75° 15′ 23″	9.420215	

Since the hypotenuse BC is less than 90°, the angle C is of the same species with the given angle B; (§. 9.) and is therefore less than a right angle.

Ex. 2. In the spherical triangle ABC, right-angled at C, let the hypotenuse AB be 63° 56′ 7″, and the angle A 45° 41′ 21″, required the sides AC and BC, and the remaining angle B?—*Ans.* AC = 55°, BC = 40°, angle B = 65° 46′ 5″.

PROBLEM II.

Given one of the sides and its adjacent angle to find the other angle, the hypotenuse, and the other side.

Ex. 1. In the triangle ABC, right-angled at A, let the side AB be 56° 30′, and the angle B, 32° 14′, required the remaining parts of the triangle?—*Ans.* BC, 60° 45′ 24″, AC, 27° 44′ 8″, angle C, 72° 52′ 46″.

Solution.

To find BC

Making complement of angle B the middle part, then AB and complement BC are the adjacent parts, therefore

$$\text{Rad} \times \text{Cos. B} = \text{Cot. BC} \times \text{Tan. AB}.$$

Hence

$$\text{Tan. AB} : \text{Rad.} :: \text{Cos. B} : \text{Cot. BC},$$

$$\text{Or Rad.} : \text{Cot. AB} :: \text{Cos. B} : \text{Cot. BC.} \quad \text{(Theor. II. Cor)}$$

Because the angle B, and the adjacent side AB, are of the same species, the hypotenuse BC is less than 90°. (§. 10.)

To find AC.

Making AB the middle part, then AC and complement of angle B are the adjacent parts; therefore

$$\text{Rad.} \times \text{Sin. AB} = \text{Tan. AC} \times \text{Cot. B}.$$

Hence

$$\text{Cot. B} : \text{Rad.} :: \text{Sin. AB} : \text{Tan. AC};$$

$$\text{Or, Rad.} : \text{Tan. B} :: \text{Sin. AB} : \text{Tan. AC.} \quad \text{(Theor. V.)}$$

The side AC and its opposite angle B being of the same species, (§. 7.) AC is therefore less than 90°.

To find angle C.

Making complement of angle C the middle part, then AC and

complement BC are the adjacent parts, and AB and complement of angle B are the opposite parts; therefore

Rad. $\times$ Cos. C $=$ Cos. AB $\times$ Sin. B.

Hence

Rad. . Cos. AB : · Sin. B : Cos. C. (Theor. IV.)

The angle C and its opposite side AB are of the same species; therefore angle C is less than a right angle.

Ex. 2. In the right-angled spherical triangle ABC, let the side AB be 42° 8′ 24″, and its adjacent angle A 64° 38′ 1″, required the remaining parts of the triangle?—*Ans.* The hypotenuse AC, 64° 39′ 51″. The other side BC, 54° 45′ 13″. The remaining angle C, 47° 55′ 54″.

PROBLEM III.

Given one of the sides, and its opposite angle, to find the adjacent angle, the hypotenuse, and the remaining side.

Ex. 1. In the triangle ABC, or *a*B*c*, right-angled at A, or *a*, let the angle B be 43° 52′, and the side AC, or *ac*, 37° 34′; required the remaining parts of the triangle?—*Ans.* In the triangle ABC, the side AB is 53° 9′ 6″, the hypotenuse BC, 61° 37′ 5″, and the angle C, 65° 26′ 40″. In the triangle *a*B*c*, the side *a*B is 126° 50′ 54″, hypotenuse B*c*, 118° 22′ 55″, and the angle *c*, 114° 33′ 20″.

Solution.

To find AB or *a*B.

Making AB or *a*B the middle part, then AC or *ac* and complement of angle B are the adjacent parts, therefore

Rad. $\times$ Sin. AB or *a*B $=$ Tan. AC or *ac* $\times$ Cot. B.

Hence

Rad. : Cot. B : : Tan. AC or *ac* · Sin. AB or *a*B. (Theor. V. Cor.)

To find BC or B*c*.

Making AC or *ac* the middle part, then complement of angle B and complement of BC or B*c* are the opposite parts, therefore

Rad. $\times$ Sin. AC or *ac* $=$ Sin. B $\times$ Sin. BC or B*c*.

Hence

Sin. B : Sin. AC or *ac* : : Rad. : Sin. BC or B*c*. (Theor. I)

To find angle C or *c*.

Making complement of angle B the middle part, then AC and complement of angle C or *c* are the opposite parts; therefore

Rad. $\times$ Cos. B $=$ Cos. AC $\times$ Sin. C or *c*.

Hence

Cos. AC : Rad. : : Cos. B : Sin. C or *c*. (Theor. IV.)

All the parts, being determined by their sines only, are ambiguous. (§. 6.)

Ex. 2. In the right-angled spherical triangle ABC, there are given the side BC equal to $26^{\circ}\ 3'\ 53''$, and the opposite angle A equal to 35°, it is required to find the hypotenuse AC, the other side AB, and the other angle C?—*Ans.* AC $= 50^{\circ}$, AB $= 44^{\circ}\ 18'\ 39''$, and angle C $= 65^{\circ}\ 46'\ 7''$.

PROBLEM IV.

Given the hypotenuse, and one side, to find the angles and the other side.

Ex. 1. In the triangle ABC, right-angled at A, let the side AB be $55^{\circ}\ 13'$, and the hypotenuse BC, $65^{\circ}\ 40'$; required the remaining parts of the triangle?—*Ans.* Angle B $= 49^{\circ}\ 22'\ 42''$, angle C, $64^{\circ}\ 20'\ 30''$; and the side AC, $43^{\circ}\ 45'\ 24''$

Solution.

To find angle B.

Making complement of angle B the middle part, then AB and complement of BC are the adjacent parts, therefore

Rad. $\times$ Cos. B $=$ Tan. AB $\times$ Cot. BC.

Hence

Rad. : Cot. BC : : Tan. AB : Cos. B. (Theor. II. Cor.)

Since the hypotenuse BC is less than a quadrant, the angle B and its adjacent side AB are of the same species, (§. 10.) therefore angle B is less than a right angle.

To find angle C.

Making AB the middle part, then complement of BC, and complement of angle C, are the opposite parts; therefore

$$\text{Rad.} \times \text{Sin. AB} = \text{Sin. BC} \times \text{Sin. C.}$$

Hence

$$\text{Sin. BC} : \text{Rad.} :: \text{Sin. AB} : \text{Sin. C.} \quad \text{(Theor. I.)}$$

Angle C and its opposite side AB are of the same species, (§. 7.) therefore angle C is less than a right angle.

To find AC.

Making complement of BC the middle part, then AB and AC are the opposite parts, therefore

$$\text{Rad.} \times \text{Cos. BC} = \text{Cos. AB} \times \text{Cos. AC.}$$

Hence

$$\text{Cos. AB} : \text{Rad.} :: \text{Cos. BC} . \text{Cos. AC.} \quad \text{(Theor. III.)}$$

Since the hypotenuse BC is less than a quadrant, the sides AB and AC are of the same species; (§. 8.) therefore AC is less than a quadrant.

Ex. 2. If the hypotenuse of a right-angled spherical triangle be $51^{\circ}\ 30'$, and the perpendicular equal to $40^{\circ}\ 18'\ 15''$, what are the angles and remaining side?—*Ans.* The other side $35^{\circ}\ 17'\ 8''$, the angle opposite the given side, $55^{\circ}\ 44'\ 36''$; the angle adjacent to it, $47^{\circ}\ 34'\ 15''$.

PROBLEM V.

Given the two sides, to find the angles and hypotenuse.

Ex. 1. In the triangle ABC, right-angled at A, let the side AB be $56^{\circ}\ 30'$, and the side AC be $27^{\circ}\ 28'$; required the remaining parts of the triangle?—*Ans.* Hypotenuse BC, $60^{\circ}\ 40'\ 40''$, angle B $31^{\circ}\ 56'\ 19''$, and angle C, $73^{\circ}\ 1'\ 24''$.

Solution.

To find BC.

Making complement of BC the middle part, then AB and AC are the opposite parts, therefore

Rad. × Cos. BC = Cos. AB × Cos. AC.

Hence

Rad. . Cos. AB : . Cos. AC . Cos. BC. (Theor. III.)

Because the oblique angles B and C are of the same species, the hypotenuse BC is less than a quadrant.

To find angle B.

Making AB the middle part, then AC and complement of angle B are the adjacent parts, therefore

Rad. × Sin. AB = Tan. AC × Cot. B.

Hence

Tan. AC . Rad. . Sin. AB . Cot. B,

Or, Rad. . Sin. AB . : Cot. AC Cot. B. (Theor. V. Cor.)

Angle B is of the same species with its opposite side AC, (§. 7.) and is therefore less than a right angle.

To find angle C.

Making AC the middle part, then AB and complement of angle C are the adjacent parts, therefore

Rad. × Sin. AC = Tan. AB × Cot. C.

Hence

Tan. AB : Rad. : : Sin. AC : Cot. C,

Or, Rad. . Sin. AC : : Cot AB : Cot. C. (Theor. V. Cor.)

Angle C is of the same species with its opposite side AB, and is therefore less than a right angle.

Ex. 2. The sides of a right-angled spherical triangle, are 65° 19′, and 54° 29′ 10″ respectively; required the angles and hypo-

tenuse?—*Ans.* The one angle $= 57^{0}\ 2'\ 19''$; the other $= 69^{0}\ 29'\ 21''$; the hypotenuse $= 75^{0}\ 57'\ 39''$.

PROBLEM VI.

Given the two angles, to find the hypotenuse and sides of the triangle.

Ex. 1. In the triangle ABC, right-angled at A, let the angle B be $36^{0}\ 32'$, and the angle C, $65^{0}\ 47'$, required the remaining parts of the triangle?—*Ans.* BC $= 52^{0}\ 37'\ 15''$; AC $= 28^{0}\ 13'\ 54''$, AB $= 46^{0}\ 26'\ 41''$.

Solution.

To find BC.

Making complement of BC the middle part, then complement of angle B, and complement of angle C, are the adjacent parts, therefore

$$\text{Rad.} \times \text{Cos. BC} = \text{Cot. B} \times \text{Cot. C}.$$

Hence

$$\text{Rad.} : \text{Cot. B} :: \text{Cot. C} : \text{Cos. BC. (Theor. VI. Cor.)}$$

Because the oblique angles B and C are of the same species, the hypotenuse BC is less than a quadrant. (§. 9.)

To find AC.

Making complement of angle B the middle part, then AC and complement of angle C are the opposite parts; therefore

$$\text{Rad.} \times \text{Cos. B} = \text{Sin. C} \times \text{Cos. AC}.$$

Hence

$$\text{Sin. C} : \text{Rad.} :: \text{Cos. B} : \text{Cos. AC. (Theor. IV.)}$$

The side AC being of the same species with its opposite angle B, (§. 7.) is less than a quadrant.

To find AB.

Making complement of angle C the middle part, then AB and complement of angle B are the opposite parts, therefore

$$\text{Rad.} \times \text{Cos. } C = \text{Sin. } B \times \text{Cos. } AB.$$

Hence

$$\text{Sin. } B : \text{Rad.} :: \text{Cos. } C : \text{Cos. } AB. \text{ (Theor. IV.)}$$

The side AB, being of the same species with its opposite angle C, is less than a quadrant.

Ex. 2. Given the oblique angles of a right-angled spherical triangle equal to 68° 29′ 48″, and 57° 16′ 1″ respectively, it is required to find the hypotenuse and sides of the triangle?—*Ans.* Hypotenuse = 75° 19′ 48″, one side = 64° 10′; the other side = 54° 28′*

Solution of oblique-angled Spherical Triangles

12. The solution of the different cases of oblique-angled spherical triangles, depends upon the following principles which, of course, apply equally to right-angled triangles.

THEOREM I

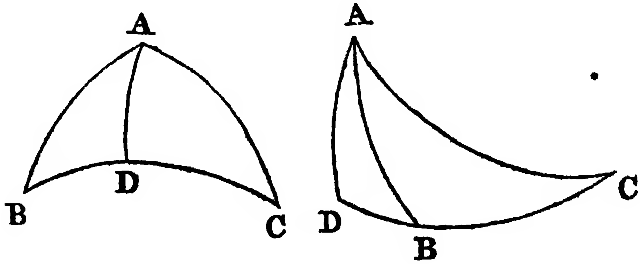
In every spherical triangle the sines of the angles are proportional to the sines of the opposite sides.

Let ABC be any spherical triangle, then,

$$\text{Sin. } B : \text{Sin. } C :: \text{Sin. } AC : \text{Sin. } AB.$$

* If one side of a spherical triangle be a quadrant, the unknown parts may be found by means of a right-angled spherical triangle, whose acute angles are the supplements of the other sides of the triangle, and whose three sides are the supplements of its angles, the hypotenuse being the supplement of the angle opposite to the quadrantal side. (See Lemma, page 140.)

From the vertex A draw the arch AD perpendicular to the opposite side BC, then in the two right-angled spherical triangles ABD, ACD we have, (§. 4. Theor. I.)



Sin. B : Rad. . Sin. AD : Sin. AB

Rad. : Sin. C . : Sin. AC : Sin. AD .

Therefore, *ex æquali*, inversely, we obtain

Sin. B : Sin. C : : Sin. AC : Sin. AB.

If the perpendicular AD fall without the triangle ABC, we have the same two proportions as above, but in one of them Sin. B then denotes Sin. ABD: Since, however, the angles ABD and ABC are supplements of each other, their sines are equal. Therefore we have in every case

Sin. B . Sin. C : : Sin. AC : Sin. AB.

THEOREM II.

In every oblique-angled spherical triangle, the cosines of the sides are directly proportional to the cosines of the segments into which the base is divided by the perpendicular let fall upon it from the opposite angle.

Let ABC be a spherical triangle, and from the vertex A draw AD perpendicular to the opposite side BC, dividing it into the two segments BD, DC, (see figures of last Theorem,) then,

Cos. AB : Cos. AC : : Cos. BD : Cos. DC.

For, from the right-angled triangles ABD, ACD we have, (§. 4. Theor. III.)

Cos. AB : Cos. BD : : (Cos. AD : Rad. : :) Cos. AC : Cos. DC, therefore, by alternation,

Cos. AB : Cos. AC . : Cos. BD . Cos. DC.

THEOREM III.

The same construction remaining, the tangents of the sides are inversely proportional to the cosines of the segments into which the vertical angle is divided by the perpendicular: that is,

Tan. AB : Tan. AC : . Cos. CAD Cos. BAD.

For, by Theor. II. §. 4, we have

Tan. AB Tan. AD · : Rad. Cos. BAD,

and Tan. AD : Tan. AC · . Cos. CAD : Rad.,

therefore, *ex æquali,* inversely, we have

Tan. AB . Tan. AC : . Cos. CAD : Cos. BAD.

THEOREM IV.

The same construction remaining the cosines of the angles at the base are directly proportional to the sines of the segments of the vertical angle: that is,

Cos. B : Cos. C . : Sin. BAD : Sin. CAD;

For, from Theor. IV. §. 4, we have

Cos. B : Sin. BAD : : (Cos. AD . Rad. :) Cos. C : Sin. CAD,

therefore Cos. B : Cos. C : : Sin. BAD : Sin. CAD.

THEOREM V.

The same construction remaining, the tangents of the angles at the base are inversely proportional to the sines of the segments of the base, that is,

Tan. B : Tan. C . . Sin. CD : Sin. BD.

For, by Theor. V. §. 4., we have

$$\text{Tan. B : Rad. : : Tan. AD . Sin. BD,}$$

$$\text{and Rad. : Tan. C : : Sin. CD : Tan. AD ;}$$

$$\text{Therefore Tan. B . Tan. C : : Sin. CD : Sin. BD.}$$

THEOREM VI.

In any spherical triangle as the tangent of half the sum of the segments of the base, is to the tangent of half the sum of the two sides, so is the tangent of half their difference, to the tangent of half the difference of the segments of the base.

Let b, c be the two sides of a spherical triangle, and m, n the segments of the base, then

$$\text{Tan. } \tfrac{1}{2}(m+n) \text{ . Tan. } \tfrac{1}{2}(b+c) : : \text{Tan. } \tfrac{1}{2}(b-c) : \text{Tan. } \tfrac{1}{2}(m-n).$$

For, by Theor. II. §. 12., we have

$$\text{Cos. } b \text{ . Cos. } c \text{ . Cos. } m : \text{Cos. } n \text{, therefore}$$

$$\text{Cos. } b + \text{Cos. } c : \text{Cos. } b - \text{Cos. } c : : \text{Cos. } m + \text{Cos. } n : \text{Cos. } m - \text{Cos. } n$$

But, by Formula XXIV. page 56.,

$$\frac{\text{Cos. } b + \text{Cos. } c}{\text{Cos. } b - \text{Cos. } c} = \frac{\text{Cot. } \tfrac{1}{2}(b+c)}{\text{Tan. } \tfrac{1}{2}(b-c)}$$

Or, $\text{Cos. } b + \text{Cos. } c : \text{Cos. } b - \text{Cos. } c \text{ . : Cot. } \tfrac{1}{2}(b+c) : \text{Tan. } \tfrac{1}{2}(b-c)$

and in like manner,

$\text{Cos. } m + \text{Cos. } n : \text{Cos. } m - \text{Cos. } n : : \text{Cot. } \tfrac{1}{2}(m+n) : \text{Tan. } \tfrac{1}{2}(m-n)$

Wherefore we have,

$\text{Cot. } \tfrac{1}{2}(b+c) : \text{Tan. } \tfrac{1}{2}(b-c) : : \text{Cot. } \tfrac{1}{2}(m+n) : \text{Tan. } \tfrac{1}{2}(m-n)$,

and by alternation, and observing that

$$\text{Cot. } \tfrac{1}{2}(b+c) : \text{Cot. } \tfrac{1}{2}(m+n) : \text{. Tan. } \tfrac{1}{2}(m+n) \text{ . Tan. } \tfrac{1}{2}(b+c),$$

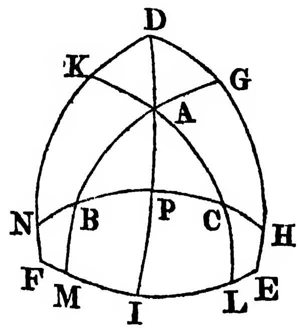
we obtain

$$\text{Tan. } \tfrac{1}{2}(m+n) \text{ . Tan. } \tfrac{1}{2}(b+c) : \text{. Tan. } \tfrac{1}{2}(b-c) \text{ . Tan. } \tfrac{1}{2}(m-n).$$

Lemma.

If the angular points of any spherical triangle be made the poles of three great circles, another triangle will be formed by their intersection, such that the sides of the one triangle will be respectively the supplements of the measures of the angles opposite to them in the other.

Let the angular points of the triangle ABC be the poles of the three great circles FE, ED, DF, which intersect each other in the points F, D, E, the sides of the triangle DEF are the supplements of the measures of the angles A, B, C, namely, FE of the angle BAC, DE of the angle ABC, and DF of the angle ACB. and again AC is the supplement of the angle DFE, AB of the angle FED, and BC of the angle EDF.



For, let each side of ABC be produced to meet the sides that contain the angle opposite to it in the triangle DEF. Then, because BC passes through the pole of each of the great circles ED, DF; the plane of the circle BC must be perpendicular to the planes of ED, DF, therefore the line of intersection of ED, DF must be perpendicular to the plane of BC, so that the point D, where that line meets the surface of the sphere, is the pole of BC. (Def. II.) In like manner it may be shown that E is the pole of AB, and F the pole of AC.

Now, since D and E are the poles of BC and AB, the arches DH and EG are quadrants (Cor. Def. II.), and DH, together with EG, that is, DE, together with GH, are equal to a semicircle. But since B is the pole of DE, GH is the measure of the angle ABC, (Cor. Def. III.), consequently DE is the supplement of the measure of the angle ABC. In the same manner DF and EF are the supplements of the measures of the angles ACB, BAC.

Again, since CK, AL are quadrants, CK, together with AL, that is, KL, together with AC, are equal to a semicircle, and since F is the pole of KL, KL is the measure of the angle DFE, therefore the measure of the angle DFE is the supplement of the side AC. In the same manner it is shown that the measures of the angles EDF, DEF are the supplements of the sides BC, AB.

Cor. Let a great circle pass through D and A, the vertices of the triangles ABC, DEF it will cut the bases BC, EF at right angles, because it passes through their poles, and it is evident that the segments BP, PC of the base in the triangle ABC will be the complements of the measures of the segments IDF, IDE of the vertical angle in the triangle DEF, taken alternately. Also the segments EI, IF of the base in the triangle DEF will be the complements of the measures of the segments PAB, PAC of the vertical angle in the triangle ABC.

THEOREM VII.

In any spherical triangle, as the cotangent of half the vertical angle is to the tangent of half the sum of the angles at the base, so is the tangent of half the difference of these angles to the tangent of half the difference, or of half the sum of the segments of the vertical angle, according as the perpendicular falls within or without the triangle.

Let B, C be the angles at the base of a spherical triangle, and p, q the segments into which the vertical angle A is divided by the perpendicular drawn from it to the opposite side, then,

$$\text{Cot.} \tfrac{1}{2}A : \text{Tan.} \tfrac{1}{2}(B + C) :: \text{Tan.} \tfrac{1}{2}(B - C) : \text{Tan.} \tfrac{1}{2}(p \mp q).$$

In the supplemental triangle formed by the intersection of the three great circles which have the angular points A, B, C for their poles, let the sides be denoted by a', b', c'; so that a' is the supplement of A, b' the supplement of B, and c' the supplement of C. (Lemma.) Also let m' and n' denote the segments into which the base a' of the supplemental triangle is divided by the perpendicular arch passing through the vertices of the two triangles, so that m' is the complement of q, and n' the complement of p. (Cor. to Lem.)

By applying Theor. VI. to the supplemental triangle, we obtain

$$\text{Tan.} \tfrac{1}{2}(180 - A) : \text{Tan.} \tfrac{1}{2}(c' + b') :: \text{Tan.} \tfrac{1}{2}(c' - b') : \text{Tan.} \tfrac{1}{2}(m' \mp n').$$

The difference or sum of m' and n' is to be taken according as the perpendicular falls within or without the triangle.

But $b' = 180^\circ - B$, $c' = 180 - C$, $m' = 90^\circ - q$, $n' = 90^\circ - p$, therefore, substituting, and observing that the tangent of an arch is the same with the tangent of its supplement, we have

$$\text{Cot.} \tfrac{1}{2}A : \text{Tan.} \tfrac{1}{2}(B + C) :: \text{Tan.} \tfrac{1}{2}(B - C) : \text{Tan.} \tfrac{1}{2}(p \mp q).$$

THEOREM VIII.

In any spherical triangle as the sine of half the sum of the two sides, is to the sine of half their difference, so is the cotangent of half the vertical angle to the tangent of half the difference of the angles at the base.

Let A denote the vertical angle of a spherical triangle, B and C the angles at the base, and b, c the sides opposite to B, C, respectively, then,

$$\text{Sin.}\ \tfrac{1}{2}(b+c) : \text{Sin.}\ \tfrac{1}{2}(b-c) :: \text{Cot.}\ \tfrac{1}{2}A : \text{Tan.}\ \tfrac{1}{2}(B-C).$$

Put p and q to denote the segments of the vertical angle as in last Theorem: From Theor. III. §. 12., we have

$$\text{Tan.}\ b : \text{Tan.}\ c :: \text{Cos.}\ p : \text{Cos.}\ q,$$

therefore

$$\text{Tan.}\ b + \text{Tan.}\ c : \text{Tan.}\ b - \text{Tan.}\ c :: \text{Cos.}\ p + \text{Cos.}\ q : \text{Cos.}\ p - \text{Cos.}\ q,$$

Hence

$$\frac{\text{Tan.}\ b + \text{Tan.}\ c}{\text{Tan.}\ b - \text{Tan.}\ c} = \frac{\text{Cos.}\ p + \text{Cos.}\ q}{\text{Cos.}\ p - \text{Cos.}\ q}.$$

But from Formulas XXIX. and XXX., page 57, and Formula XIII., page 55, we have

$$\frac{\text{Tan.}\ b + \text{Tan.}\ c}{\text{Tan.}\ b - \text{Tan.}\ c} = \frac{\text{Sin.}\ (b+c)}{\text{Sin.}\ (b-c)} = \frac{\text{Sin.}\ \tfrac{1}{2}(b+c)\ \text{Cos.}\ \tfrac{1}{2}(b+c)}{\text{Sin.}\ \tfrac{1}{2}(b-c)\ \text{Cos.}\ \tfrac{1}{2}(b-c)},$$

and from Formula XXIV., page 56, we have

$$\frac{\text{Cos.}\ p + \text{Cos.}\ q}{\text{Cos.}\ p - \text{Cos.}\ q} = \frac{\text{Cot.}\ \tfrac{1}{2}A}{\text{Tan.}\ \tfrac{1}{2}(p \mp q)}$$

the negative or positive sign of q being taken according as the perpendicular falls within or without the triangle: Therefore

$$\frac{\text{Sin.}\ \tfrac{1}{2}(b+c)\ \text{Cos.}\ \tfrac{1}{2}(b+c)}{\text{Sin.}\ \tfrac{1}{2}(b-c)\ \text{Cos.}\ \tfrac{1}{2}(b-c)} = \frac{\text{Cot.}\ \tfrac{1}{2}A}{\text{Tan.}\ \tfrac{1}{2}(p \mp q)}. \quad (1.)$$

Again, because $\text{Sin.}\ b : \text{Sin.}\ c :: \text{Sin.}\ B : \text{Sin.}\ C$, therefore

$$\text{Sin.}\ b + \text{Sin.}\ c : \text{Sin.}\ b - \text{Sin.}\ c :: \text{Sin.}\ B + \text{Sin.}\ C : \text{Sin.}\ B - \text{Sin.}\ C,$$

and

$$\frac{\text{Sin. } b + \text{Sin. } c}{\text{Sin. } - b \text{ Sin. } c} = \frac{\text{Sin. B} + \text{Sin. C}}{\text{Sin. B} - \text{Sin. C}}$$

Hence, by Formulas I. and II., page 54, and Formula XIX., page 56, we obtain

$$\frac{\text{Sin. } \frac{1}{2}(b + c) \text{ Cos. } \frac{1}{2}(b - c)}{\text{Sin. } \frac{1}{2}(b - c) \text{ Cos. } \frac{1}{2}(b + c)} = \frac{\text{Tan. } \frac{1}{2}(\text{B} + \text{C})}{\text{Tan. } \frac{1}{2}(\text{B} - \text{C})}. \quad (2.$$

Multiplying the corresponding sides of equations (1.) and (2.), and rejecting the common factors, we obtain

$$\frac{\text{Sin}^2 \frac{1}{2}(b + c)}{\text{Sin}^2 \frac{1}{2}(b - c)} = \frac{\text{Cot. } \frac{1}{2}\text{A Tan. } \frac{1}{2}(\text{B} + \text{C})^2}{\text{Tan. } \frac{1}{2}(p \mp q) \text{ Tan. } \frac{1}{2}(\text{B} - \text{C})}.$$

But from last theorem we find

$$\frac{\text{Cot. } \frac{1}{2}\text{A}}{\text{Tan. } \frac{1}{2}(\text{B} - \text{C})} = \frac{\text{Tan. } \frac{1}{2}(\text{B} + \text{C})}{\text{Tan. } \frac{1}{2}(p \mp q)}:$$

Therefore

$$\frac{\text{Cot}^2 \frac{1}{2}\text{A}}{\text{Tan}^2 \frac{1}{2}(\text{B} - \text{C})} = \frac{\text{Cot. } \frac{1}{2}\text{A Tan. } \frac{1}{2}(\text{B} + \text{C})}{\text{Tan. } \frac{1}{2}(p \mp q) \text{ Tan. } \frac{1}{2}(\text{B} - \text{C})} = \frac{\text{Sin}^2 \frac{1}{2}(b + c)}{\text{Sin}^2 \frac{1}{2}(b - c)}.$$

Hence

$$\frac{\text{Sin. } \frac{1}{2}(b + c)}{\text{Sin. } \frac{1}{2}(b - c)} = \frac{\text{Cot. } \frac{1}{2}\text{A}}{\text{Tan. } \frac{1}{2}(\text{B} - \text{C})},$$

And

$$\text{Sin. } \frac{1}{2}(b + c) : \text{Sin. } \frac{1}{2}(b - c) :: \text{Cot. } \frac{1}{2}\text{A} : \text{Tan. } \frac{1}{2}(\text{B} - \text{C}).$$

THEOREM IX.

In any spherical triangle, as the cosine of half the sum of the two sides is to the cosine of, half their difference, so is the cotangent of half the vertical angle to the tangent of half the sum of the angles at the base.

Let A, B, C, b, c, p, q, denote as before, then

$$\text{Cos. } \frac{1}{2}(b + c) : \text{Cos. } \frac{1}{2}(b - c) :: \text{Cot. } \frac{1}{2}\text{A} : \text{Tan. } \frac{1}{2}(\text{B} + \text{C}).$$

For in the demonstration of the preceding theorem, it was shewn that

$$\frac{\text{Sin. } \frac{1}{2}(b+c) \text{ Cos. } \frac{1}{2}(b+c)}{\text{Sin. } \frac{1}{2}(b-c) \text{ Cos. } \frac{1}{2}(b-c)} = \frac{\text{Cot. } \frac{1}{2}\text{A}}{\text{Tan. } \frac{1}{2}(p \mp q)};$$

And

$$\frac{\text{Sin. } \frac{1}{2}(b+c) \text{ Cos. } \frac{1}{2}(b-c)}{\text{Sin. } \frac{1}{2}(b-c) \text{ Cos. } \frac{1}{2}(b+c)} = \frac{\text{Tan. } \frac{1}{2}(\text{B}+\text{C})}{\text{Tan. } \frac{1}{2}(\text{B}-\text{C})}.$$

Multiplying the former of these equations by the reciprocal of the latter, and rejecting the common factors, we obtain

$$\frac{\text{Cos}^2 \frac{1}{2}(b+c)}{\text{Cos}^2 \frac{1}{2}(b-c)} = \frac{\text{Cot. } \frac{1}{2}\text{A Tan. } \frac{1}{2}(\text{B}-\text{C})}{\text{Tan. } \frac{1}{2}(\text{B}+\text{C}) \text{ Tan. } \frac{1}{2}(p \mp q)}.$$

But from Theorem VII.,

$$\frac{\text{Cot. } \frac{1}{2}\text{A}}{\text{Tan. } \frac{1}{2}(\text{B}+\text{C})} = \frac{\text{Tan. } \frac{1}{2}(\text{B}-\text{C})}{\text{Tan. } \frac{1}{2}(p \mp q)}:$$

Therefore

$$\frac{\text{Cot}^2 \text{ A}}{\text{Tan}^2 \frac{1}{2}(\text{B}+\text{C})} = \frac{\text{Cot. } \frac{1}{2}\text{A Tan. } \frac{1}{2}(\text{B}-\text{C})}{\text{Tan. } \frac{1}{2}(\text{B}+\text{C}) \text{ Tan. } \frac{1}{2}(p \mp q)} = \frac{\text{Cos}^2 \frac{1}{2}(b+c)}{\text{Cos}^2 \frac{1}{2}(b-c)}.$$

Hence, it is evident that

$$\text{Cos. } \tfrac{1}{2}(b+c) : \text{Cos. } \tfrac{1}{2}(b-c) : : \text{Cot. } \tfrac{1}{2}\text{A} : \text{Tan. } \tfrac{1}{2}(\text{B}+\text{C}).$$

THEOREM X.

In any spherical triangle, as the sine of half the sum of the angles at the base, is to the sine of half their difference, so is the tangent of half the base to the tangent of half the difference of the two sides.

Let a denote the base of a spherical triangle, B, C, the angles adjacent to it, and b, c, the other two sides; then,

$$\text{Sin. } \tfrac{1}{2}(\text{B}+\text{C}) : \text{Sin. } \tfrac{1}{2}(\text{B}-\text{C}) : : \text{Tan. } \tfrac{1}{2}a : \text{Tan. } \tfrac{1}{2}(b-c).$$

In the supplemental triangle, let the sides be denoted by a', b', c', and the angles by A′, B′, C′; so that a' is the supplement of A, and

A' of a; b' the supplement of B; and B' of b; and c' the supplement of C, and C' of c.

By applying Theor. VIII. to the supplemental triangle, we find

Sin. $\frac{1}{2}(c' + b')$: Sin. $\frac{1}{2}(c' - b')$: : Cot. $\frac{1}{2}$A' : Tan. $\frac{1}{2}$(C' — B').

Substituting for A', B', C', b', c', their equals, and observing that the sine and tangent of an arch are the same with those of its supplement, we obtain

Sin. $\frac{1}{2}$(B + C) : Sin. $\frac{1}{2}$(B — C) : : Tan. $\frac{1}{2}a$: Tan. $\frac{1}{2}(b - c)$.

THEOREM XI.

In any spherical triangle, as the cosine of half the sum of the angles at the base, is to the cosine of half their difference, so is the tangent of half the base to the tangent of half the sum of the two sides.

Let a, b, c, B, C, denote the same things as in last theorem; then,

Cos. $\frac{1}{2}$(B + C) : Cos. $\frac{1}{2}$(B — C) : : Tan. $\frac{1}{2}a$: Tan. $\frac{1}{2}(b + c)$.

For, by applying Theor. IX. to the supplemental triangle, we have,

Cos. $\frac{1}{2}(c' + b')$: Cos. $\frac{1}{2}(c' - b')$: : Cot. $\frac{1}{2}$A' : Tan. $\frac{1}{2}$(B' + C').

Hence by substituting, we find,

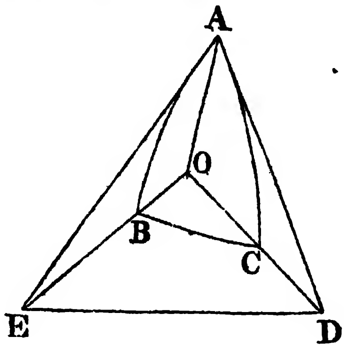
Cos. $\frac{1}{2}$(B + C) : Cos. $\frac{1}{2}$(B — C) : : Tan. $\frac{1}{2}a$: Tan. $\frac{1}{2}(b + c)$.

THEOREM XII.

In every spherical triangle, as the rectangle contained by the sines of the sides, is to the excess of the rectangle contained by the radius and the cosine of the base, above the rectangle contained by the cosines of the sides, so is the radius to the cosine of the vertical angle.

Let ABC be a spherical triangle; and let its sides be denoted by a, b, c, and the opposite angles by A, B, C respectively; then,

$$\text{Sin. } b \text{ Sin. } c : \text{Rad. Cos. } a - \text{Cos. } b \text{ Cos. } c :: \text{Rad.} : \text{Cos. A}.$$



Let O be the centre of the sphere. In the plane of the circle OAB, draw AE the tangent, and OE the secant of the arch AB $= c$; also in the plane of the circle OAC; draw AD the tangent and OD the secant of the arch AC $= b$, and join DE. Then in the triangles DOE, DAE, (Pl. Trig. §. 7. Theor. V.)

$$\frac{\text{Cos. O}}{\text{R}} \times 2\text{DO} \times \text{OE} = \text{DO}^2 + \text{OE}^2 - \text{DE}^2$$

$$\frac{\text{Cos. A}}{\text{R}} \times 2\text{DA} \times \text{AE} = \text{DA}^2 + \text{AE}^2 - \text{DE}^2,$$

Hence by substracting, and observing that the triangles OAE, OAD, have each a right angle at A, we obtain,

$$\frac{\text{Cos. O}}{\text{R}} \times \text{DO} \times \text{OE} - \frac{\text{Cos. A}}{\text{R}} \times \text{DA} \times \text{AE} = \text{AO}^2.$$

Therefore

$$\frac{\text{Cos. O}}{\text{R}} \times \frac{\text{DO}}{\text{DA}} \times \frac{\text{OE}}{\text{AE}} \quad \frac{\text{AO}}{\text{DA}} \times \frac{\text{AO}}{\text{AE}} = \frac{\text{Cos. A}}{\text{R}}.$$

But from the two right-angled triangles ADO, AEO, we obtain

$$\frac{\text{DO}}{\text{DA}} = \frac{\text{Rad.}}{\text{Sin. DOA}} = \frac{\text{R}}{\text{Sin. } b}, \frac{\text{OE}}{\text{AE}} = \frac{\text{R}}{\text{Sin. } c}, \frac{\text{AO}}{\text{DA}} = \frac{\text{Rad.}}{\text{Tan. } b}, = \frac{\text{Cos. } b}{\text{Sin. } b},$$

and $\frac{\text{AO}}{\text{AE}} = \frac{\text{Cos. } c}{\text{Sin. } c}$.

Wherefore, by substituting, we have

$$\frac{\text{Cos. } a \times \text{R} - \text{Cos. } b \text{ Cos. } c}{\text{Sin. } b \text{ Sin. } c} = \frac{\text{Cos. A}}{\text{R}}.$$

Whence

Sin. b Sin. c : Rad. Cos. a — Cos. b Cos. c : : Rad. : Cos. A.

THEOREM XIII.

In any spherical triangle, as the rectangle contained by the sines of the two sides is to the rectangle contained by the sines of half the sum and half the difference of the base and the difference of the two sides, so is the square of the radius to the square of the sine of half the vertical angle.

Let a, b, c, denote the three sides of a spherical triangle, and A its vertical angle opposite to the base a, then

$$\text{Sin.}\,b\ \text{Sin.}\,c : \text{Sin.}\,\tfrac{1}{2}(a+b-c)\ \text{Sin.}\,\tfrac{1}{2}(a-b+c) :: \text{Rad}^2 : \text{Sin}^2\,\tfrac{1}{2}\text{A}.$$

For, by last theorem, we have

$$\frac{\text{Cos.}\,a \times \text{R} - \text{Cos.}\,b\ \text{Cos.}\,c}{\text{Sin.}\,b\ \text{Sin.}\,c} = \frac{\text{Cos. A}}{\text{Rad.}};$$

therefore, multiplying both sides of this equation by Rad., and subtracting the results from R; observing also that $\text{R}^2 - \text{R Cos. A} = 2\text{Sin}^2\,\frac{1}{2}\text{A}$, (Formula XVIII., page 56,) we find

$$\frac{\text{R Sin.}\,b\ \text{Sin.}\,c + \text{R Cos.}\,b\ \text{Cos.}\,c - \text{R}^2\ \text{Cos.}\,a}{\text{Sin.}\,b\ \text{Sin.}\,c} = \frac{2\text{Sin}^2\,\frac{1}{2}\text{A}}{\text{R}}.$$

But, by Formula XII., page 55,

$$\text{R (Sin.}\,b\ \text{Sin.}\,c + \text{Cos.}\,b\ \text{Cos.}\,c) = \text{R}^2\ \text{Cos.}\,(b - c);$$

therefore

$$\frac{\text{R}^2\ (\text{Cos.}\,(b-c) - \text{Cos.}\,a)}{\text{Sin.}\,b\ \text{Sin.}\,c} = \frac{2\text{Sin}^2\,\frac{1}{2}\text{A}}{\text{R}};$$

and by Formula IV., page 54,

$$\text{R}^2\ (\text{Cos.}\,(b-c) - \text{Cos.}\,a) = 2\text{R Sin.}\,\tfrac{1}{2}(a+b-c)\ \text{Sin.}\,\tfrac{1}{2}(a-b+c)$$

Wherefore

$$\frac{\text{Sin.}\,\frac{1}{2}(a+b-c)\ \text{Sin.}\,\frac{1}{2}(a-b+c)}{\text{Sin.}\,b\ \text{Sin.}\,c} = \frac{\text{Sin}^2\,\frac{1}{2}\text{A}}{\text{R}^2}:$$

And

Sin. b Sin. c : Sin. $\frac{1}{2}(a+b-c)$ Sin. $\frac{1}{2}(a-b+c)$: : R^2 : $\text{Sin}^2\ \frac{1}{2}A$.

THEOREM XIV.

In any spherical triangle, as the rectangle contained by the sines of the two sides is to the rectangle contained by the sines of half the sum and half the difference of the base and the sum of the sides, so is the square of the radius to the square of the cosine of half the vertical angle.

Let a, b, c, be the three sides of a spherical triangle, and A the vertical angle opposite the base a, then,

Sin. b Sin. c : Sin. $\frac{1}{2}(b+c+a)$ Sin. $\frac{1}{2}(b+c-a)$: : R^2 . $\text{Co}^{\cdot 2}\ \frac{1}{2}A$.

For, since

$$\frac{\text{R Cos. } a - \text{Cos. } b \text{ Cos. } c}{\text{Sin. } b \text{ Sin. } c} \quad \frac{\text{Cos. A}}{\text{R}}$$

multiplying both sides by R, and adding the results to R, observing also that $R^2 + \text{R Cos. A} = 2\text{Cos}^2\ \frac{1}{2}A$, we obtain

$$\frac{\text{R (Sin. } b \text{ Sin. } c - \text{Cos. } b \text{ Cos. } c) + R^2 \text{ Cos. } a}{\text{Sin. } b \text{ Sin. } c} = \frac{2\text{Cos}^2\ \frac{1}{2}A}{R}$$

that is

$$\frac{R^2 (\text{Cos. } a - \text{Cos. } (b+c))}{\text{Sin. } b \text{ Sin. } c} = \frac{2\text{Cos}^2\ \frac{1}{2}A}{R},$$

hence

$$\frac{\text{Sin. } \frac{1}{2}(b+c+a) \text{ Sin. } \frac{1}{2}(b+c-a)}{\text{Sin. } b \text{ Sin. } c} = \frac{\text{Cos}^2\ \frac{1}{2}A}{R^2}$$

Wherefore

Sin. b Sin. c : Sin. $\frac{1}{2}(b+c+a)$ Sin. $\frac{1}{2}(b+c-a)$: : R^2 : $\text{Cos}^2\ \frac{1}{2}A$.

THEOREM XV.

In any spherical triangle, as the rectangle contained by the sines of the angles at the base is to the rectangle contained by the co-

sines of half the sum and half the difference of the vertical angle and the sum of the angles at the base, so is the square of the radius to the square of the sine of half the base of the triangle.

Let B, C, denote the angles at the base of a spherical triangle, A the vertical angle, and a the opposite side or base, then,

Sin. B Sin. C : Cos. $\frac{1}{2}$(B + C + A) Cos. $\frac{1}{2}$(B + C − A) : : R^2 : $\text{Sin}^2\ \frac{1}{2}a$.

For, let a', b', c', denote the sides of the supplemental triangle, and A′ the angle opposite a', so that a' is the supplement of A, and A′ of a, b' the supplement of B, and c' the supplement of C: then from Theor. XIV., we have

Sin. b' Sin. c' : Sin. $\frac{1}{2}(b' + c' + a')$ Sin. $\frac{1}{2}(b' + c' - a')$: : R^2 : $\text{Cos}^2\ \frac{1}{2}A'$.

Substituting for a', b', c', A′, their equals, and taking the sine and cosine of the arch or angle for the sine and cosine of its supplement, we obtain

$$\text{Sin. B Sin. C : Sin.}\left(180^\circ + \left(90^\circ - \frac{B+C+A}{2}\right)\right)\text{Cos.}\ \tfrac{1}{2}(B+C-A)$$

$$: : \text{Rad}^2 : \text{Sin}^2\ \tfrac{1}{2}a.$$

But, if P be any arch or angle,

$$\text{Sin.}\ (180^\circ + (90^\circ - P)) =$$

$$\frac{\text{Sin. }180^\circ\ \text{Cos. }(90 - P) + \text{Cos. }180^\circ\ \text{Sin. }(90^\circ - P)}{R};$$

therefore, since Sin. $180^\circ = 0$, Cos. $180^\circ = -$ R, and Sin. $(90^\circ - P) =$ Cos. P, we have

$$\text{Sin.}\ (180^\circ + (90^\circ - P)) = - \text{Cos. P.}$$

Hence, attending only to the numerical value of the cosine, the above proportion becomes

Sin. B Sin. C : Cos $\frac{1}{2}$(B + C + A) Cos. $\frac{1}{2}$(B + C − A) : : Rad^2 : $\text{Sin}^2\ \frac{1}{2}a$.

THEOREM XVI.

In any spherical triangle, as the rectangle contained by the sines of the angles at the base is to the rectangle contained by the cosines of half the sum and half the difference of the vertical angle and the difference of the angles at the base, so is the square of the radius to the square of the cosine of half the base of the triangle.

Let A, B, C, and a denote the same things as in last theorem, then

$$\text{Sin. B Sin. C} : \text{Cos.} \tfrac{1}{2}(\text{A} + \text{B} - \text{C}) \ \text{Cos.} \tfrac{1}{2}(\text{A} - \text{B} + \text{C}) :: \text{Rad}^2 \ \text{Cos}^2 \tfrac{1}{2}a.$$

For, by applying Theor. XIII. to the supplemental triangle, we have

$$\text{Sin. } b' \ \text{Sin. } c' : \text{Sin.} \tfrac{1}{2}(a' + b' - c') \ \text{Sin.} \tfrac{1}{2}(a' - b' + c') :: \text{R}^2 : \text{Sin}^2 \tfrac{1}{2}\text{A}',$$

and, by substituting as before, we have

$$\text{Sin. B Sin. C} \ . \ \text{Cos.} \tfrac{1}{2}(\text{A} + \text{B} - \text{C}) \ \text{Cos.} \tfrac{1}{2}(\text{A} - \text{B} + \text{C}) \ .: \text{Rad}^2 : \text{Cos}^2 \tfrac{1}{2}a.$$

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13. We proceed now to the application of the preceding theorems, to the solution of the several cases of oblique-angled spherical triangles. Every oblique-angled triangle may be resolved into two right-angled triangles, and the unknown parts found by the principles applicable to the solution of right-angled triangles. But a much better solution may be obtained, without the aid of a perpendicular, from Theorems I., VIII., IX., &c....XVI.

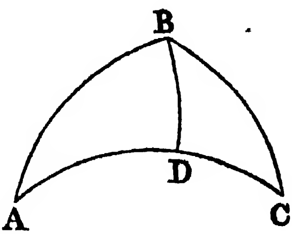
PROBLEM I.

Given two sides, and the included angle of an oblique-angled spherical triangle, to find the other angles and the remaining side.

Ex. 1. In the oblique-angled triangle ABC, let the side AB be 48° 30′, the side AC 68° 56′, and the angle A 36° 14′; required the remaining parts of the triangle?

Solution.

From B, either of the required angles, draw the arch BD perpendicular to AC the opposite side, then, in the right-angled triangle ABD, we have given the hypotenuse and the angle A. Hence,



To find AD and DC.

As radius	10.000000	
To Tan. AB, 48° 30′	10.053192	
So is Cos. A, 36° 14′	9.906667	(Theor. II. §. 4.)
To Tan. AD, 42° 21′ 21″	9.959859	

Because AB is less than a quadrant, AD and angle A are of the same species, (§. 10.) so that AD is less than a quadrant.

AC = 68° 56′
AD = 42° 21′ 21″

DC = 26° 34′ 39″

To find BC.

As Cos. AD, 42° 21′ 21″	9.868630	
To Cos. DC, 26° 34′ 39″	9.951498	
So is Cos. AB, 48° 30′	9.821265	(Theor. II. §. 12.
To Cos. BC, 36° 41′ 9″	9.904133	

Because AB is less than a quadrant, therefore AD, DB, are of the same species: but DC is of the same species with AD: hence BD and DC are of the same species, and BC is therefore less than a quadrant. (§. 8.)

To find angle C.

As Sin. DC, $26^\circ\ 34'\ 39''$	9.650703	
To Sin. AD, $42^\circ\ 21'\ 21''$	9.828488	
So is Tan. A, $36^\circ\ 14'$	9.864975	(Theor. V. §. 12.)
To Tan. C, $47^\circ\ 48'\ 58''$	10.042760	

Since each of the angles A and C is of the same species with the perpendicular, (§. 7.) C is of the same species with A, and is therefore less than a right angle.

To find angle B.

As Sin. AB, $48^\circ\ 30'$	9.874456	
To Sin. AC, $68^\circ\ 56'$	9.969957	
So Sin. C, $47^\circ\ 48'\ 58''$	9.869815	(Theor. I. §. 12.)
To Sin. Supp. B, $67^\circ\ 24'\ 17''$	9.965316	
180°		

Angle B $= 112^\circ\ 35'\ 43''$

If, from C, a perpendicular were let fall upon the opposite side AB, the perpendicular would be found to fall without the triangle. Hence the angle A and the supplement of B are each of the same species with the perpendicular (§. 7.), therefore the angles A and B are of different species, so that the angle B is greater than a right angle.

The unknown parts of the triangle may be found without the aid of the perpendicular arch BD, by proceeding as follows:

To find the angles B and C.

As Sin. $\frac{1}{2}$(AC + AB) $58^\circ\ 43'$	9.931768	
To Sin. $\frac{1}{2}$(AC — AB) $10^\circ\ 13'$	9.248883	
So is Cot. $\frac{1}{2}$ angle A, $18^\circ\ 7'$	10.485223	(Theor. VIII. §. 12.)
To Tan. $\frac{1}{2}$(B — C) $32^\circ\ 23'\ 22''$	9.802338	

Again,

As Cos. $\frac{1}{2}$(AC + AB) 58° 43′	9.715394	
To Cos. $\frac{1}{2}$(AC — AB) 10° 13′	9.993059	
So is Cot. $\frac{1}{2}$angle A, 18° 7′	10.485223	(Theor. IX. §. 12.)
To Tan. $\frac{1}{2}$(B + C) 80° 12′ 20″	10.762888	
.. $\frac{1}{2}$(B — C) 32° 23′ 22″		
Angle B = 112° 35′ 42″		
Angle C = 47° 48′ 58″		

To find CB.

As Sin. angle C, 47° 48′ 58″	9.869815	
To Sin. angle A, 36° 14′	9.771643	
So is Sin. AB, 48° 30′	9.874456	(Theor. I. §. 12.)
To Sin. BC, 36° 41′ 9″	9.776284	

Ex. 2. In the spherical triangle ABC, there are given the side AB = 114° 30′, the side BC = 56° 40′, and the angle ABC = 62° 52′; required the remaining parts of the triangle?—*Ans.* Remaining side AC, 83° 10′ 28″, angle A, 48° 29′ 26″, angle C, 125° 21′ 12″.

PROBLEM II.

Given two angles, and the side between them, to find the remaining angle and the other sides.

Ex. 1. In the oblique-angled triangle ABC, let the side AB be 62° 13′, the angle A, 35° 26′, and the angle B, 116° 40′; required the remaining parts of the triangle?—*Ans.* The side AC = 84° 17′; BC = 40° 12′ 21″; angle C = 52° 36′ 51″.

Solution.

From B, either of the given angles, draw the arch BD perpendicular to the opposite side AC, (See fig. Prob. I.), then,

To find angles ABD, CBD.

Rad. : Cos. AB : : Tan. A : Cot. ABD. (Theor. VI. §. 4.)

Angle CBD = ABC — ABD.

To find BC.

Cos. CBD : Cos. ABD : : Tan. AB : Tan. BC. (Theor. III. §. 12.)

The side BC is less than a quadrant, because the angle CBD, and the angle A which determines the species of the perpendicular BD, are of the same species. (§. 7. and §. 10.)

To find angle C.

Sin. ABD : Sin. CBD : : Cos. A : Cos. C. (Theor. IV. §. 12.)

Angles A and C are of the same species, each being of the same species with the perpendicular BD, therefore angle C is less than a right angle.

To find AC.

Sin. A : Sin. B : : Sin. BC : Sin. AC. (Theor. I. §. 12.)

If a perpendicular arch were drawn from the angle A to the opposite side BC, the angle included between that perpendicular and the side AC would be found to be of the same species with the supplement of angle B; therefore AC is less than a quadrant. (§. 7. and §. 10.)

Or thus, without drawing a perpendicular.

To find AC and BC.

Sin. $\frac{1}{2}$(B + A) : Sin. $\frac{1}{2}$(B — A) : : Tan. $\frac{1}{2}$AB : Tan. $\frac{1}{2}$(AC — BC). (Theor. X. §. 12.)

Cos. $\frac{1}{2}$(B + A) . Cos. $\frac{1}{2}$(B — A) : : Tan. $\frac{1}{2}$AB : Tan. $\frac{1}{2}$(AC + BC). (Theor. XI. §. 12.)

Half the sum, and half the difference of the sides being thus obtained, the sides AC, BC, themselves are easily found.

To find angle C.

Sin. BC : Sin. AB : : Sin. A : Sin. C. (Theor. I. §. 12.)

Ex. 2. Given the side BC, 117° 25′ 54″, the angle B, 70°, and the angle C, 122°, to find the other parts of the spherical tri-

angle?—*Ans.* The remaining angle BAC, 100° 42′ 45″; the side AC, 58° 5′ 4″, the side AB, 180°.

PROBLEM III.

Given the two sides, and an angle opposite to one of them, to find the other angles and the remaining side.

Ex. 1. In the oblique-angled triangle ABC, let the side AB be 51° 6′, the side BC, 42° 17′, and the angle BAC, 37° 25′; required the remaining parts of the triangle?—*Ans.* Angle C = 44° 39′ 18″; angle B = 118° 10′ 29″, and side AC = 77° 26′ 43″.

Solution.

To find angle C.

Sin. BC : Sin. AB : : Sin. A : Sin C. (Theor. I. §. 12.)

Angle C is ambiguous.

To find angle ABC.

From B, draw BD perpendicular to AC; then,

R : Cos. AB : : Tan. A : Cot. ABD; (Theor. VI. §. 4.)

and Tan. BC : Tan. AB : : Cos. ABD : Cos. DBC. (Theor. III. §. 12.)

Angle ABC = ABD $\pm$ DBC.

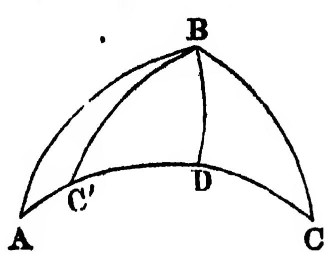
The ambiguous sign + or — renders ABC ambiguous.

To find AC.

Sin. A . Sin. ABC : : Sin. BC : Sin. AC. (Theor. I. §. 12.)

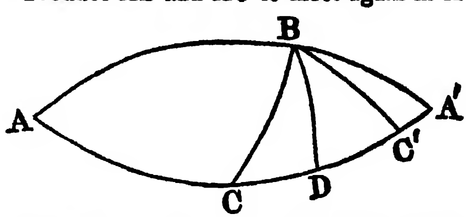
Angle ABC being ambiguous, AC is also ambiguous.

Note.—In this problem, as in the analogous problem of plane triangles, there are, within certain limits, two triangles which satisfy the conditions. First, let us suppose the given angle A less than 90°, and the side AB also less than 90°: then, since the angle A and the side DB of the right-angled triangle ABD are of the



same species, (§. 7.) DB is less than 90°; so that BD is the shortest distance of the point B from the arch AC, and taking DC′ = DC, the oblique arches BC, BC′ are equal, and encrease as the points C and C′ recede from D. When BC is greater than BD, but less than BA, there will therefore be two triangles, ABC, ABC′, each of which satisfies the conditions of the question: but if BC be greater than BA, the point C′ will fall beyond A, so that there will be only one triangle ABC.

Again, let us suppose the angle A less than 90°, but the side AB greater than 90°. Produce AB and AC to meet again in A′: then it is evident, that when BC is greater than BD, but less than BA′, there will be two triangles, ABC, ABC′, but when BC exceeds BA′, the point C′ will fall beyond A′, so that there can be but one triangle ABC.



When the sum of AB and BC is less than 180°, there will therefore be two triangles, ABC, and ABC′; but when their sum is greater than 180°, there will be only one triangle ABC.

Considering in the same manner the case in which the given angle A is greater than 90°, we may determine under what circumstances the same *data* admit of two triangles, and when they admit only of one.

Upon the whole, let A denote the given angle of the triangle, a its opposite side, and c the other given side: then,

If A be less than 90°, and c less than 90°, and a greater than c, there is but one triangle: but if a is less than c, there are two triangles.

If A be less than 90°, and c greater than 90°, and $a + c$ greater than 180°, there is only one triangle; but if $a + c$ be less than 180°, there are two triangles.

If A be greater than 90°, and c less than 90°, and $a + c$ greater than 180°, there are two triangles; but if $a + c$ be less than 180°, there is but one triangle.

If A be greater than 90°, and c greater than 90°, and a greater than c, there are two triangles, but if a be less than c, there is but one triangle.

Or thus, without drawing a perpendicular.

To find angle C.

Sin. BC : Sin. AB : : Sin. A : Sin. C. (Theor. I. §. 12.)

To find angle B.

Sin. $\frac{1}{2}$(AB — BC) : Sin. $\frac{1}{2}$(AB + BC) : : Tan. $\frac{1}{2}$(C — A) : Cot. $\frac{1}{2}$B. (Theor. VIII. §. 12.)

To find AC.

Sin. A : Sin. ABC : : Sin. BC : Sin. AC. (Theor. I. §. 12.)

Ex. 2. Let the side AC be 114° 30′, the side AB, 56° 40′, and the angle B, opposite to the former, 125°, 20′, required the side BC, and the remaining angles A and C?—*Ans.* The side BC, 83° 11′ 52″; angle C, 48° 30′ 24″; and angle A, 62° 53′ 59″.

PROBLEM IV.

Given two angles and a side opposite to one of them, to find the other sides, and the remaining angle.

Ex. 1. In the oblique-angled triangle ABC, let the angle A be 31° 3′, the angle B, 129° 4′, and the side AC, 75° 4′; required the remaining parts of the triangle?—*Ans.* The side BC, 39° 55′ 56″; the side AB, 44° 53′ 40″; and angle C, 34° 33′ 6″.

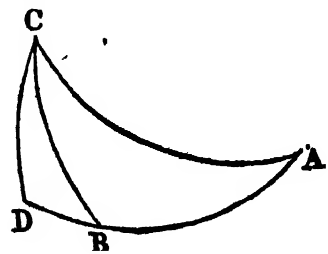
Solution.

To find BC.

Sin. ABC : Sin. A : : Sin. AC : Sin. BC. (Theor. I. §. 12.)

To find AB.

From the unknown angle C, draw CD perpendicular to AB; then



Rad. : Cos. A : : Tan. AC : Tan. AD ; (Theor. II. §. 4.)
and Tan. B : Tan. A : : Sin. AD : Sin. BD. (Theor. V. §. 12.)

$$AB = AD - BD.$$

To find angle ACB.

Sin. BC : Sin. AB : . Sin. A : Sin. ACB. (Theor. I. §. 12.)

Note.—In this problem, as in the preceding, there are, within certain limits, two triangles which satisfy the conditions. The rules for determining whether in any particular case the *data* admit of two triangles or of only one, may be deduced by considering the supplemental triangle. They are as follows :

Let A, B denote the two given angles, and b the side opposite to the angle B. Then,

If b be greater than 90°, and A greater than 90°, and B less than A, there is only one triangle ; but if B be greater than A, there are two triangles.

If b be greater than 90°, and A less than 90°, and B + A less than 180°, there is only one triangle ; but if B + A be greater than 180°, there are two triangles.

If b be less than 90°, and A greater than 90°, and B + A less than 180°, there will be two triangles ; but if B + A be greater than 180°, there will be only one triangle.

If b be less than 90°, and A less than 90°, and B less than A, there will be two triangles ; but if B be greater than A, there will be only one triangle.

Applying this last maxim to the above example, it appears that the *data* admit of only one triangle.

Or thus, without drawing a perpendicular.

To find BC.

Sin. ABC : Sin. A : : Sin. AC : Sin. BC, (Theor. I. §. 12.)

To find angle ACB.

Sin. $\frac{1}{2}$(AC — BC) : Sin. $\frac{1}{2}$(AC + BC) : : Tan. $\frac{1}{2}$(B — A) : Cot. $\frac{1}{2}$ACB. (Theor. VIII. §. 12.)

To find AB.

Sin. A : Sin. ACB : : Sin. BC : Sin. AB. (Theor. I. §. 12.)

Ex. 2. If one side of a spherical triangle be 79° 17′ 14″, its opposite angle 62° 34′ 6″, also another angle of the triangle 50°; it is required to determine the unknown parts of the triangle?—*Ans.* One of the remaining sides 58°; the other remaining side 110°, the remaining angle 121° 54′ 56″.

PROBLEM V.

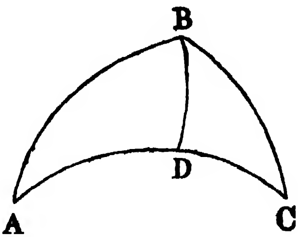
Given the three sides, to find the angles.

Ex. 1. In the oblique-angled triangle ABC, let the side AB be 53° 30′, the side AC, 74° 56′, and the side BC, 45° 48′; required the three angles of the triangle?—*Ans.* Angle A, 45° 39′ 25″, angle B, 105° 34′ 33″, and angle C, 53° 18′ 44″.

Solution.

From B, one of the angles of the triangle, draw BD perpendicular to AC, the opposite side.

To find the segments AD and DC.



Tan. $\frac{1}{2}$AC : Tan. $\frac{1}{2}$(AB + BC) . : Tan. $\frac{1}{2}$(AB BC) : Tan. $\frac{1}{2}$(AD — DC). (Theor. VI. §. 12.)

When the perpendicular falls without the triangle, this proportion gives $\frac{1}{2}$(AD + DC).

$$\tfrac{1}{2}(AD + DC) + \tfrac{1}{2}(AD - DC) = AD$$

$$\tfrac{1}{2}(AD + DC) - \tfrac{1}{2}(AD - DC) = DC.$$

To find angle A.

Tan. AB : Tan. AD : : Rad. : Cos. A. (Theor. II. §. 4.)

To find angle C.

Tan. BC : Tan. DC : : Rad. : Cos. C. (Theor. II. §. 4.)

To find angle B.

$\left.\begin{matrix}\text{Sin. BC}\\ \text{Sin. AB}\end{matrix}\right\}$: Sin. AC : : $\left\{\begin{matrix}\text{Sin. A}\\ \text{Sin. C}\end{matrix}\right\}$: Sin. B. (Theor. I. §. 12.)

Or thus, without drawing a perpendicular.

To find angle A.

Sin. AB Sin. AC : Sin. $\frac{1}{2}$(BC + AC — AB) Sin. $\frac{1}{2}$(BC — AC + AB) : : Rad^2 : Sin^2 $\frac{1}{2}$A. (Theor. XIII. §. 12.)

Or,

Sin. AB Sin. AC : Sin. $\frac{1}{2}$(AC + AB + BC) Sin. $\frac{1}{2}$(AC + AB — BC) : : Rad^2 : Cos^2 $\frac{1}{2}$A. (Theor. XIV. §. 12.)

Angles B and C must be found in the same manner; or, by Theor. I. §. 12.

To find angle B.

Sin. BC : Sin. AC : . Sin. A : Sin. B.

To find angle C.

Sin. BC : Sin. AB : : Sin. A : Sin. C.

Ex. 2. The three sides of a spherical triangle, are 65° 40′, 89° 46′ 45″, and 54° 39′ 14″ respectively; it is required to determine the three angles?—*Ans.* The angles are respectively equal to 59° 50′ 22″, 108° 23′ 36″, and 50° 42′ 55″.

PROBLEM VI.

Given the three angles, to find the sides.

Ex. 1. In the oblique-angled triangle ABC, let the angle A be 46°, 20′, the angle B, 123° 56′, and the angle C, 64° 36′; required the sides of the triangle?—*Ans.* The side AC = 113° 38′ 37″; AB = 85° 50′ 37″; BC = 53° 0′ 12″.

Solution.

From B, one of the angles of the triangle draw BD perpendicular to the opposite side AC. (See fig. Prob. V.)

To find the segments ABD, DBC, into which ABC is divided by the perpendicular.

Cot. $\frac{1}{2}$B . Tan. $\frac{1}{2}$(A + C) . : Tan. $\frac{1}{2}$(C — A) : Tan. $\frac{1}{2}$(ABD — DBC). (Theor. VII. §. 12.)

When the perpendicular falls without the triangle, this proportion gives $\frac{1}{2}$(ABD + DBC):

Then,

$$\tfrac{1}{2}(ABD + DBC) + \tfrac{1}{2}(ABD - DBC) = ABD.$$
$$\tfrac{1}{2}(ABD + DBC) - \tfrac{1}{2}(ABD - DBC) = DBC.$$

To find AB

Rad. : Cot. A . Cot. ABD . Cos. AB. (Theor. VI. Cor. §. 4.)

To find BC.

Rad . Cot. C . Cot. DBC . Cos. BC.

To find AC

$$\left.\begin{matrix}\text{Sin. C}\\ \text{Sin. A}\end{matrix}\right\} : \text{Sin. B} : : \left\{\begin{matrix}\text{Sin. AB}\\ \text{Sin. BC}\end{matrix}\right\} : \text{Sin. AC.}$$ (Theor. I. §. 12.)

Or thus, without drawing a perpendicular.

To find AB.

Sin. A Sin. B : Cos. $\frac{1}{2}$(A + B + C) Cos. $\frac{1}{2}$(A + B — C) : : R^2 : Sin^2 $\frac{1}{2}$AB. (Theor. XV. §. 12.)

Or

Sin. A Sin. B : Cos. $\frac{1}{2}$(B — A + C) Cos. $\frac{1}{2}$(B — A — C) : : R^2 : Cos^2 $\frac{1}{2}$AB. (Theor. XVI. §. 12.)

The sides BC and AC may be found in the same manner; or, by Theor. I. §. 12.

To find BC.

Sin. C · Sin. A : : Sin. AB : Sin. BC.

To find AC.

Sin. C : Sin. B . : Sin. AB : Sin. AC.

Ex. 2. In the oblique-angled spherical triangle ABC, there are given the angle A, 131° 35′, the angle B, 63° 30′ and the angle C, 59° 25′; it is required to find the sides of the triangle?—*Ans.* The side AB, 71° 28′ 40″, the side AC, 80° 17′ 56″, the side BC, 124° 31′ 39″ *.

APPLICATION

OF

SPHERICAL TRIGONOMETRY

TO THE

SOLUTION OF ASTRONOMICAL PROBLEMS.

Prob. I. Supposing the obliquity of the ecliptic, or the sun's greatest declination, to be 23° 27′ 30″, it is required to find his right ascension and declination, when his place in the ecliptic is 18° 24′, Taurus?—*Ans.* Right ascension, 45° 56′ 11″. Declination, 17° 19′ 7″ N.

Prob. II. In latitude 55° 58′ N., on a particular day in the season of spring, the sun's meridian altitude was observed to be

* The answers to the examples under the several problems, being carried to seconds, it may be proper to remark, that the different methods of solution, may sometimes give results different from each other by one or two seconds This arises from the tables giving the sines, tangents, &c. to no more than six decimal places.

49° 15′ 24″; required his place in the ecliptic?—*Ans.* The sun's place is 11° 16′ 15″ of Taurus.

Prob. III. If the sun's place in the ecliptic be 18° 24′ of Taurus, it is required to find the time of his rising and setting on that day, and also the point of the compass upon which he rises and sets, supposing the latitude of the place to be 55° 58′ N.?—*Ans.* The sun rises E. 32° 8′ N. at 4^h 10′ morning; and sets W. 32° 8′ N. at 7^h 50′ evening*.

Prob. IV. In latitude 60° 4′ N., the sun being north of the equator, his altitude at six o'clock in the evening, was found to be 18° 19′, required the declination and azimuth of the sun at that time?—*Ans.* Sun's declination, 21° 15′ 45″ N. Azimuth 79° 0′ 41″ from the north.

Prob. V. At Edinburgh, in latitude 55° 58′ N., on the longest day, 21st June, what is the sun's altitude, and what is the hour, when he is due east or west, supposing the obliquity of the ecliptic to be 23° 27′ 30″?—*Ans.* The sun is due east at 7^h 8′ 10″ morning, and due west at 4^h 51′ 50″ afternoon. Altitude at that time, 28° 42′ 33″.

Prob. VI. In what latitude does the sun rise at 4^h 30′ A. M., when his declination is 21° 16′ N.?—*Ans.* In latitude 44° 30′ 55″ N.

Prob. VII. At London, in latitude 51° 32′ N., the sun's true altitude was found, in the afternoon, to be 36° 10′, his declination on that day being 16° 34′ N.; required the apparent time, and also the sun's azimuth?—*Ans.* Apparent time, 3^h 28′ 6″. Sun's azimuth, 69° 22′ 15″ from the south.

Prob. VIII. At what time does twilight begin and end at Edinburgh, in latitude 55° 58′ N., when the sun's place in the ecliptic is 19° 18′ of Libra,—supposing the inclination of the ecliptic to be 23° 27′ 30″, and that twilight continues till the sun is 18° below the horizon?—*Ans.* Morning twilight begins at 4^h 35′ 29″; and evening twilight ends at 7^h 24′ 31″.

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* It is to be observed, that here no allowance is made for the daily change of declination, nor for the effects of parallax and refraction.

PROB. IX. In latitude $51^\circ\ 32'$, at $2^h\ 55'\ 20''$ P. M. apparent time, the sun's altitude was observed to be $40^\circ\ 29'$, required the sun's azimuth and declination?—*Ans* Azimuth, $60^\circ\ 58'\ 47''$ from the south. Declination, $16^\circ\ 11'\ 21''$ N.

PROB. X. Supposing the sun's declination to be $18^\circ\ 30'$ N., and that at $10^h\ 11'\ 26''$ A M, his true altitude above the horizon of a place in the northern hemisphere, was found to be $52^\circ\ 35'$, required the latitude of the place of observation?—*Ans.* The latitude is $48^\circ\ 51'$ N.

Note.—It is evident, that of these six things, the latitude, the altitude of the sun, his declination, the horary angle, the azimuth, and the angle at the sun,—any three being given, the rest may be found.

PROB. XI. At Edinburgh, in Lat. $55^\circ\ 58'$ N, and Long $3^\circ\ 12'$ W, on the 26th February 1816. at some hour of the night, the star Arcturus was observed in the eastern hemisphere, elevated above the horizon, $28^\circ\ 46'$, at what hour was the observation made?—the right ascension of the star being $14^h\ 7'\ 16''$, its declination $20^\circ\ 8'\ 38''$ N., the sun's right ascension for noon at Greenwich, as found by the Nautical Almanack, $22^h\ 34'\ 17''$, and the daily change of the sun's right ascension $3'\ 46''$.—*Ans.* Apparent time of observation, $10^h\ 58'\ 45''$ P. M.

PROB XII What is the distance between the fixed stars Aldebaran in Taurus, and Procyon in Canis Minor, the right ascension of the former being $66^\circ\ 20'\ 24''$, and its declination $16^\circ\ 7'\ 51''$ N., —the right ascension of the latter being $112^\circ\ 24'\ 54''$, and its declination $5^\circ\ 41'\ 26''$ N?—*Ans.* Distance $= 46^\circ\ 19'\ 3''$.

PROB. XIII. Let the longitude of a particular star be supposed to be $198^\circ\ 27'$, and its latitude $31^\circ\ 2'$ N., it is required to find its right ascension and declination, also the angle contained by the circle of latitude and the circle of declination which pass through that star?—*Ans.* Right ascension, $209^\circ\ 11'\ 6''$. Declination, $21^\circ\ 24'\ 22''$. Angle at the star, $23^\circ\ 55'\ 44''$.

PROB. XIV. Supposing the place of the sun to be $18^\circ\ 24'$ of Taurus, and that, at twenty minutes past nine o'clock in the evening, the true altitude of a star, which is observed in the south-east quarter, is found to be $43^\circ\ 15'$ above the horizon, the angle between the vertical circle on which the star is observed, and the meridian being $47^\circ\ 24'$, and the latitude of the place of observa-

tion being 58° 51′, it is required to determine the right ascension and declination of the star, and also the time of its passing the meridian?—*Ans* Right ascension, 220° 33′ 55″. Declination, 19° 21′ 6″. Time of passing the meridian, 11^h 38′ 31″ in the evening.

Prob. XV. Suppose the distance of a comet or new star, which is to the north of the ecliptic, to be 65° 47′ from Sirius, whose latitude is 39° 33′ S., and longitude 3^s 11° 13′, and 51° 6′ from Procyon, whose latitude is 15° 58′ S., and longitude 3^s 22° 55′, it is required to find the latitude and longitude of the comet or star?—*Ans*. Lat. 22° 51′ 43″ N. Long 78° 57′ 43″.

Prob XVI. At a place in the northern hemisphere, the sun's declination being 19° 39′ 12″ N., the true altitude of his centre, in the forenoon was found to be 38° 20′ 30″, and at the end of an hour and a half afterwards, 50° 26′ 10″, required the latitude of the place?—*Ans*. 51° 32′ N. nearly.

Prob. XVII. At 9^h 23′ 20″, A. M. apparent time, the true altitude of the sun's centre was 34° 29′, and at 11^h 9′ 32″, the altitude was 42° 19′, required the latitude and declination?—*Ans*. Latitude 57° 7′ N. Declination 10° 27′ N.

Prob XVIII. On 1st July 1812, in latitude 57° 9′ N., and longitude 2° 8′ W., the stars Vega and Altair were observed east of the meridian on the same vertical circle at 10^h 9′ P. M. *per* watch; required the apparent time of observation and the error of the watch the right ascension of Vega being 277° 38′ 48″, its declination, 38° 36′ 47″ N., the right ascension of Altair, 295° 24′ 23″, its declination, 8° 22′ 45″ N., also the right ascension of the sun for noon at Greenwich, as found by the Nautical Almanack, 6^h 40′ 53 9″, and the daily change of the sun's right ascension, 4′ 8″?—*Ans*. Apparent time, 10^h 11′ 43″, Watch slow, 2′ 43″.

Prob. XIX. Required to determine the true distance between the sun and moon, from the following *data*, viz. Moon's apparent altitude, 22° 15′. Sun's apparent altitude, 21° 35′. Apparent distance, 119° 20′ 34″. Moon's parallax in altitude, 53′ 41″. Moon's refraction at the observed altitude, 2′ 19″. Sun's parallax in altitude, 8″. Sun's refraction at observed altitude 2′ 24″?—*Ans*. True distance 118° 46′ 48″.

AN

ABSTRACT OF THE RULES

FOR THE

MENSURATION OF PLANE SURFACES

AND OF

SOLIDS.

The term *Mensuration*, is usually employed to denote a system of rules and methods, by which numerical measures of geometrical quantities are obtained.

In every practical application of mathematics, it is requisite to express magnitudes of all kinds by numbers. For this purpose, some determinate magnitude of the same kind with that which is to be measured, must be assumed as a measuring unit, and the number expressing how often this unit is contained in the said magnitude, is the numerical value or measure of the magnitude.

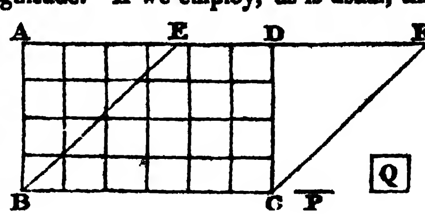
MENSURATION OF PLANE SURFACES.

PROBLEM I.

To find the area of a parallelogram, whether it be a square, a rectangle, a rhombus, or a rhomboid.

Rule 1.—Multiply the length by the perpendicular breadth, and the product will be the area.

Demonstration. The measuring unit of surfaces, may be of any determinate figure and magnitude. If we employ, as is usual, the square described upon the measuring unit of lines, it is evident, that, in order to find how often this square is contained in the rectangle ABCD, it is only necessary to multiply together the numbers



which express how often the linear unit is contained in its length and breadth. But the parallelogram EBCF is equal to the rectangle; hence the reason of the rule is manifest

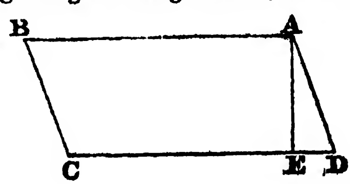
Ex. 1. Required the area of a square, whose side is $19\frac{1}{2}$ feet?—*Ans.* 380.25 feet.

2. Required the area of a rectangle, whose length is 134.75 chains, and breadth, 9.25 chains?—*Ans.* 124 ac. 2 ro. 23 po.

3. Required the area of a parallelogram, whose length is 49 feet 9 inches, and perpendicular breadth is 7 feet 3 inches?—*Ans.* $360\frac{11}{16}$ feet, or 360 feet 8′ 3″.

RULE 2.—As radius,
To sine of any angle of the parallelogram,
So is the product of the sides containing the angle,
To the area of the parallelogram.

Demonstration. For, in the right-angled triangle ADE, Rad. : Sin. D . . AD : AE, and multiplying the two last terms of this proportion by DC, we obtain Rad. Sin. D . · AD × DC : AE × DC, but AE × DC = area of the parallelogram, hence, the reason of the rule is obvious.



Ex. 1. Suppose the sides AD and DC to be 27 feet, and 47.25 feet respectively, and the included angle D 38° 20′; required the area of the parallelogram?—*Ans.* 791.27 square feet.

2. Suppose the sides of a parallelogram to be 79.75 yards, and 84.32 yards respectively, and the included angle 46° 35′; required the area?—*Ans.* 4884.5 square yards.

PROBLEM II.

To find the area of a triangle.

RULE 1.—Multiply any one of its sides by the perpendicular let fall upon it from the opposite angle, and half the product will be the area.

For every triangle is half of a parallelogram of the same base and altitude.

Ex. 1. What is the area of a triangle ABC, whose base BC is 54, and perpendicular AD is 29.52 chains?—*Ans.* 79 acres, 2 roods, 32 64 poles.

2. What is the expense of paving a triangular court, at 4s. 6d. *per* square yard, one of its sides being 48 feet 6 inches, and the perpendicular from the opposite angle, $30\frac{1}{3}$ feet?—*Ans.* £18 : 9 : $9\frac{3}{4}$.

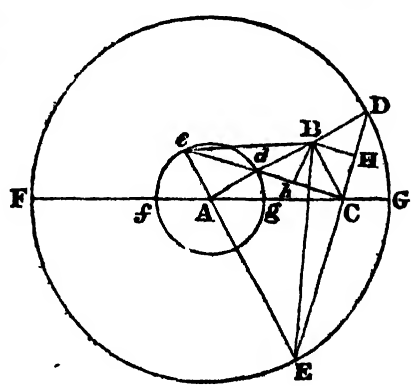
Rule 2.—As radius,
To sine of any angle of a triangle;
So is the product of the sides containing the angle,
To twice the area of the triangle.

This rule follows immediately from Rule 2. Prob. I.

Ex. What is the area of a triangle ABC, whose two sides AB, BC are 45 and 56.75 feet, and the included angle B, 36° 45′?—*Ans.* 763 98 square feet.

Rule 3.—When the three sides are given. Add together the three sides, and take one half of the sum. Next, from the said half sum, subtract each side severally, thus obtaining three remainders; lastly, multiply the said half sum and these three remainders all together, and extract the square root of the last product, for the area of the triangle.

Demonstration. Let ABC be any triangle, produce AB any one of its sides, and take BD and B*d*, each equal to BC, join CD and C*d*, and through A draw a line parallel to BC, meeting CD and C*d*, produced in E and *e*, thus the angle AED will be equal to the angle BCD, that is, to the angle BDC or ADC, and hence AE = AD: And in like manner, because the angle A*ed* is equal to BC*d*, that is, to the angle B*d*C or A*de*, therefore A*d* = A*e*. On A as a centre at the distance AD or AE, describe a circle meeting AC in F and G; and on the same centre with the distance A*d* or A*e*, describe another circle meeting AC in *f*, *g*, and draw BH and B*h* perpendicular to CD and C*d*.



Then, because BD, BC, and Bd are equal, the point C is in the circumference of a circle, of which Dd is the diameter; therefore CD and Cd are bisected in H and h, and the angle DCd is a right angle, and hence the figure CHBh is a rectangle, so that Bh = CH = $\frac{1}{2}$CD and BH = Ch = $\frac{1}{2}$Cd. Join BE and Be, then the triangle ABC is equal to each of the triangles BEC and BeC, but the triangle BEC = $\frac{1}{2}$EC × BH, that is $\frac{1}{4}$EC × Cd, and in like manner the triangle BeC = $\frac{1}{2}e$C × Bh, that is to $\frac{1}{4}e$C × CD. Therefore the triangle ABC = $\frac{1}{4}$EC × Cd, and also $\frac{1}{4}e$C × CD. Now, since CD : Cd : . CE × CD : CE × Cd, and also CD : Cd : : Ce × CD . Ce × Cd, therefore CE × CD : CE × Cd : : Ce × CD . Ce × Cd, that is, because CE × CD = FC × CG and Ce × Cd = fC × Cg,

$$FC \times CG : CE \times Cd : . Ce \times CD . fC \times Cg,$$

which last proportion, by taking one-fourth part of each of its terms, and substituting the triangle ABC instead of its equivalent values $\frac{1}{4}$CE × Cd and $\frac{1}{4}$Ce × CD, gives us

$$\tfrac{1}{2}FC \times \tfrac{1}{2}CG : \text{trian. } ABC : . \text{ trian. } ABC : \tfrac{1}{2}fC \times \tfrac{1}{2}Cg.$$

Hence trian^2 ABC = $\frac{1}{2}$FC × $\frac{1}{2}$CG × $\frac{1}{2}f$C × $\frac{1}{2}$Cg, and

$$\text{trian. } ABC = \sqrt{\tfrac{1}{2}FC \times \tfrac{1}{2}CG \times \tfrac{1}{2}fC \times \tfrac{1}{2}Cg}.$$

Now, since FA or AG = AD = AB + BC if we put AB + BC + AC = 2S, we have $\frac{1}{2}$FC = S, $\frac{1}{2}$CG = $\frac{1}{2}$(2S — 2AC) = S — AC, $\frac{1}{2}f$C = $\frac{1}{2}$(AC + (AB — BC)) = $\frac{1}{2}$(2S — 2BC) = S — BC, and $\frac{1}{2}$Cg = $\frac{1}{2}$(AC — (AB — BC)) = $\frac{1}{2}$(2S — 2AB) = S — AB. By substituting these values in the above expression for the area of the triangle ABC, we obtain,

$$\text{triangle } ABC = \sqrt{S \times (S - AC) \times (S - BC) \times (S - AB)}$$

This formula, when expressed in words, is exactly the rule above stated.

Ex. 1. Required the area of a triangle, whose three sides are 49.52, 75 and 37.46 chains respectively?—*Ans.* 81 acres, 2 roods, 3 poles.

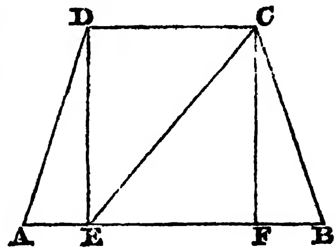
2. The three sides of a triangular field are 46 chains 37 links, 34 chains 4 links, and 51 chains 39 links, required the area?—*Ans.* 77 acres, 0 roods, 21.57 poles.

PROBLEM III.

To find the area of a trapezoid.

Rule.—Add together the two parallel sides, then multiply the sum by the perpendicular breadth or distance between, and half the product will be the area.

Demonstration. By dividing the trapezoid ABCD into right-angled triangles, we have the area $= AED + DEC + CEF + CBF = \frac{1}{2}DE \times AE + \frac{1}{2}DE \times DC + \frac{1}{2}FC \times FE + \frac{1}{2}CF \times FB = \frac{1}{2}DE \times (AE + DC + EF + FB) = \frac{1}{2}DE \times (AB + DC)$, from which the rule is sufficiently obvious.



Ex. 1. Required the area of a trapezoid ABCD, whose parallel sides CD and AB are 9.5 and 28.25 chains, and perpendicular breadth DE, 19.7 chains ?—*Ans.* 37 acres, 0 roods, 29.4 poles.

2. Given the parallel sides of a trapezoid equal to 140 3 feet, and 75.45 feet, also the perpendicular distance equal to 69.75 feet, required the area ?—*Ans.* 7524.28125 square feet.

3. How many square feet in a plank 13 inches broad at one end, and 15 inches at the other, the length being 16 feet 5 inches ?—*Ans.* 19 feet 1 inch 10″.

PROBLEM IV.

To find the area of any trapezium.

RULE.—Divide the trapezium into two triangles by a diagonal, then find the areas of these triangles, and add them together.

Note.—If two perpendiculars be let fall on the diagonal from the two opposite angles, the sum of these perpendiculars being multiplied by the diagonal, half the product will be the area of the trapezium. The reason of this rule is evident.

Ex. 1. Required the area of a trapezium of which the diagonal is 49.7 chains, and the perpendiculars falling upon it from the opposite angles 25.9 and 14.5 chains ?—*Ans.* 100 acres, 1 rood, 23.04 poles.

2. In the four-sided field ABCD, on account of obstacles in the two sides AB, CD, and in the perpendiculars the following measures only could be taken, namely, the two sides BC, 265, and AD, 220 yards, the diagonal AC, 378 yards, and the two distances of the perpendiculars from the ends of the diagonal, namely, AE, 100, and CF, 70 yards, required the area ?—*Ans.* 17 acres, 2 roods, 21 poles.

PROBLEM V.

To find the area of an irregular polygon.

RULE.—Draw diagonals, dividing the figure into triangles and trapeziums, then find the areas of all these separately, and add them together for the area of the polygon. The manner of applying this rule is evident.

PROBLEM VI.

To find the area of a regular polygon.

RULE.—Multiply the perimeter of the polygon or the sum of its sides, by the perpendicular drawn from the centre upon one of the sides, and take half the product for the area. This rule is evidently nothing more than a particular mode of applying the rule of the preceding problem.

Ex. 1. Required the area of a regular pentagon ABCDE, whose side AB or CD, &c. is 27 feet, and perpendicular HK is 19.25 feet? —*Ans.* 144.375 square yards.

2. Required the area of a regular hexagon, of which the side is 49 yards?—*Ans.* 6238 square yards, nearly.

PROBLEM VII.

To find the diameter and circumference of a circle, the one from the other*.

RULE 1.—As 7 is to 22, so is the diameter to the circumference nearly.

As 22 is to 7, so is the circumference to the diameter nearly.

RULE 2.—As 113 is to 355, so is the diameter to the circumference nearly.

As 355 is to 113, so is the circumference to the diameter nearly.

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* For the demonstration of the rules for finding the diameter, circumference, and area of a circle, which are not here demonstrated, see Book I. of the Supplement to the first six books of Euclid's Elements of Geometry, by the late Professor Playfair.

RULE 3.—As 1 is to 3.1416, so is the diameter to the circumference nearly.

As 3.1416 is to 1, so is the circumference to the diameter nearly.

Ex. 1. Required the circumference of a circle, whose diameter is 34 feet?—*Ans.* 106.814 feet.

2. The circumference of a circle is 16 yards, required the diameter?—*Ans.* 5.093 yards.

3 Supposing the earth to be an exact sphere, required its circumference, the diameter being 7936 miles?—*Ans.* 24932 miles nearly.

PROBLEM VIII.

To find the length of any arch of a circle.

RULE*.—As 180 is to the number of degrees in the arch, so is 3 1416 times the radius to its length —as is evident from rule 3d, of the preceding problem.

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* The following is a very convenient method of approximating to the length of an arch of a circle

RULE —From 8 times the chord of half the arch subtract the chord of the whole arch and $\frac{1}{3}$ of the remainder will be the length of the arch nearly

This rule may be briefly demonstrated thus Let a denote an arch of a circle, then, from the series expressing the sine of an arch in terms of the arch, we have, (radius being unity),

$$\text{Sin } \tfrac{1}{2}a = \tfrac{1}{2}a - \frac{a^3}{48} + \frac{a^5}{3840} - \&c.$$

Therefore, if the arch a be small, so that a^5 is a very small quantity, we obtain,

$$\text{Sin } \tfrac{1}{2}a = \tfrac{1}{2}a - \frac{a^3}{48} \text{ nearly.}$$

In like manner, we have,

$$\text{Sin } \tfrac{1}{4}a = \tfrac{1}{4}a - \frac{a^3}{384} \text{ nearly}$$

Eliminating by means of the two last equations, the quantity a^3, the resulting equation is

$$16 \text{ Sin. } \tfrac{1}{4}a - 2\text{Sin. } \tfrac{1}{2}a = 3a$$

But $16 \text{ Sin } \tfrac{1}{4}a = 8 \text{ chord } \tfrac{1}{2}a$, and $2\text{Sin. } \tfrac{1}{2}a = \text{chord } a$. therefore,

$$8 \text{ chord } \tfrac{1}{2}a - \text{chord } a = 3a$$

The same conclusion will evidently be obtained, whatever be the radius of the circle

Ex. 1. Required the length of the arch AEB, whose chord AB is 9, the radius AC or BC being 12 feet?—*Ans.* 9.2256 feet.

2. Suppose the chord AB equal to 25, and radius AC, 19.25 chains, required the length of the arch?—*Ans.* 27.21 chains.

PROBLEM IX.

To find the area of a circle.

RULE 1.—Multiply half the diameter by half the circumference, and the product will be the area.

RULE 2.—Multiply the square of the diameter by .7854, and the product will be the area.

Demonstration. If the diameter of a circle be unity, the circumference is known to be 3.1416 nearly. Hence by Rule I. the area is $\frac{1}{2} \times \frac{1}{2} \times 3.1416 = .7854$. But the areas of circles are to one another as the squares of their diameters. Hence, if D be the diameter of any circle whatever, we have $1^2 : D^2 :: .7854 : D^2 \times .7854$ = the area of the circle whose diameter is D.—From which the rule is evident.

Ex. 1. What is the area of a circle, whose diameter is 26 feet?—*Ans.* 530.93 square feet.

2. Required the area of a circular field, whose circumference is 694 yards?—*Ans.* 7 acres, 3 roods, 27 poles.

3. Required the area of a ring between the circumference of two concentric circles, their diameters being 20 and 15 inches?—*Ans.* 137.445 square inches.

4. The expense of inclosing a circular court at 8s. *per* yard, amounted to £320, required the expense of paving it at 6d. *per* square yard?—*Ans.* £1273 : 4 : 9.

PROBLEM X.

To find the area of a sector of a circle.

RULE 1.—Multiply the radius by half the arc of the sector, and the product will be the area, as is evident from the first rule given for finding the area of a circle.

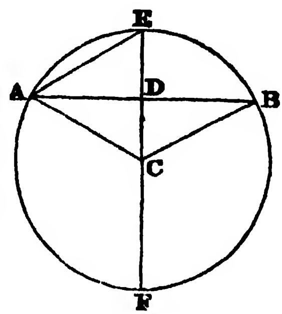
RULE 2.—As 360 is to the degrees in the arc of the sector, so is the area of the whole circle, to the area of the sector.

Ex. 1. Required the area of a circular sector, whose arc contains $24^{\circ}\ 30'$, the radius of the circle being 7 feet?—*Ans.* 10.476 square feet.

2. Required the area of a sector of a circle, the arc containing 14°, and the whole circumference being 108 yards?—*Ans.* 36.096 square yards.

PROBLEM XI.

To find the area of a segment of a circle.



RULE.—Find the area of the sector having the same arch with the segment by the last problem.—Find also the area of the triangle contained by the chord of the segment, and the two radii of the sector.—Then take the sum of these two for the answer when the segment is greater than a semicircle, or take their difference when it is less than a semicircle.

Ex. 1. To find the area of the segment AEBDA, its chord AB being 25, and the radius AC or BC 19?—*Ans.* 80.34.

2. Required the area of the segment AEBDA, the arch of the segment being 29°, and the radius 27 feet?—*Ans.* 7.78 square feet.

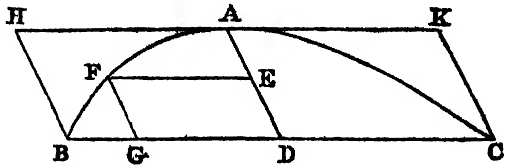
PROBLEM XII.

To find the area of any segment of a parabola.

RULE.—Multiply the base of the segment by its height, and two-thirds of the product will be the area.

Demonstration. Let BAC be a parabola of which BC is a given ordinate and AD the corresponding abscissa. From F any point in the curve draw FG parallel to AD, and FE parallel to BD.

Put AD $= a$, BD $= b$, GD or FE $= x$, and FG or DE $= y$. Then from the nature of the curve we have,



AE : AD : : FE^2 : DB^2, that is, $a - y : a :: x^2 : b^2$; hence we obtain $y = a\left(1 - \frac{x^2}{b^2}\right)$.

Suppose now BD to be divided into m equal parts; each part will be equal to $\frac{1}{m}b$. Let x become successively equal to $\frac{1}{m}b$, $\frac{2}{m}b$, $\frac{3}{m}b$, &c.... $\frac{m}{m}b$, and let the corresponding values of y be y', y'', y''',&c.... $y^{(m)}$, then we obtain,

$$\begin{array}{ll}
\text{when } x = 0, & y = a \\
x = \frac{1}{m}b, & y' = a\left(1 - \frac{1^2}{m^2}\right) \\
x = \frac{2}{m}b, & y'' = a\left(1 - \frac{2^2}{m^2}\right) \\
x = \frac{3}{m}b, & y''' = a\left(1 - \frac{3^2}{m^2}\right) \\
\text{\&c.} & \text{\&c.} \\
x = \frac{m}{m}b, & y^{(m)} = a\left(1 - \frac{m^2}{m^2}\right)
\end{array}$$

Taking the sum of these equations, and observing that $1^2 + 2^2 + 3^2 + 4^2 + \&c.... + m^2 = \frac{1}{3}m^3 + \frac{1}{2}m^2 + \frac{1}{6}m$, (*See Simpson's Algebra, Sect. XIV.*) we obtain

$$y + y' + y'' + \&c.... + y^{(m)} = a(m + 1) - a\,\frac{\frac{1}{3}m^3 + \frac{1}{2}m^2 + \frac{1}{6}m}{m^2}$$

Let the angle ADB be denoted by φ, and multiply both sides of this last equation by $\frac{1}{m}b$ Sin. φ, and it becomes,

$$\frac{1}{m}b \text{ Sin. } \varphi\,(y + y' + y'' + \&c.... + y^{(m)}) = ab \text{ Sin. } \varphi + \frac{1}{m}ab \text{ Sin. } \varphi$$
$$- ab \text{ Sin. } \varphi\left(\frac{1}{3} + \frac{\frac{1}{2}m + \frac{1}{6}}{m^2}\right).$$

Now, it is evident that the left hand side of this result is the sum of a series of parallelograms, which approaches nearer and nearer to the area of half the parabolic segment BAC, according as the number m becomes greater. Let us suppose, then, that m is infinite, and we find the area of half the segment equal to $\frac{2}{3}\,ab \operatorname{Sin} \varphi$, or to two-thirds of the parallelogram AHBD. Hence it appears that every segment of a parabola is two-thirds of its circumscribing parallelogram, and the reason of the rule is obvious.

Ex 1. Required the area of a parabolic segment, of which the base is 15 feet 10 inches, and the height 7 feet 8 inches?—*Ans.* 80 Sq. f. $133\frac{1}{3}$ Sq. in.

2 The base or double ordinate of a parabolic segment is 38, the abscissa 14, and the angle which the ordinate makes with the abscissa $75^{0}\ 19'$, required the area of the segment?—*Ans.* 343.084.

PROBLEM XIII

To find the area of an ellipse.

RULE—Multiply the product of the two axes by the number 7854, and the result will be area nearly.*

Demonstration. Let ACB be a semiellipse, of which AB is the greater axis, and CE half of the less axis. Upon AB describe a semicircle meeting EC produced in D. Through any point F in

---

* If we put the transverse axis of an ellipse $= t$, the conjugate $= c$, and $1 - \frac{c^2}{t^2} = d$, also the number 3 1416 $= \pi$, then the periphery is expressed by the following formula

$$\pi t \left\{ 1 - \frac{d}{2^2} - \frac{3d^2}{2^2\ 4^2} - \frac{3^2 \cdot 5d^3}{2^2\ 4^2 \cdot 6^2} - \frac{3^2\ 5^2\ 7d^4}{2^2\ 4^2\ 6^2\ 8^2} - \&c \right\}$$

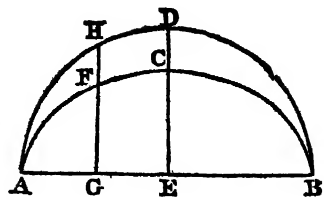
Or, we have the following approximations,

I $\frac{1}{2}\pi \times (t + c)$

II $\pi \times \sqrt{\frac{1}{2}(t^2 + c^2)}$

III $\frac{1}{2}\pi \times \left\{ 3\sqrt{\frac{1}{2}(t^2 + c^2)} - \frac{1}{4}(3t + P) \right\}$, where P is the principal parameter

the ellipse draw HFG perpendicular to AB; and put $2AE = a$, $2EC = b$, $AG = x$, $FG = y$, and $GH = y'$. Then from the nature of the curves we have GH : GF : : ED : EC, or $y' : y :: a . b$; therefore $y = \frac{b}{a}y'$.



Suppose AB to be divided into m equal parts, each part will be equal to $\frac{1}{m}a$. Multiply each side of the equation $y = \frac{b}{a}y'$ by $\frac{1}{m}a$ and it becomes

$$\frac{1}{m}ay = \frac{b}{a} \times \frac{1}{m}ay'.$$

If x be now supposed to become successively equal to $\frac{1}{m}a$, $\frac{2}{m}a$, $\frac{3}{m}a$, &c.... $\frac{m}{m}a$, the corresponding values of $\frac{1}{m}ay$ will give a series of rectangles of which the sum will approach nearer and nearer to the area of the semiellipse ACB according as the number m becomes greater; and the corresponding values of $\frac{1}{m}ay'$ will give a series of rectangles of which the sum approaches nearer and nearer to the area of the semicircle ADB according as m becomes greater. Hence, when m is supposed infinite, we obtain

$$\text{Semiellipse} = \frac{b}{a} \times \text{Semicircle}.$$

Therefore the area of an ellipse is equal to the area of a circle described on the greater axis multiplied by $\frac{b}{a}$, or the ellipse is to the circle as the less axis is to the greater. But the area of a circle described on a, the greater axis, is equal to $.7854\,a^2$. Wherefore the area of the ellipse is

$$.7854\,a^2 \times \frac{b}{a} = .7854\,ab.$$

This result gives the rule.

Ex. 1. The greater diameter of an ellipse is 50 feet, the less is 40 feet; what is the area in square yards?—*Ans.* $174\frac{1}{2}$ sq. yds. nearly.

2. The transverse axis of an ellipse is $24\frac{1}{4}$ inches, its eccentricity $8\frac{1}{8}$ inches; what is its area?—*Ans.* 2 sq. f. 54.827 sq. in.

PROBLEM XIV.

To find the area of a segment of an ellipse cut off by an ordinate to either axis.

Rule.—Find the area of the corresponding segment of the circle described on that axis of the ellipse to which the base of the segment is perpendicular: Then, as the axis to which the base is perpendicular is to the other axis so is the area of the circular segment to the area of the elliptic segment.

For, in the same manner as it was demonstrated in last problem that the whole ellipse is to the circle described on the greater axis as the less axis is to the greater, it may also be demonstrated that any segment cut off by an ordinate to either axis has to the corresponding segment of the circle described on that axis the same ratio which the other axis has to that axis.

Ex. 1. Required the area of a segment of an ellipse of which the base is perpendicular to the less axis, the height being 12 inches, and the axes of the ellipse 80 and 60 inches.—*Ans.* $536\frac{3}{4}$ sq. inches nearly.

2. What is the area of a segment of an ellipse cut off by a straight line perpendicular to the greater axis, the height of the segment being 10 feet, the greater axis of the ellipse 35 feet, and the less 25 feet?—*Ans.* 161.878 sq. feet.

PROBLEM XV.

To find the area of a sector of an ellipse, the corresponding ordinate being perpendicular to either axis.

Rule.—Suppose the ordinate (produced if necessary) to meet the circle described on that axis to which the ordinate is perpendicular, and find the area of the circular sector corresponding to the arch of the circle which it cuts off. Then as the axis to which the

ordinate is perpendicular is to the other axis so is the area of the circular sector to the area of the elliptic sector.

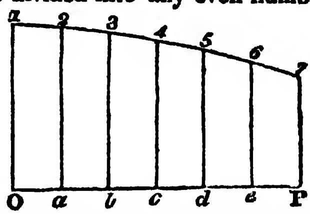
The reason of this rule is evident from what has already been demonstrated.

Ex. The ordinate to the greater axis of an ellipse is 6 feet 8 inches, the greater axis is 18 feet, and the less 10 feet; what is the area of each of the two sectors into which the ellipse is divided by straight lines drawn from the centre to the extremities of the ordinate?—*Ans.* Area of the greater sector 108.534 sq. feet. Area of the less sector 32.838 sq. feet.

PROBLEM XVI.

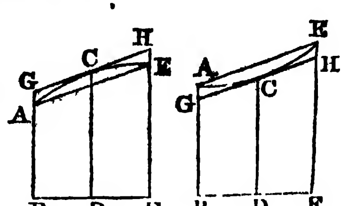
To find the area of any curvilinear figure, nearly, by the method of equidistant ordinates.

RULE. -Let the right line OP be divided into any even number of equal parts, as O*a*, *ab*, *bc*, &c. and let perpendiculars be raised from these points, as O1, *a*2, *b*3, *c*4, &c. put A for the sum of the extreme ordinates, O1 and P7, B for the sum of the second, fourth, and other even ordinates, *a*2, *c*4, &c. and C for the sum of all the rest.—Then the area of the figure is expressed by the formula $\frac{A + 4B + 2C}{3} \times D$, where D represents the common distance of the equidistant ordinates.



Demonstration. Let ACE be an arch of a parabola and BF any straight line. From the points A and E let fall the perpendiculars AB, EF perpendicular to BF. Bisect BF in D, and from D draw DC at right angles to BF and let it meet the arch ACE in the point C. Draw GH a tangent to the parabola at C and meeting BA, FE, or BA, EF produced, in G, H.

From the nature of the curve, AGHE is a parallelogram, and the segment ACE is two-thirds of AGHE, or twice the parallelogram AGHE is equal to three times the parabolic segment ACE.



Add each of these to three times the trapezoid ABFE if GH falls above AE, or, take each of them from three times the same trapezoid if GH falls below AE, and we have twice the trapezoid GBFH together with the trapezoid ABFE equal to three times the space ACEFB.

Now, by Prob. III., the trapezoid ABFE is equal to $\frac{1}{2}(AB + EF) \times BF = (AB + EF) \times BD$, and twice the trapezoid GBFH is equal to $\frac{1}{2}(BG + FH) \times 2BF = 4CD \times BD$. therefore three times the space ACEFB is equal to $(AB + 4CD + EF) \times BD$. Hence we obtain

$$\text{Space ACEFB} = \frac{AB + 4CD + EF}{3} \times BD.$$

Now, whatever be the nature of the curve proposed, let the line OP be divided into such an even number of equal parts (suppose six) that the portions of the curve intercepted between the ordinates O1 and $b3$, $b3$ and $d5$, $d5$ and P7 may without sensible error, be considered as parabolic arches. Then putting $O1 = h$, $a2 = i$, $b3 = k$, $c4 = l$, $d5 = m$, $e6 = n$, $P7 = o$, and Oa, or ab, &c. $= p$, we will obtain,

$$\begin{aligned} \text{The space } 1Ob3 &= \tfrac{1}{3}(h + 4i + k)\,p \\ \text{———— } 3bd5 &= \tfrac{1}{3}(k + 4l + m)\,p \\ \text{———— } 5dP7 &= \tfrac{1}{3}(m + 4n + o)\,p \end{aligned}$$

Taking the sum of these equations and observing that $h + o = A$, $i + l + n = B$, $k + m = C$, and $p = D$, we have the whole curvilinear surface equal to

$$\frac{A + 4B + 2C}{3} \times D.$$

This is the formula given above.

Ex. 1. Required the area of a quadrant, of which the radius is 1 ?—*Ans.* .7854.

2. What is the area of a hyperbola FDM, of which the abscissa FM is 10, the semiordinate MD, 12, and the semitransverse 15 ?—*Ans.* 75.2468.

3. Let AF, AE be the assymptotes, and C the vertex of an equilateral hyperbola; also let D be a point in that branch of the curve which is adjacent to AE: From C and D let CB, DE be drawn perpendicular to AE; required the area of the space BCDE, supposing AB or BC equal to 1, and BE also equal to 1 ?—*Ans.* .69316.

4. The length of the base of a field curvilinear on one side is 720 links, and upon it are erected seven equidistant ordinates of 200, 225, 230, 246, 260, 280, and 300 links, required the area of the field?—*Ans.* 1 acre, 3 roods, 7.488 poles.

OF LAND SURVEYING.

The art of surveying consists in determining the boundaries and contents of an extended surface.—When performed in the completest manner, it ascertains the positions of all the remarkable objects which are situated within the range of observation, it measures their mutual distances and relative heights, and thus furnishes an accurate plan of the surface. It is seldom, however, that the operations of the land-surveyor are carried to such minuteness; and the principal object at which he generally aims, is to trace the chief boundaries, and to determine the superficial contents of each field.—For this purpose, the several fields are divided into large triangles, or other determinate figures; various instruments, such as a chain, theodolite, &c. are employed for measuring those lines and angles which are accessible; inaccessible lines and angles are calculated by Trigonometry, and the surfaces of the different portions into which the fields are supposed to be divided, are computed by the rules laid down in the preceding abstract.

In measuring hilly grounds, the sloping surface should be reduced to the horizontal, which is easily accomplished by considering the ratio of the hypotenuse of a right-angled triangle to the base, viz. the ratio of the radius, to the cosine of the angle of acclivity or declivity.—In this ratio all the hypotenusal distances are to be reduced before the calculation is begun.

The practice of this useful art can be learned most successfully under the direction of a professional man in the field.

MENSURATION OF SOLIDS.

PROBLEM I.

To find the surface of a prism.

Rule.—Multiply the sum of the perpendicular breadths of the sides of the prism by its length or height, and the product will be the surface of all the sides. When the whole surface is required we must add to this product the area of the two ends.

Note.—If it is a right prism whose surface is required, the sum of the perpendicular breadths of the sides will evidently be equal to the perimeter of the end of the solid.

Ex. 1. Required the whole surface of a rectangular parallelopiped whose length is 12 feet, and its base $2\frac{1}{2}$ feet by 3 feet 9 inches?—*Ans.* $168\frac{3}{4}$ sq. feet.

2. Required the surface of a cube whose edge is 6 feet 2 inches? —*Ans.* $228\frac{1}{6}$ sq. feet.

3. Required the whole surface of an oblique triangular prism of which the base is equilateral, the length of the prism being 9 feet, each side of the base 2 feet 9 inches, and the angle between its base and one of its faces which is rectangular 74° 16′?—*Ans.* 79.413 sq. feet

4. How many square yards are there in the walls of a room of a prismatic form whose height is $10\frac{1}{2}$ feet, and circumference 56 feet?—*Ans.* $65\frac{1}{3}$ sq. yards.

PROBLEM II.

To find the surface of a cylinder.

Rule.—Multiply the circumference of its base by its length, and the product will be the curve surface; to which add the area of the ends if the whole surface is required.

A cylinder may be considered as a round prism, or as the limit of all the prisms whose bases are figures inscribed in, or circumscribed about the circular ends of the cylinder. Hence the reason of the rule is obvious.

Ex 1. What is the whole surface of a cylinder 17 feet long, the diameter of the base being 19 inches?—*Ans.* $88\frac{1}{2}$ sq. feet nearly.

2. If the length of a roller be 5 feet 9 inches, and the diameter of its base 1 foot 8 inches, how often does it revolve in rolling an acre?—*Ans.* $1446\frac{5}{6}$ nearly.

PROBLEM III.

To find the surface of a pyramid.

Rule.—Find separately the areas of the triangles which form its sides and their sum will be the convex surface of the solid. To

this sum add the area of the base when the whole surface is required.

Note.—If the pyramid is regular it is evident that the convex surface will be found by multiplying the perimeter of the base by the slant height of the solid, and taking half the product.

Ex. 1. Required the whole surface of a regular triangular pyramid, each side of the base of which is 2 feet 8 inches, and the perpendicular from the vertex on a side of the base 22½ feet?—*Ans.* 93.08 sq. feet.

2. Required the whole surface of a regular square pyramid, each side of the base being 5 feet and the perpendicular height of the solid 17 feet 4 inches?—*Ans.* 200 sq. feet nearly.

3. What is the expense of polishing the upright surface of a pyramidal stone whose slant height is 21 feet and each side of its pentagonal base 30 inches, the polishing being supposed to cost 8d. per square foot?—*Ans.* £4 : 7 : 6.

PROBLEM IV.

To find the surface of a right cone.

RULE.—Multiply the circumference of the base by the slant height and half the product is the curve surface of the cone: to which the area of the base must be added when the whole surface is required.

For the curve surface of a right cone is evidently equal to a circular sector of which the radius is equal to the slant height of the cone and the arch equal to the circumference of the base of the cone. The surface of an oblique cone is not quadrable: indeed no rule has yet been found that will even lead to a practical approximation to its area.

Ex. 1. Required the whole surface of a cone whose slant height is 16 feet and the diameter of its base 6½ feet?—*Ans.* 196.546 sq. feet.

2. What will the painting of a conical spire amount to at 8d. per square yard, supposing the circumference of the base 64 feet, and the perpendicular height 118 feet?—*Ans.* £14 : 0 . 8¾

PROBLEM V.

To find the surface of the frustum of a right pyramid or cone.

Rule.—Add together the perimeters of the two ends of the frustum; multiply the sum by the slant height, and half the product is the surface. Add the areas of the two ends if required.

For it is evident that the faces of the pyramidal frustum are so many trapezoids which have the corresponding sides of the ends of the frustum for their opposite sides and the slant height of the frustum for the perpendicular breadth of each. The conical frustum, again, is the limit of the pyramidal frustums whose bases are figures inscribed in or circumscribed about the circular ends of the conical frustum. Hence the reason of the rule, in both cases, is obvious.

Ex. 1. Required the whole surface of the frustum of a square pyramid, the side of the one end being 40 inches, and that of the other 26 inches, also the slant height of the frustum 10 feet?—*Ans.* 125 sq. feet 116 sq. inches.

2. Required the convex surface of the frustum of a cone, the diameters of the bases being $3\frac{1}{2}$ feet and 5 feet 7 inches respectively, and the slant height 12 feet?—*Ans.* 171.217 sq. feet.

3. From a right cone the diameter of the base of which is 4 feet, and whose perpendicular height is 30 feet, the upper part was cut off by a plane parallel to the base; required the whole superficial content of the remaining frustum, the perpendicular height of the part cut off being 12 feet?—*Ans.* 173.265 sq. feet.

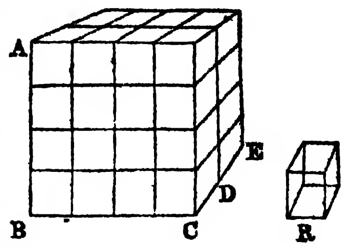
PROBLEM VI.

To find the solid content of any prism or cylinder.

Rule.—Multiply the area of the base or end by the perpendicular height; the product is the solid content.

Demonstration. The measuring unit of solids may, like that of surfaces, be of any determinate figure and magnitude. The cube of which the length, breadth, and thickness is equal to the measuring unit of lines is employed for expressing numerically the

solid content of bodies. Now, let the square of which AB, BC are two adjacent sides be equal to the area of the base of a right prism or cylinder; it is evident that if CD, the height of the prism or cylinder, be equal to the measuring unit of lines, whatever number of times the square AB·BC contains the measuring unit of surfaces, the prism or cylinder will contain R, the measuring unit of solids, the same number of times. for upon each



square of the measuring unit of lines contained in the square AB·BC, as a base, will stand a cube equal to R. Again, if CE, the perpendicular height of the prism or cylinder, be equal to twice the measuring unit of lines, the prism or cylinder will contain R twice as often as the area of the base contains the measuring unit of surfaces: and, generally, putting P to denote how often the measuring unit of lines is contained in the perpendicular height of the right prism or cylinder, we have the solidity expressed by $P \cdot BC^2$. But all prisms and cylinders having equal bases and altitudes are equal to one another. Hence the reason of the rule is manifest.

Ex. 1. The sides of the base of a right triangular prism are 10, 14, and 17 inches, and the length is 4 feet 7 inches, what is the solid content?—*Ans.* 2 cubic feet 392.8 cubic inches.

2. What is the solidity of a cube whose side is 17 feet 3 inches? —*Ans.* $5132\frac{6}{64}$ cubic feet.

3. A piece of timber is 16 inches square at the end, and 20 feet long, what is its solid content, and suppose a solid foot is to be cut off the end of it, at what distance from the end must it be cut? —*Ans.* Solid content $35\frac{5}{9}$ cubic feet. Distance from the end $6\frac{3}{4}$ inches.

4. The diameter of the base of a cylinder $20\frac{3}{4}$ inches, and its height 12 feet; what is its solid content?—*Ans.* 28.1803 cubic feet.

5. If the outside diameter of an iron roller be 2 feet, the thickness of the metal $1\frac{1}{2}$ inch, and the length of the roller 3 feet 9 inches, what is its whole weight, supposing a cubic inch of iron to weigh $4\frac{1}{4}$ ounces?—*Ans.* 11 cwt. 1 qr. 7 lbs. 6.046 oz.

PROBLEM VII.

To find the solid content of a pyramid or cone.

RULE.—Multiply the area of the base by the perpendicular height, and one third of the product is the solid content.

For every pyramid or cone is one third of a prism of equal base and altitude.

Ex. 1. Required the solidity of a triangular pyramid, the sides of its base being 6, 7, 8 inches, and its height 19 feet 4 inches?—*Ans.* 1572.43 cubic inches.

2. How many solid feet are there in a square pyramidal stone, each side of the base of which is 26 inches and its slant height 9 feet?—*Ans.* 13 cubic feet 1695.113 cubic inches.

3. The diameter of the base of a cone is 8 feet and its height $24\frac{1}{2}$ feet, required its solid content?—*Ans.* $410\frac{1}{3}$ cubic feet nearly.

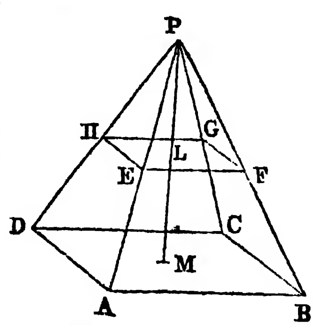
PROBLEM VIII.

To find the solid content of the frustum of a pyramid or cone.

RULE.—Find a mean proportional between the areas of the two ends, (that is, the square root of their product;) multiply the sum of this mean proportional and the areas of the ends by the perpendicular height of the frustum, and one third of the product will be the solid content.

Demonstration. Let ABCD be the base, and EFGH the top of the frustum, P the vertex of the pyramid, and PM the perpendicular on the base meeting the top in L.

Putting S^2 and s^2 for the areas of the base and top of the frustum, we have the solid content of the whole pyramid equal to $\frac{1}{3}PM \times S^2$, and the solid content of the part PEFGH cut off by the plane EEGH equal to $\frac{1}{3} PL \times s^2$. Hence the solid content of the frustum is equal to



$$\tfrac{1}{3}PM \times S^2 - \tfrac{1}{3}PL \times s^2 = \tfrac{1}{3}(PL + LM) S^2 - \tfrac{1}{3}PL \times s^2$$
$$= \tfrac{1}{3}LM \times S^2 + \tfrac{1}{3}PL \times (S - s) \cdot (S + s).$$

Now, because the top and bottom of the frustum are similar figures we have $S^2 : s^2 :: AB^2 : EF^2$; therefore $S : s :: AB : EF$. But, it is evident from similar triangles that $AB : EF :: (AP : EP : .)\ PM : PL$: hence $S : s :: PM : PL$, and $S - s : s :: LM : PL$; wherefore $PL \times (S - s) = LM \times s$. By substituting this value of $PL \times (S - s)$ in the expression which we have found for the solidity of the frustum, we obtain

$$\tfrac{1}{3}LM \times S^2 + \tfrac{1}{3}LM \times s\,(S + s) = \tfrac{1}{3}LM \times (S^2 + Ss + s^2).$$

If it now be considered that $Ss = \sqrt{S^2\, s^2}$ is a mean proportional between the areas of the two ends of the frustum, it is evident that this formula when expressed in words gives the rule.

Ex. 1. If each side of the greater end of a piece of squared timber be 38 inches, each side of the less end 16 inches, and the length 20 feet; what is its solid content?—*Ans.* $106\frac{2}{3}\frac{?}{}$ cubic feet.

2. The height of a pillar which is the frustum of a cone is 14 feet, and the diameters of the two ends are 4 feet and 2 feet inches respectively, how many cubic feet are there in the pillar? —*Ans.* 131.2027 cubic feet.

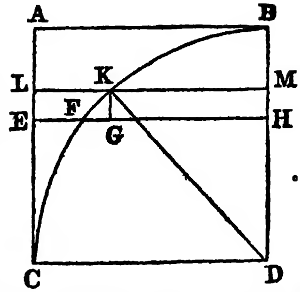
3. Each side of the greater base of the frustum of a hexagonal pyramid is 13 inches, each side of the less base 8 inches, and the length of the frustum 2 feet, how many cubic feet does it contain? —*Ans.* 4.05348 cubic feet.

PROBLEM IX.

To find the surface of a sphere, or of any segment or zone of it.

RULE.—Multiply the circumference of the sphere by the height of the part required, and the product will be the curve surface whether it be a segment, a zone, or the whole sphere.

Demonstration. Let BCD be a quadrant and BACD a square formed by drawing the tangents BA, CA to intersect each other in A. If the figure be supposed to revolve round BD as an axis the quadrantal arch BC will describe the surface of a hemisphere, and the straight line AC will describe the surface of a cylinder. Take FK a very small arch, and draw LKM, EFH perpendicular to BD or AC and therefore parallel to each other. Draw also KG perpendicular to EH, and join DK.



By its revolution round BD as an axis, the line EL will describe the surface of a cylinder having for its base a circle of which LM is the radius: hence if we put π to denote the circumference of a circle of which the diameter is unity, we will have for the surface generated by the line EL the expression $2\pi LM \times EL$.

Again, since FK is very small it may be considered as a straight line, and the surface which it generates, in revolving round BD as an axis, may be considered as the surface of a conical frustum having for its bases two circles of which FH and KM are the radii. Therefore the surface generated by FK is equal to $\pi(FH + KM) \times FK$: But since FK is very small FH and KM are nearly equal to one another. Wherefore for the surface generated by FK we have $2\pi KM \times FK$. Now since each of the angles MKG, DKF is a right angle, if the angle DKG be taken away from each, the remaining angles MKD, FKG are equal. Hence the right angled triangles MKD, FKG are equiangular and similar: Wherefore

$$KM : KD \text{ or } LM \,..\, KG \text{ or } EL : FK$$

$$\text{and } KM \times FK = LM \times EL.$$

By multiplying both sides of this equation by 2π, they become the expressions which we have found for the surfaces generated by FK and EL which must therefore be equal to one another.

Thus it appears that the corresponding indefinitely small elements of the spherical and cylindrical surfaces are always equal, and hence that any finite portions of them comprehended between planes perpendicular to the axis BD will be equal, so that the truth of the rule is evident.

Ex. 1. Required the surface of a globe whose diameter is 23 inches?—*Ans* 11.541 sq. feet.

2. What is the convex surface of a segment 5 inches in height, and cut off from the same globe?—*Ans.* 361.284 sq. inches.

3. How many square miles are there in the surface of the earth, supposing it a perfect sphere of which the diameter is 7936 miles, also in the surface of each of its zones, supposing the obliquity of the ecliptic to be 23° 27′ 30″?—

		Square miles
Ans. The surface	of the whole globe	197858269.594
	of the torrid zone	78763846.791
	of each temperate zone	51370848.055
	of each frigid zone	8176363.346

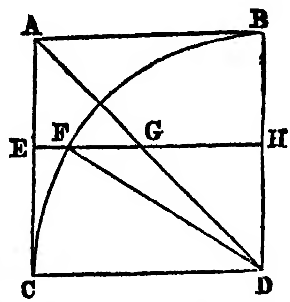
PROBLEM X.

To find the solid content of a sphere.

RULE 1.—Multiply the area of a great circle of the sphere by the diameter, and two-thirds of the product is the solid content.

RULE 2.—Multiply the cube of the diameter by .5236, and the product is the content.

Demonstration. Let BCD be a quadrant, and ABDC a square described on the radius BD: join AD. If the figure be supposed to revolve round BD as an axis, it is evident that the quadrant will describe a hemisphere, the square a cylinder, and the triangle ADB a cone. Draw any straight line EH parallel to DC, meeting the lines AC, AD, BD in E, G, H, and the arch of the quadrant in the point F. It is evident that the square of FD is equal to the squares of FH, HD because the angle at H is a right angle. But FD is equal to EH, and HD is equal to GH, therefore $EH^2 = FH^2 + GH^2$, and the circle described with the radius EH will be equal to the two circles described with the radii FH, GH. Hence the sections of the sphere and cone made by a plane perpendicular to BD are always equal to the section of the cylinder made by the same plane. If we conceive, then, the three solids to be made up of very thin cylinders having these sections for their bases, it follows that any portion of the cylinder comprehended between two planes parallel to its base will be equal to the sum of the corresponding portions of the hemisphere and cone.



Let S be the solidity of the segment of the hemisphere, described by the revolution of the portion BFH of the quadrant s the solidity of the corresponding portion of the cylinder, and s' the solidity of the corresponding portion of the cone: also let h denote their common height BH, d the diameter of the sphere, and $\frac{1}{4}\pi$ the area of a circle whose diameter is unity. Because AB is equal to CD, and GH to HD or to BD — BH we have the two bases of the frustum of the cone equal to two circles whose diameters are d and $d - 2h$ respectively.

Now, by Prob. VIII, we have

$$s' = \tfrac{1}{4}\pi\,(d^2 + d\,(d - 2h) + (d - 2h)^2) \times \tfrac{1}{3}h =$$
$$\tfrac{1}{4}\pi\,(d^2h - 2dh^2 + \tfrac{4}{3}h^3).$$

But $S = s - s'$, and by Prob. VI, $s = \frac{1}{4}\pi d^2 h$, hence we obtain

$$S = \tfrac{1}{4}\pi\,(2dh^2 - \tfrac{4}{3}h^3) = \tfrac{2}{3} \times \tfrac{1}{4}\pi\,(3d - 2h)\,h^2.$$

If we now suppose $h = d$ we obtain for the whole sphere $\frac{2}{3} \times \frac{1}{4}\pi d^2 d$. But $\frac{1}{4}\pi d^2 d$ is the solid content of a cylinder circumscribed about the sphere. Hence every sphere is two-thirds of the circumscribing cylinder.

Again, spheres are to one another as the cubes of their diameters: hence if D be the diameter of any sphere, S its solid content, and s the solid content of a sphere whose diameter is unity, we have

$$1^3 : D^3 :: s : S, \text{ and } S = s \times D^3.$$

But since the area of a circle whose diameter is unity is .7854, it is evident that $s = .7854 \times 1 \times \frac{2}{3} = .5236$. Therefore $S = .5236 \times D^3$.

Thus the reason of both rules is obvious.

Ex. 1. What is the solid content of a globe of which the diameter is 17 feet?—*Ans.* 2572.4468 cubic feet.

2. If the circumference of a globe be 37 feet 9 inches, what is its solid content?—*Ans.* 908.444 cubic feet.

PROBLEM XI.

To find the solid content of a spherical segment.

RULE.—From three times the diameter of the sphere, subtract twice the height of the segment, multiply the remainder by the square of the height, and that product by .5236, and the last product is the solidity.

For, it was proved in demonstrating Rule I. of last Problem, that if S be the solid content of a segment of a sphere of which d is the diameter, and h the height of the segment, we have

$$S = \tfrac{2}{3} \times \tfrac{1}{4}\pi\,(3d - 2h) \times h^2.$$

This formula expressed in words gives the rule.

Ex. 1. What is the solid content of the segment of a sphere whose height is 9 feet, the diameter of the sphere being 30 feet? —*Ans.* 3053.6352 cubic feet.

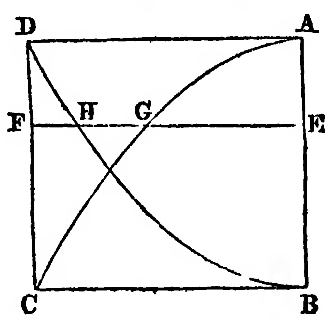
2. The diameter of the base of a segment of a sphere is 28 feet, and the height of the segment $6\frac{1}{2}$ feet, required its solid content? —*Ans.* 2145.0028 cubic feet.

PROBLEM XII.

To find the solid content of a paraboloid, or solid produced by a parabola revolving about its axis.

Rule.—Multiply the area of the base by the height, and half the product will be the solid content.

Demonstration. Let AGC, BHD be two equal and similar semi-parabolas having AB for their common axis. Draw AD, BC perpendicular to AB and meeting the parabolas in D and C, join DC. it is evident that ABCD is a rectangle. If the figure ABCD be supposed to revolve on AB as an axis, the two parabolas will describe two equal paraboloids, and the rectangle will describe a cylinder. Let P denote the parameter of each parabola, and draw EF parallel to DA or BC, and meeting the two parabolas in G and H, and the sides of the rectangle in F and E.



From the properties of the parabola we have $EG^2 = P \cdot AE$ and $EH^2 = P \cdot BE = P\ (AB - AE) = P \cdot AB - P \cdot AE$, therefore $EG^2 + EH^2 = P \cdot AB$. But $EF^2 = (AD^2$ or BC^2 $=)\ P \cdot AB$. Hence $EF^2 = EG^2 + EH^2$, and the circle described with the radius EF is equal to the two circles described with the radii EG, EH. By conceiving, therefore, the paraboloids and cylinder to be composed of very thin cylinders which have these circles for their bases, and reasoning in the same manner as in the case of the segment of a sphere, we find that the cylinder described by the rectangle ABCD is equal to the two equal paraboloids described by the parabolas AGC, BHD, or to double of one of them. Hence the reason of the rule is obvious.

Ex. 1. Required the solid content of a paraboloid whose height is 29 feet and the diameter of its base $15\frac{1}{2}$ feet?—*Ans.* 2736.039 cubic feet.

2. The height of a parabaloid is 6 feet 10 inches, and the parameter of the axis of the parabola, by the revolution of which it was described, is 5 feet, what is the solid content?—*Ans.* 366.738 cubic feet.

PROBLEM XIII.

To find the solid content of the frustum of a paraboloid.

RULE.—Add together the areas of the circular ends, then multiply the sum by the height of the frustum, and half the product is the solid content.

Demonstration.—Let H be the height of the frustum, A the area of its greater circular end, and a that of its less. also let $H + h$ be the height of the whole paraboloid. Then by the preceding problem the solid content of the whole paraboloid is equal to $\frac{1}{2}A(H + h)$ and the solid content of the part cut off is equal to $\frac{1}{2}ah$, therefore the solid content of the frustum is equal to

$$\tfrac{1}{2}A(H + h) - \tfrac{1}{2}ah, \text{ or } \tfrac{1}{2}AH + \tfrac{1}{2}(A - a)h.$$

But since the squares of the semiordinates of a parabola are to one another as their distances from the vertex, and circles are to one another as the squares of their diameters or radii, we have $A \,.\, a :: H + h : h$, hence $Ah = a(H + h)$, and $(A - a)h = aH$. Substituting this value for $(A - a)h$ in the above expression, we obtain for the solid content of the frustum $\frac{1}{2}H(A + a)$. Hence the reason of the rule is obvious.

Ex. Required the solidity of the frustum of a paraboloid, supposing its height to be 7 feet 8 inches, the diameter of its greater circular end 4 feet 3 inches, and that of its less 3 feet?—*Ans.* 81.477 cubic feet.

PROBLEM XIV.

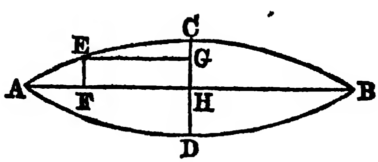
To find the solid content of a parabolic spindle, or solid generated by the revolution of an arch of a parabola about an ordinate to the axis.

RULE.—Multiply the area of the middle section by the length of the solid, and eight-fifteenths of the product will be the content.

Demonstration. Let ACB be an arch of a parabola, and let it revolve about the ordinate AB. Let CHD be the axis of the parabola, and from E any point in the curve draw EF parallel to CH and EG parallel to AH. Put HF $= x$, EF $= y$, CH $= p$, and AH $= q$. Then from the nature of the parabola we have $q^2 : x^2 :: p : p - y$; therefore $px^2 = q^2 p - q^2 y$. From this equation we find

$$y^2 = p^2\left(1 - \frac{x^2}{q^2}\right)^2 = p^2 \left(1 \quad 2\,\frac{x^2}{q^2} + \frac{x^4}{q^4}\right).$$

Suppose AH to be divided into m equal parts; then each part will be equal to $\frac{1}{m}q$. Let x become successively equal to $\frac{1}{m}q$, $\frac{2}{m}q$, $\frac{3}{m}q$, &c.... $\frac{m}{m}q$, and let the corresponding values of y be y', y'', y''', &c.... $y^{(m)}$, then we have by substituting in the above equation,



$$\begin{array}{ll}
\text{when } x = 0, & y^2 = p^2 \\
\text{——} \; x = \frac{1}{m}q, & y'^2 = p^2 \left(1 - 2\cdot\frac{1^2}{m^2} + \frac{1^4}{m^4}\right) \\
\text{——} \; x = \frac{2}{m}q, & y''^2 = p^2 \left(1 - 2\cdot\frac{2^2}{m^2} + \frac{2^4}{m^4}\right) \\
x = \frac{}{m}q, & y'''^2 = p^2 \left(1 - 2\cdot\frac{3^2}{m^2} + \frac{3^4}{m^4}\right) \\
\text{\&c.} & \text{\&c.} \\
\text{——} \; x = \frac{m}{m}q, & \overset{(m)}{y^2} = p^2 \left(1 - 2\cdot\frac{m^2}{m^2} + \frac{m^4}{m^4}\right)
\end{array}$$

Taking now the sum of these equations, observing that $1^2 + 2^2 + 3^2 + \&c.....+ m^2 = \frac{1}{3}m^3 + \frac{1}{2}m^2 + \frac{1}{6}m$, and $1^4 + 2^4 + 3^4 + 4^4 + \&c.....+ m^4 = \frac{1}{5}m^5 + \frac{1}{2}m^4 + \frac{1}{3}m^3 - \frac{1}{30}m$, we obtain

$$y^2 + y'^2 + y''^2 + y'''^2 + \&c....\overset{(m)}{y^2} = p^2\left(m + 1 - \frac{\frac{2}{3}m^3 + m^2 + \frac{1}{3}m}{m^2} + \frac{\frac{1}{5}m^5 + \frac{1}{2}m^4 + \frac{1}{3}m^3 - \frac{1}{30}m}{m^4}\right).$$

Now, if both sides of this last equation be multiplied by $\frac{1}{m}q\pi$, (π being the circumference of a circle whose diameter is unity,) it is evident that $\frac{1}{m}q\pi \left(y^2 + y'^2 + y''^2 + \&c....+ \overset{(m)}{y^2}\right)$ is equal to the sum of a series of cylinders having the same altitude $\frac{1}{m}q$, and for their bases the circles described with radii equal to the successive values of y. The sum of these cylinders approaches nearer and nearer to the solid content of half of the parabolic spindle according as m becomes greater. The right hand side of the above equation when multiplied by $\frac{1}{m}q\pi$ gives us for the sum of the cylinders

$$\pi q p^2 \left(1 + \frac{1}{m} - \frac{2}{3} - \frac{m + \frac{1}{3}}{m^2} + \frac{1}{5} + \frac{\frac{1}{2}m^3 - \frac{1}{3}m^2 - \frac{1}{30}}{m^4}\right).$$

Let m be now supposed infinitely great and we will have the sum of the cylinders equal to

$$\pi q p^2 \left(1 - \frac{2}{3} + \frac{1}{5}\right) = \frac{8}{15}\pi p^2 q.$$

But when m is infinitely great the sum of the cylinders is equal to half of the parabolic spindle. Hence the reason of the rule is obvious.

Ex Required the solid content of a parabolic spindle, of which the length is 80 inches and the greatest diameter 2 feet 8 inches? —*Ans.* 19,858 cubic feet.

PROBLEM XV.

To find the solid content of a frustum of a parabolic spindle, one of the ends of the frustum passing through the centre of the spindle.

RULE.—Add into one sum eight times the square of the diameter of the greater end, and three times the square of the diameter of the less end, and four times the product of the diameters; multiply the sum by the length, and this product again by .05236, and the result will be the content.

Demonstration. Put CH $= p$, AH $= q$, (see Fig. of preceding Prob.) and let r be the semidiameter of the less end of the frustum, and t its length. Then from the nature of the parabola we have $p \quad r : p :: t^2 : q^2 = \frac{pt^2}{p-r}$. Substitute this value of q in the equation $y^2 = p^2\left(1 - 2 \cdot \frac{x^2}{q^2} + \frac{x^4}{q^4}\right)$ it becomes $y^2 = p^2\left(1 - 2 \cdot \frac{p-r}{p} \cdot \frac{x^2}{t^2} + \frac{(p-r)^2}{p^2} \frac{x^4}{t^4}\right)$.

By supposing t to be divided into m equal parts, and x to be successively equal to $\frac{1}{m}t, \frac{2}{m}t, \frac{3}{m}t$, &c.....$\frac{m}{m}t$, and proceeding in the same manner as we did in demonstrating the rule for finding the content of the whole spindle, we obtain for the solid content of the frustum this expression,

$$\pi p^2 t\left(1 - \tfrac{2}{3} \frac{p-r}{p} + \tfrac{1}{5} \cdot \frac{(p-r)^2}{p^2}\right) = \frac{\pi}{15} t\,(8p^2 + 4pr + 3r^2),$$

or taking the diameters instead of the radii of the ends the multiplier becomes $\frac{\frac{1}{4}\pi}{15} = .05236$. Hence the reason of the rule is obvious.

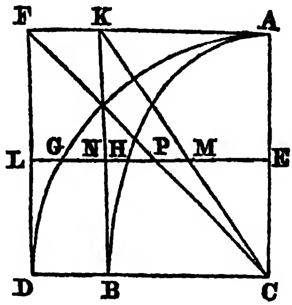
Ex. Supposing the diameters of the ends of a frustum of a parabolic spindle to be 10 feet and 6 feet 2 inches, its length to be 12 feet, and one end to pass through the centre of the spindle; what is its solid content?—*Ans.* 729.32244 cubic feet.

PROBLEM XVI.

To find the solid content of a spheroid, or solid generated b the rotation of an ellipse about either axis.

Rule.—Multiply the fixed axis by the square of the revolving axis, and that product again by the number .5236, and the last product will be the solid content.

Demonstration. Let ABC be an elliptic quadrant, and AKBC its circumscribing rectangle. From C as a centre with the distance CA describe the quadrant AGD, and let AFDC be its circumscribing square. Join CF, CK, and draw any line EL parallel to FA or DC, meeting FD in L, the arch AD in G, KB in N, the elliptic arch AB in H, CF in P, and CK in M. Then, from the nature of the ellipse GE . EH . : (DC . CB) LE : EN, and alternately GE : LE . : EH : EN, therefore GE^2 . LE^2 . EH^2 : EN^2.



Again, from similar triangles PE . ME : : (FA . AK .) LE EN, and alternately PE LE . . ME . EN, therefore

PE^2 . LE^2 : : ME^2 . EN^2

But GE^2 LE^2 . EH^2 . EN^2

Wherefore $PE^2 + GE^2$. LE^2 . . $ME^2 + EH^2$ EN^2

Now $PE^2 + EG^2 = LE^2$, hence also $ME^2 + EH^2 = EN^2$. By supposing, then, the figure to revolve about AC as an axis, and reasoning in the same manner as we did in demonstrating the rule for finding the solid content of a sphere and its segment, we are led to the conclusion that the semi-spheroid described by the revolution of the elliptic quadrant ABC, together with the cone described by the revolution of the triangle CKA, is equal to the cylinder described by the revolution of the rectangle AKBC. Hence the spheroid described by the ellipse revolving about the greater axis is equal to two-thirds of its circumscribing cylinder: And by describing a quadrant from C as a centre with the distance CB, and supposing the figure to revolve on BC as an axis, it may also be demonstrated that the spheroid generated by the revolution of the ellipse on its less axis is equal to two-thirds of its circumscribing cylinder. Hence the reason of the rule is obvious.

Ex 1. The axes of an oblong spheroid are 50 inches and 30 inches, what is its solid content ?—*Ans.* 13 cubic feet 1098 cubic inches.

2. What is the solid content of an oblate spheroid, whose longer axis is 55 feet and shorter axis 33 feet ?—*Ans.* 52268.37 cubic feet.

3. The axes of an oblong spheroid are 55 feet and 33 feet, required its solid content?—*Ans.* 31361.022 cubic feet

PROBLEM XVII

To find the solid content of the frustum of a spheroid, its ends being perpendicular to the fixed axis, and one of them passing through the centre.

RULE.—To the area of the less end add twice that of the greater, multiply the sum by the altitude of the frustum, and one-third of the product will be the content.

Note.—This rule applies also to the frustum of a sphere.

Demonstration. From the demonstration of the rule for finding the solid content of a spheroid, it is evident that a frustum comprehended between two planes perpendicular to the fixt axis, one of which passes through the centre is equal to the difference of the corresponding portions of the circumscribing cylinder and of the cone generated by the revolution of the triangle KCA. (See Fig of preceding Prob)

Put, then, H equal to the height of the frustum, D equal to the diameter of the greater end, and d equal to that of the less. we have for the square of the diameter of the base of the portion of the cone $D^2 - d^2$. Hence if $\frac{1}{4}\pi$ denote the area of a circle whose diameter is unity, we have for the solid content of the portion of the cylinder $\frac{1}{4}\pi D^2 \times H$; and for the solid content of the corresponding portion of the cone $\frac{1}{4}\pi(D^2 - d^2) \times \frac{1}{3}H$. Therefore the solid content of the frustum of the spheroid is equal to

$$\tfrac{1}{4}\pi D^2 \times H - \tfrac{1}{4}\pi(D^2 - d^2) \times \tfrac{1}{3}H = (\tfrac{1}{4}\pi \times 2D^2 + \tfrac{1}{4}\pi d^2) \times \tfrac{1}{3}H.$$

Hence the reason of the rule is evident.

Ex. 1. Suppose the greater end of the frustum of a spheroid to be 18 inches in diameter, the less 10 inches, and the length 14 inches, required the solid content?—*Ans.* 1 cub. foot 1013.569 cub in.

2. What is the solid content of the frustum of a sphere whose diameter is 2 feet, the height of the frustum being 9 inches?—*Ans.* 1.9144 cubic foot.

GAUGING.

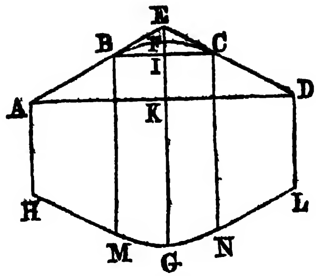
GAUGING is that branch of Mensuration which treats of the method of computing the content of any cask. For this purpose it is necessary to consider casks as of some determinate figure. They are usually considered as of one or other of the four following forms.

1. The middle frustum of a spheroid.
2. The middle frustum of a parabolic spindle.
3. The two equal frustums of a paraboloid.
4. The two equal frustums of a cone.

A cask of any one of these forms may be gauged by help of the rules given for finding the solid content of the particular frustum whose form the cask is supposed to have. But by assuming as a hypothesis that one-third of a cask at each end is nearly the frustum of a cone, and that the middle part may be considered as the middle frustum of a parabolic spindle, we obtain the following general rule by which the content of any cask whatever may be nearly computed in wine, ale, or imperial gallons.

GENERAL RULE.—Add into one sum 39 times the square of the bung diameter, 25 times the square of the head diameter, and 26 times the product of the diameters. Multiply the sum by the length and the product by .00034, then the last product, divided by 9, will give the wine gallons, and divided by 11 will give the ale gallons. For imperial gallons, multiply the said sum by the length, and the product by ·00003123.

Demonstration. Let AHLD be a section of the cask made lengthwise by a plane passing through the centre. Produce AB and DC, the right parts of the section of the side, to meet in E, join BC and AD, and from B, E, C draw BM, EG, CN, perpendicular to AD, and let EG meet the curve BC in F, the straight line BC in I, and AD in K. Let l denote the length of the cask, b the bung diameter and h the head diameter Then since AB, DC have the same direction as the parabolic curve BFC they must be tangents to it, therefore FI $= \frac{1}{2}$EI. But BI $= \frac{1}{3}$AK by hypothesis,



and by similar triangles BI : EI :: AK : EK, therefore EI $= \frac{1}{3}$EK, hence FI $= \frac{1}{2}$EI $= \frac{1}{6}$EK $= \frac{1}{5}$FK $= \frac{1}{10}(b - h)$, so that the common diameter BM $=$ FG $-$ 2FI $= b - \frac{1}{5}(b - h) = \frac{1}{5}(4b + h)$, which put equal to c. Now by the rules for finding the solid content of parabolic spindles and of conic frustums we obtain for the middle part of the cask the expression, ($\frac{1}{4}\pi$ being the area of the circle whose diameter is unity,)

$$\frac{8b^2 + 4bc + 3c^2}{15} \times \frac{\frac{1}{4}l\pi}{2} = \frac{328b^2 + 44bh + 3h^2}{25 \times 45} \times \frac{1}{4}l\pi$$

and for the two ends, the expression,

$$\frac{c^2 + ch + h^2}{3} \times \frac{\frac{1}{2}l\pi}{3} = \frac{160b^2 + 280bh + 310h^2}{25 \times 45} \times \frac{1}{4}l\pi.$$

Taking the sum of these two expressions, and reducing, we obtain the solid content of the cask in cubic inches expressed by this formula,

$$(39b^2 + 26bh + 25h^2)\,\frac{\frac{1}{4}l\pi}{90} \text{ nearly}$$

The factor $\frac{\frac{1}{4}\pi}{90}$ or $\frac{.7854}{90}$ being divided by 231, (the cubic inches in a wine gallon,) gives $\frac{.00034}{9}$ the multiplier for wine gallons, and since 231 is to 282 as 9 to 11 nearly, $\frac{.00034}{11}$ will be the multiplier for ale gallons. The wine gallon is to the imperial gallon as .82673 is to 1, and $\frac{00034}{9} \times 82673 = .00003123$ which is the multiplier for imperial gallons.

Wrong

Ex. 1. Suppose the bung-diameter of a cask having the form of the middle frustum of a spheroid, to be 32 inches, the head-diameter 24 inches, and the length 40 inches, what is the content in ale, in wine, and in imperial gallons?—*Ans.* By the rule for finding the solid content of the frustum of a spheroid the content of the cask

is found to be 97.44 ale gallons, or 118.95 wine gallons, or 98.34 imperial gallons. By the general rule the content is 91.87 ale gallons, or 112.28 wine gallons, or 92.82 imperial gallons.

2. The bung-diameter of a cask whose form is that of the middle frustum of a parabolic spindle is 36 inches, the head diameter 20 inches, and the length 45 inches; required its content in wine gallons?—*Ans.* By the rule for finding the solid content of the frustum of a parabolic spindle the content of the cask is 147.37 wine gallons. By the general rule it is 134.75 wine gallons.

3. Suppose a cask to have the form of two equal frustums of a paraboloid, the length 20 inches, the bung diameter 16 inches, and the head diameter 12 inches, required its content in ale gallons?—*Ans.* By the rule for finding the solid content of the frustum of a paraboloid the content of the cask is 11.14 ale gallons. By the general rule it is 11 48 ale gallons.

4. Required the content, in imperial gallons, of a cask in the form of two equal frustums of a cone, its bung diameter being 17 inches, its head diameter 10 inches, and its length 19 inches?—*Ans.* By the rule for finding the solid content of the frustum of a cone the content of the cask is 9.9515 imperial gallons. By the general rule it is 10.794 imperial gallons.

EXPLANATION

OF THE

TABLES OF INTEREST, ANNUITIES, &c.

1. The first of these Tables (page 80) gives the sum to which £1 will amount, if improved at compound interest, for any number of years not exceeding fifty, at 3, 4, and 5 per cent. The amount of any other sum may therefore be found, by multiplying the amount of £1 for the given time and rate by the principal, expressed in pounds.

Thus the amount of £12 : 10 · 6, or £12.525 for 15 years, at 5 per cent., is 2.07893 × 12.525 = £26.0386 nearly.

2. The Table entitled *Present Value of £1 Compound Interest*, gives the sum which, improved at compound interest, will amount to £1 in any given number of years not exceeding fifty, or the sum which should be paid down immediately, as an equivalent for £1, to be paid at the expiration of the given term of years.

The present value of any given sum is to be found by multiplying the present value of £1 by that sum expressed in pounds.

For example, £35 to be received ten years hence, is equivalent to .744094 × 35 = £26.0433, to be paid down immediately.

3. *Amount of £1 Annuity*, and *Present Value of £1 Annuity Compound Interest.* (page 81.) The nature of these Tables, and their application to any other sum, is sufficiently obvious.

Thus, an annuity of £10, if forborn for seven years, reckoning interest at 4 per cent., will amount to 7.8983 × 10 = £78.983, or £79 nearly.

Again, the present value of an annuity of £10, to be paid at the end of each year, for seven years, is 6.0021 × 10 = £60.021, or £60 nearly.

4. The Tables of the Probabilities of Life, formed from the Registers of Carlisle and Northampton, (pages 82 and 83), are the basis of the important and extensive Theory of Life Annuities, Life Insurances, and of every calculation affected by the uncertainty of human life. Their object is to indicate by numbers, according to the doctrine of CHANCES, the probability that a person of any given age shall live to any other given age. This probability is expressed by a fraction of which the denominator is the Tabular number of persons alive at the first given age, and the numerator the Tabular number alive at the other given age.

Thus, let it be required to find what is the probability that a person aged 40 shall live to the age of 65. By the Carlisle Table, it appears that out of 10,000 persons born, 5075 attain the age of 40, and 3018 attain the age of 65: Therefore the probability required is $\frac{3018}{5075} = .595$, and any advantage, the enjoyment of which depends on the person completing his 65th year, will be less valuable than if its enjoyment were absolutely certain in the proportion of .595 to 1, or of 595 to 1000.

By the Northampton Tables, the same probability will be the fraction $\frac{1632}{3635} = .449$: this result differs from the other, and shows that the probability of human life at different places and under different circumstances, varies considerably.

Again, let it be required to find what is the probability that a person aged 40 will die before he completes his 65th year. By the Carlisle Table, we see that of 5075 persons alive at the age of 40, only 3018 live to the age of 65: therefore 2057 die between the two given ages. Hence, the probability of the person dying within that period is $\frac{2057}{5075} = .405$.

Since it amounts to *certainty* (which is expressed by an unit), that the person will be either alive or dead at the proposed age, if either of the fractions which express the probability of his being alive or dead be subtracted from unity, the remainder will be the fraction which expresses the other probability.

5. *Expectation of Life according to the Carlisle and Northampton Tables of Probabilities.* (page 84.)

These Tables show the average duration of life of a number of persons at any given age. Thus, from the Carlisle Table, it appears, that supposing a great number of persons to be each 15 years of age, they will, one with another, on an average, live 45

years longer, or to the age of 60. According to the Northampton Table, persons of the age of 15, may, one with another, expect an addition of 36.51 years of life.

6. *Value of an Annuity of £1 on a single Life, according to the Carlisle and Northampton Tables of Probabilities.* (pages 85 and 88.)

These Tables show the sum that should be paid down immediately by a person of a given age, for an annuity of £1, to be continued throughout life, or, in other words, the number of years purchase that should be given immediately for a life annuity, reckoning the improvement of money at 3, 4, or 5 per cent.

Thus, suppose a person that has just completed his 35th year wishes to purchase a life annuity, and that the interest of money is reckoned at 4 per cent., the annuity appears to be worth 16.041 years purchase, according to the Carlisle Table, or 14.039 according to the Northampton Table of Probabilities: that is, the Annuitant must pay down £16 . 0 · 10, according to the former Table, and £14 . 0 : 9¾, according to the latter, for every pound of annuity which he is to receive.

7. *Value of an Annuity of £1, on Two Joint Lives, according to the Carlisle and Northampton Tables of Probabilities.* (pages 86 and 89)

These Tables shew immediately the value of an annuity of £1 to continue during the joint lives of two persons whose ages are multiples of 5. Thus, if the ages be 25 and 40, it appears from the Northampton Table, (page 89), that reckoning interest at 3 per cent., an annuity payable, while both continue alive, is worth 11.854 years purchase.

If one or both the ages be not multiples of 5, the value of the annuity must be found by interpolation. The result will indeed not be perfectly correct, but probably near enough the truth for any practical purpose. The approximate value is to be found upon this hypothesis:

If one and the same life be combined successively with five others that differ from it by numbers of years, which form an arithmetical series, whose common difference is one, the values of annuities on these five combinations of lives will form nearly an arithmetical progression.

Admitting this hypothesis, we may find the value of an annuity on the joint continuance of two lives aged 48 and 55, reckoning interest at 5 per cent., by the Carlisle Table, as follows

By combining the age 55 with the ages 45, 46, 47, 48, 49, 50,

the tabular values, by our hypothesis, ought to form an arithmetical progression. We know the two extremes, viz.

Ages.	*Values.*	
45 and 55	8.870	First term.
50 and 55	8.528	Fifth term.

Difference of extremes = 0.342

The intermediate value, which we want, is the third term of this decreasing series, it will therefore be less than the first by $\frac{3}{5}$ of the difference of the extremes, that is by $\frac{3}{5} \times .342 = .205$, this subtracted from 8.870, the first term, leaves 8.665 for the value of an annuity to continue while two persons aged 48 and 45 are both alive.

Example 2.—What is the present value of an annuity on two lives aged 27 and 46, reckoning interest at 4 per cent., according to the Carlisle Table?

We first, proceeding as in the former example, find the values of annuities for the two combinations of lives 25, 46, and 30, 46. These are.

Ages.	*Values.*
25 and 46	12.344
30 and 46	12 101

Difference 0.243

Here we have given the first and last of five terms of a decreasing geometrical series, to find the third, viz. the value for the ages 27 and 46. Two-fifths of their difference .243, is .097, and this subtracted from the first, leaves 12.247, or 12.25 nearly, for the value required.

8. *Lengths of Circular Arcs.* (page 90.)

By this Table, an arc of any number of degrees, minutes, &c., may be expressed in parts of the radius, and conversely.

Example 1.—To find the length of an Arc of 57° 17′ 44″ 48‴.

57°	.9948377
17′	49451
44″	2133
48‴	39

The sum 1.0000000

Hence it appears that the arc is equal to the radius.

EXAMPLE 2.—To find the degrees, minutes, &c. in the Arc 1, which is equal to the radius.

Given length	1.0000000
57° (next less in table)	.9948377
	51623
17′ (next less) - - -	49451
	2172
44″ (next less) - - -	2133
48‴ - - - - - - -	39

9. *Common and Hyperbolic Logarithms.* (page 91.)

This Table serves to convert Common into Hyperbolic Logarithms, and the contrary.

EXAMPLE 1.—To find the Hyperbolic Logarithm answering to the Common Logarithm 0.9562425.

Com. Log.	*Hyp. Log.*
0.9 - - -	2.0723266
54 - -	.1243396
24 - -	5526
25 -	58
0.9542425	2.1972246 answer.

EXAMPLE 2.—To find the common Logarithm answering to the Hyperbolic Logarithm 2.1972246.

	Hyp. Log.
Given	2.1972246
0.9 - - -	2.0723266
	1248980
54 - -	1243396
	5584
24 - -	5526
25 -	58
0.9542425 answer	

10. *Areas of the Segments of a Circle.* (page 91.)

In this Table, each number in the column of Areas is the area of the Circular Segment whose height, or versed sine of its half arc, is the number immediately on the left of it; the diameter of the circle being 1, and the whole area .785398.

11. *Table for finding the difference between the True and Apparent Level,* (page 93.)—Shewing how much must be deducted from the apparent level, in order to find the true level at any distance.

12. The Table of Refraction (page 94) shows how much is to be subtracted from the corresponding apparent altitude. The greatest refraction is 33′ 0″, at the horizon, and diminishes gradually to the zenith, where it is nothing. In observing the altitudes of terrestrial objects the refraction should be taken into account, otherwise the result will be incorrect.

13. *Depression or Dip of the Horizon.* (page 94.)—This correction is necessary, on account of the observer's elevation above the surface of the sea, when altitudes taken with the quadrant are too great by a quantity to be taken from this table, with the height of the observer's eye in feet, to be subtracted from the observed altitude.

14. *Dip of the Sea at different distances from the observer* (page 94.)—When the part of the horizon, directly under the sun, is obstructed by land, and distance from shore, less than five or six miles, the object brought down to the line separating sea and land, the dip is to be taken from this table, with the height of the eye at the top, and distance in miles in the side column.

15. This Table (page 95) shows by inspection the number of links to be subtracted from each chain, in rising or sloping ground, according to the several degrees of altitude or depression, for reducing them to horizontal lines. The first column contains the degrees and minutes of ascent or descent, the second contains the links to be deducted, and the third the inches corresponding to the number of links adjoining.

If the sloping length be 1200, and angle of acclivity 17° 15′, the table shows that 4½ links are to be deducted from every chain, which shortens the distance by 54 links.

16. *Polygon Tables.* (page 95.)—Multiply the square of the side of the given polygon by the number opposite its name in the table, the product is the area.

The angle OAP is half the angle of the polygon, if one fourth of the tangent or number opposite the name be multiplied by the number of sides in the figure, it will give the tabular area of the polygon in the table.

17. *Surfaces and Solidities of the regular bodies.* (page 96.)—To find the superficies, multiply the proper tabular area by the square of the linear edge.

To find the solidity, multiply the tabular solidity by the cube of the linear edge, for the solid content.

18. *The number of Miles in a Degree of Longitude, at different distances from the Equator.* (page 96.)—This is a very useful Table on many occasions,—the number of miles in a degree of longitude, at any number of degrees from the equator, is found by inspection.

A

TABLE

CONTAINING THE

LOGARITHMS OF ALL NUMBERS,

FROM AN UNIT TO 10,000.

Numbers from 1 to 100 and their Logarithms with Indices.

N	Log	N	Log	N	Log	N.	Log.
1	0.000000	26	1.414973	51	1.707570	76	1.880814
2	0.301030	27	1.431364	52	1.716003	77	1.886491
3	0.477121	28	1.447158	53	1.724276	78	1.892095
4	0.602060	29	1.462398	54	1.732394	79	1.897627
5	0.698970	30	1.477121	55	1.740363	80	1.903090
6	0.778151	31	1.491362	56	1.748188	81	1.908485
7	0.845098	32	1.505150	57	1.755875	82	1.913814
8	0.903090	33	1.518514	58	1.763428	83	1.919078
9	0.954243	34	1.531479	59	1.770852	84	1.924279
10	1.000000	35	1.544068	60	1.778151	85	1.929419
11	1.041393	36	1.556303	61	1.785330	86	1.934498
12	1.079181	37	1.568202	62	1.792392	87	1.939519
13	1.113943	38	1.579784	63	1.799341	88	1.944483
14	1.146128	39	1.591065	64	1.806180	89	1.949390
15	1.176091	40	1.602060	65	1.812913	90	1.954243
16	1.204120	41	1.612784	66	1.819544	91	1.959041
17	1.230449	42	1.623249	67	1.826075	92	1.963788
18	1.255273	43	1.633468	68	1.832509	93	1.968483
19	1.278754	44	1.643453	69	1.838849	94	1.973128
20	1.301030	45	1.653213	70	1.845098	95	1.977724
21	1.322219	46	1,662758	71	1.851258	96	1.982271
22	1.342423	47	1.672098	72	1.857332	97	1.986772
23	1.361728	48	1.681241	73	1.863323	98	1.991226
24	1.380211	49	1.690196	74	1.869232	99	1.995635
25	1.397940	50	1.698970	75	1.875061	100	2.000000
N	Log	N	Log	N	Log	N.	Log.

N. B.—In the following part of the Table the Indices are omitted, as they are easily supplied, being always, each of them, in the case of whole or mixed numbers, an unit less than the number of figures in the integral part of the corresponding natural number. If the number is a decimal, the index is negative, and is always an unit greater than the number of cyphers between the decimal point and the first significant figure of the decimal.

N	0	1	2	3	4	5	6	7	8	9	D
100	000000	000434	000868	001301	001734	002166	002598	003029	003461	003891	432
101	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
102	8600	9026	9451	9876	010300	010724	011147	011570	011993	012415	424
103	012837	013259	013680	014100	4521	4940	5360	5779	6197	6616	419
104	7033	7451	7868	8284	8700	9116	9532	9947	020361	020775	416
105	021189	021603	022016	022428	022841	023252	023664	024075	4486	4896	412
106	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
107	9384	9789	030195	030600	031004	031408	031812	032216	032619	033021	404
108	033424	033826	4227	4628	5029	5430	5830	6230	6629	7028	400
109	7426	7825	8223	8620	9017	9414	9811	040207	040602	040998	396
110	041393	041787	042182	042576	042969	043362	043755	044148	044540	044932	393
111	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	389
112	9218	9606	9993	050380	050766	051153	051538	051924	052309	052694	386
113	053078	053463	053846	4230	4613	4996	5378	5760	6142	6524	382
114	6905	7286	7666	8046	8426	8805	9185	9563	9942	060320	379
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117	8186	8557	8928	9298	9668	070038	070407	070776	071145	071514	369
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132	120574	120903	121231	121560	121888	122216	122544	122871	123198	3525	328
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163	212188	212454	2720	2986	3252	3518	3783	4049	4314	4579	266
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177	7973	8219	8464	8709	8954	9198	9443	9687	9932	250176	245
178	250420	250664	250908	251151	251395	251638	251881	252125	252368	2610	243
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180	255273	255514	255755	255996	256237	256477	256718	256958	257198	257439	241
181	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
182	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214	238
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189	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
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191	281033	281261	281488	281715	281942	282169	2396	2622	2849	3075	227
192	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
193	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
194	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
195	290035	290257	290480	290702	290925	291147	291369	291591	291813	292034	222
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200	301030	301247	301464	301681	301898	302114	302331	302547	302764	302980	217
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205	311754	311966	2177	2389	2600	2812	3023	3234	3445	3656	211
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208	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
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210	322219	322426	322633	322839	323046	323252	323458	323665	323871	324077	206
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222	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
223	8305	8500	8694	8889	9083	9278	9472	9666	9860	350054	194
224	350248	350442	350636	350829	351023	351216	351410	351603	351796	1989	193
225	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
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227	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
228	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
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233	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
234	9216	9401	9587	9772	9958	370143	370328	370513	370698	370883	185
235	371068	371253	371437	371622	371806	1991	2175	2360	2544	2728	184
236	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
237	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
238	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
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251	9674	9847	400020	400192	400365	400538	400711	400883	401056	401228	173
252	401401	401573	1745	1917	2089	2261	2433	2605	2777	2949	172
253	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
254	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
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260	414973	415140	415307	415474	415641	415808	415974	416141	416308	416474	167
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288	9392	9543	9694	9845	9995	460146	460296	460447	460597	460748	151
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293	6868	7016	7164	7312	7460	7608	7756	7904	8052	8200	148
294	8347	8495	8643	8790	8938	9085	9233	9380	9527	9675	148
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296	471292	471438	1585	1732	1878	2025	2171	2318	2464	2610	146
297	2756	2903	3049	3195	3341	3487	3633	3779	3925	4071	146
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311	2760	2900	3040	3179	3319	3458	3597	3737	3876	4015	139
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328	5874	6006	6139	6271	6403	6535	6668	6800	6932	7064	132
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331	9828	9959	520090	520221	520353	520485	520615	520745	520876	521007	131
332	521138	521269	1400	1530	1661	1792	1922	2053	2183	2314	131
333	2444	2575	2705	2835	2966	3096	3226	3356	3486	3616	130
334	3746	3876	4006	4136	4266	4396	4526	4656	4785	4915	130
335	5045	5174	5304	5434	5563	5693	5822	5951	6081	6210	129
336	6339	6469	6598	6727	6856	6985	7114	7243	7372	7501	129
337	7630	7759	7888	8016	8145	8274	8402	8531	8660	8788	129
338	8917	9045	9174	9302	9430	9559	9687	9815	9943	530072	128
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349	2825	2950	3074	3199	3323	3447	3571	3696	3820	3944	124
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351	5307	5431	5555	5678	5802	5925	6049	6172	6296	6419	124
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353	7775	7898	8021	8144	8267	8389	8512	8635	8758	8881	123
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356	1450	1572	1694	1816	1938	2060	2181	2303	2425	2547	122
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365	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	119
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
367	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	118
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	118
370	568202	568319	568436	568554	568671	568788	568905	569023	569140	569257	117
371	9374	9491	9608	9725	9842	9959	570076	570193	570309	570426	117
372	570543	570660	570776	570893	571010	571126	1243	1359	1476	1592	117
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
375	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	116
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381	580925	581039	1153	1267	1381	1495	1608	1722	1836	1950	114
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398	9883	9992	600101	600210	600319	600428	600537	600646	600755	600864	109
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403	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
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405	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	107
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407	9594	9701	9808	9914	610021	610128	610234	610341	610447	610554	107
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410	612784	612890	612996	613102	613207	613313	613419	613525	613630	613736	106
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415	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
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417	620136	620240	620344	620448	620552	620656	620760	620864	620968	1072	104
418	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
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427	630428	630530	630631	630733	630835	630936	1038	1139	1241	1342	102
428	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
429	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
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431	4477	4578	4679	4779	4880	4981	5081	5182	5283	5383	100
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458	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
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461	3701	3795	3889	3983	4078	4172	4266	4360	4454	4548	94
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463	5581	5675	5769	5862	5956	6050	6143	6237	6331	6424	94
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468	670246	670339	670431	670524	670617	670710	670802	670895	0988	1080	93
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470	672098	672190	672283	672375	672467	672560	672652	672744	672836	672929	92
471	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
472	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
473	4861	4953	5045	5137	5228	5320	5412	5503	5595	5687	92
474	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
475	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
476	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
477	8518	8609	8700	8791	8882	8973	9064	9155	9246	9337	91
478	9428	9519	9610	9700	9791	9882	9973	680063	680154	680245	91
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487	7529	7618	7707	7796	7886	7975	8064	8153	8242	8331	89
488	8420	8509	8598	8687	8776	8865	8953	9042	9131	9220	89
489	9309	9398	9486	9575	9664	9753	9841	9930	690019	690107	89
490	690196	690285	690373	690462	690550	690639	690728	690816	690905	690993	89
491	1081	1170	1258	1347	1435	1524	1612	1700	1789	1877	88
492	1965	2053	2142	2230	2318	2406	2494	2583	2671	2759	88
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494	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
495	4605	4693	4781	4868	4956	5044	5131	5219	5307	5394	88
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497	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
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503	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
504	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
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506	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
507	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
508	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
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518	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
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522	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
523	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
524	9331	9414	9497	9580	9663	9745	9828	9911	9994	720077	83
525	720159	720242	720325	720407	720490	720573	720655	720738	720821	0903	83
526	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
527	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
528	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
529	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
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533	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
534	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
535	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
536	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
537	9974	730055	730136	730217	730298	730378	730459	730540	730621	730702	81
538	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
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540	732394	732474	732555	732635	732715	732796	732876	732956	733037	733117	80
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542	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
543	4800	4880	4960	5040	5120	5199	5279	5359	5439	5519	80
544	5599	5679	5759	5838	5918	5998	6078	6157	6237	6317	80
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546	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
547	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
548	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
549	9572	9651	9731	9810	9889	9968	740047	740126	740205	740284	79
550	740363	740442	740521	740600	740678	740757	740836	740915	740994	741073	79
551	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
552	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
553	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
554	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
555	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
556	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
557	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
558	6634	6712	6790	6868	6945	7023	7101	7179	7256	7334	78
559	7412	7489	7567	7645	7722	7800	7878	7955	8033	8110	78
560	748188	748266	748343	748421	748498	748576	748653	748731	748808	748885	77
561	8963	9040	9118	9195	9272	9350	9427	9504	9582	9659	77
562	9736	9814	9891	9968	750045	750123	750200	750277	750354	750431	77
563	750508	750586	750663	750740	0817	0894	0971	1048	1125	1202	77
564	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
565	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
566	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
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568	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
569	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
570	755875	755951	756027	756103	756180	756256	756332	756408	756484	756560	76
571	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
572	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
573	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
574	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
575	9668	9743	9819	9894	9970	760045	760121	760196	760272	760347	75
576	760422	760498	760573	760649	760724	0799	0875	0950	1025	1101	75
577	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
578	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
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582	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
583	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
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585	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
586	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
587	8638	8712	8786	8860	8934	9008	9082	9156	9230	9303	74
588	9377	9451	9525	9599	9673	9746	9820	9894	9968	770042	74
589	770115	770189	770263	770336	770410	770484	770557	770631	770705	0778	74
590	770852	770926	770999	771073	771146	771220	771293	771367	771440	771514	74
591	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
592	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
593	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
594	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
595	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
596	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
597	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
598	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
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605	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
606	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
607	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
608	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
609	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
610	785330	785401	785472	785543	785615	785686	785757	785828	785899	785970	71
611	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
612	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
613	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
614	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
615	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
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617	790285	790356	790426	790496	790567	790637	0707	0778	0848	0918	70
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623	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
624	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
625	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
626	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
627	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
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631	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648	69
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633	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
634	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	69
635	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
636	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
637	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
638	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
639	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68
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641	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
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643	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
644	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
645	9560	9627	9694	9762	9829	9896	9964	810031	810098	810165	67
646	810233	810300	810367	810434	810501	810569	810636	0703	0770	0837	67
647	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
648	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
649	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
650	812913	812980	813047	813114	813181	813247	813314	813381	813448	813514	67
651	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
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653	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
654	5578	5644	5711	5777	5843	5911	5976	6042	6109	6175	66
655	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
656	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
657	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
658	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
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660	819544	819610	819676	819741	819807	819873	819939	820004	820070	820136	66
661	820201	820267	820333	820399	820464	820530	820595	0661	0727	0792	66
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664	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
665	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
666	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
667	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
668	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
669	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
670	826075	826140	826204	826269	826334	826399	826464	826528	826593	826658	65
671	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
672	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
673	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
674	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
675	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
676	9947	830011	830075	830139	830204	830268	830332	830396	830460	830525	64
677	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
678	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
679	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
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683	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
684	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
685	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
686	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
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688	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
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691	9478	9541	9604	9667	9729	9792	9855	9918	9981	840043	63
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695	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
696	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
697	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
698	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
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701	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
702	6337	6399	6461	6523	65[illegible]	6646	6708	6770	6832	6894	62
703	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
704	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
705	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
706	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
707	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
708	850033	850095	850156	850217	850279	850340	850401	850462	850524	850585	61
709	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
710	851258	851320	851381	851442	851503	851564	851625	851686	851747	851809	61
711	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
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713	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
714	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
715	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
716	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
717	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
718	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
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723	9138	9198	9258	9318	9379	9439	9499	9559	9619	9679	60
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725	860338	860398	860458	860518	860578	0637	0697	0757	0817	0877	60
726	0937	0996	1056	1116	1176	1236	1295	1355	1415	1475	60
727	1534	1594	1654	1714	1773	1833	1893	1952	2012	2072	60
728	2131	2191	2251	2310	2370	2430	2489	2549	2608	2668	60
729	2728	2787	2847	2906	2966	3025	3085	3144	3204	3263	60
730	863323	863382	863442	863501	863561	863620	863680	863739	863799	863858	59
731	3917	3977	4036	4096	4155	4214	4274	4333	4392	4452	59
732	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
733	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
734	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
735	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
736	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
737	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
738	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
739	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
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741	9818	9877	9935	9994	870053	870111	870170	870228	870287	870345	59
742	870404	870462	870521	870579	0638	0696	0755	0813	0872	0930	58
743	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
744	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
745	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
746	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
747	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
748	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
749	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
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751	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
752	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
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754	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
755	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
756	8522	8579	8637	8694	8752	8809	8866	8924	8981	9039	57
757	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
758	9669	9726	9784	9841	9898	9956	880013	880070	880127	880185	57
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761	1385	1442	1499	1556	1613	1670	1727	1784	1841	1898	57
762	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
763	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
764	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
765	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
766	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
767	4795	4852	4909	4965	5022	5078	5135	5192	5248	5305	57
768	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
769	5926	5983	6039	6096	6152	6209	6265	6321	6378	6434	56
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771	7054	7111	7167	7223	7280	7336	7392	7449	7505	7561	56
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774	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
775	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
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782	3207	3262	3318	3373	3429	3484	3540	3595	3651	3706	56
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784	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
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786	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
787	5975	6030	6085	6140	6195	6251	6306	6361	6416	6471	55
788	6526	6581	6636	6692	6747	6802	6857	6912	6967	7022	55
789	7077	7132	7187	7242	7297	7352	7407	7462	7517	7572	55
790	897627	897682	897737	897792	897847	897902	897957	898012	898067	898122	55
791	8176	8231	8286	8341	8396	8451	8506	8561	8615	8670	55
792	8725	8780	8835	8890	8944	8999	9054	9109	9164	9218	55
793	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
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797	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
798	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
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804	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
805	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
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812	9556	9609	9663	9716	9770	9823	9877	9930	9984	910037	53
813	910191	910144	910197	910251	910304	910358	910411	910464	910518	0571	53
814	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
815	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
816	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
817	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
818	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
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833	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
834	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
835	1686	1738	1790	1842	1894	1946	1998	[illegible]	2102	2154	52
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837	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
838	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
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841	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
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843	5828	5879	5931	5982	6034	6085	6137	6188	6239	6291	51
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845	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
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849	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
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875	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
876	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
877	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
878	3495	3544	3593	3643	3692	3742	3791	3841	3890	3939	49
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881	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
882	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
883	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
884	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
885	6943	6992	7041	7090	7139	7189	7238	7287	7336	7385	49
886	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
887	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
888	8413	8462	8511	8560	8608	8657	8706	8755	8804	8853	49
889	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
890	949390	949439	949488	949536	949585	949634	949683	949731	949780	949829	49
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892	950365	950414	950462	0511	0560	0608	0657	0706	0754	0803	49
893	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
894	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
895	1823	1872	1920	1969	2017	2066	2114	2163	2211	2259	48
896	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
897	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
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903	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
904	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
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906	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
907	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
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916	1895	1942	1990	2038	2085	2132	2180	2227	2275	2322	47
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920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212	47
921	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
922	4731	4778	4825	4872	4919	4966	5013	5060	5108	5155	47
923	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
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925	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
926	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
927	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
928	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
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930	968483	968530	968576	968623	968670	968716	968763	968810	968856	968903	47
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935	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
936	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
937	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
938	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
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942	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
943	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
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945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
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955	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412	45
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957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
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971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
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974	8559	8601	8648	8693	8737	8782	8826	8871	8916	8960	45
975	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
976	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
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981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
983	2554	2598	2642	2686	2730	2771	2819	2863	2907	2951	44
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985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030	44
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
995	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
998	9130	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43
N	0	1	2	3	4	5	6	7	8	9	D

A

TABLE

OF

LOGARITHMIC

SINES, TANGENTS, AND SECANTS,

FOR EVERY DEGREE AND MINUTE OF THE QUADRANT.

N. B.—The minutes in the left-hand column of each page, increasing downwards, belong to the degrees at the top, and those increasing upwards, in the right-hand column, belong to the degrees below.

b

M	Sine	D	Cosec.	Tang	D	Cotang	Secant	D	Cosine	
0	0 000000		Infinite	0 000000		Infinite	10 000000		10 000000	60
1	6 463726	501717	13 536274	6 463726	501717	13 536274	000000	00	000000	59
2	764756	293485	235244	764756	293483	235244	000000	00	000000	58
3	940847	208231	059153	940847	208231	059153	000000	00	000000	57
4	7 065786	161517	12 934214	7 065786	161517	12 934214	000000	00	000000	56
5	162696	131968	837304	162696	131969	837304	000000	00	000000	55
6	241877	111575	758123	241878	111578	758122	000001	01	9 999999	54
7	308821	96653	691176	308825	99653	691175	000001	01	999999	53
8	366816	85254	633184	366817	85254	633183	000001	01	899999	52
9	417968	76263	582032	417970	76263	582030	000001	01	999999	51
10	463725	68988	536275	463727	68988	536273	000002	01	999998	50
11	7 505118	62981	12 491882	7 505120	62981	12 494880	10 000002	01	9 999998	49
12	542906	57936	457094	542909	57933	457091	000003	01	999997	48
13	577668	53641	422332	577672	53642	422328	000003	01	999997	47
14	609853	49938	390147	609857	49939	390143	000004	01	999996	46
15	639816	46714	360184	639820	46715	360180	000004	01	999996	45
16	667845	43881	332156	667849	43882	332151	000005	01	999995	44
17	691173	41372	305827	694179	41373	305821	000005	01	999995	43
18	718997	39135	281003	719004	39136	280997	000006	01	999994	42
19	742477	37127	257523	742483	37128	257516	000007	01	999993	41
20	764754	35315	235246	764761	35136	235239	000007	01	999993	40
21	7 785943	33672	12 214057	7 785951	33673	12 214049	10 000008	01	9 999992	39
22	806146	32175	193854	806155	32176	193845	000009	01	999991	38
23	825451	30805	174549	825460	30806	174540	000010	01	999990	37
24	843934	29517	156066	843944	29519	156056	000011	02	999989	36
25	861662	28388	138338	861671	28390	138326	000011	02	999988	35
26	878695	27317	121305	878708	27318	121292	000012	02	999988	34
27	895085	26323	101915	895099	26325	101901	000013	02	999987	33
28	910879	25399	089121	910894	25401	089106	000014	02	999986	32
29	926119	21538	073881	926134	24540	073866	000015	02	999985	31
30	910812	23733	059158	910858	23735	059142	000017	02	999983	30
31	7 955082	22980	12 011918	7 955100	22981	12 044900	10.000018	02	9 999982	29
32	968870	22273	031130	968889	22275	031111	000019	02	999981	28
33	982233	21608	017767	982254	21610	017747	000020	02	999980	27
34	995198	20981	004802	995219	20983	004781	000021	02	999979	26
35	8 007787	20390	11 992213	8 007809	20392	11 992191	000023	02	999977	25
36	020021	19831	979979	020045	19833	979955	000024	02	999976	24
37	031919	19302	968081	031945	19305	968055	000025	02	999975	23
38	043501	18801	956499	043527	18803	956473	000027	02	999673	22
39	051781	18325	945219	054809	18327	915191	000028	02	999972	21
40	065776	17872	931224	065806	17874	931194	000029	02	999971	20
41	8 076500	17441	11 923500	8 076531	17441	11 923469	10.000031	02	9 999969	19
42	086965	17031	913035	086997	17034	913003	000032	02	999968	18
43	097183	16639	902817	097217	16642	902783	000034	02	999966	17
44	107167	16265	892833	107202	16268	892797	000036	03	999964	16
45	116926	15908	883074	116963	15910	883037	000037	03	999963	15
46	126471	15566	873529	126510	15568	873490	000039	03	999961	14
47	135810	15238	864190	135851	15241	861149	000041	03	999959	13
48	111953	14921	855047	144996	14927	855004	000042	03	999958	12
49	153907	14622	846093	153952	14625	846048	000044	03	999956	11
50	162681	14333	837319	162727	14336	837273	000046	03	999954	10
51	8 171280	14054	11 828720	8 171328	14057	11 828672	10 000048	03	9 999952	9
52	179713	13786	820287	179763	13790	820237	000050	03	999950	8
53	187985	13529	812015	188036	13532	811964	000052	03	999948	7
54	196102	13280	803898	196156	13284	803844	000054	03	999946	6
55	204070	13041	795930	204126	13044	795874	000056	03	999944	5
56	211895	12810	788105	211953	12814	788047	000058	04	999942	4
57	219581	12587	780419	219641	12590	780359	000060	04	999940	3
58	227134	12372	772866	227195	12376	772805	000062	04	999938	2
59	234557	12164	765443	234621	12168	765379	000064	04	999936	1
60	241855	11963	758145	241921	11967	758079	000066	04	999934	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

89 Degrees.

M	Sine	D.	Cosec.	Tang	D	Cotang.	Secant	D	Cosine	
0	8.241855	11963	11.758145	8.241921	11967	11.758079	10.000066	04	9.999934	60
1	249033	11768	750967	249102	11772	750898	000068	04	999932	59
2	256094	11580	743906	256165	11584	743835	000071	04	999929	58
3	263042	11398	736958	263115	11402	736885	000073	04	999927	57
4	269881	11221	730119	269956	11225	730044	000075	04	999925	56
5	276614	11050	723386	276691	11054	723309	000078	04	999922	55
6	283243	10883	716757	283323	10887	716677	000080	04	999920	54
7	289773	10721	710227	289856	10726	710144	000082	04	999918	53
8	296207	10565	703793	296292	10570	703708	000085	04	999915	52
9	302546	10413	697454	302634	10418	697366	000087	04	999913	51
10	308794	10266	691206	308884	10270	691116	000090	04	999910	50
11	8.314954	10122	11.685046	8.315046	10126	11.684954	10.000093	04	9.999907	49
12	321027	9982	678973	321122	9987	678878	000095	04	999905	48
13	327016	9847	672984	327114	9851	672886	000098	04	999902	47
14	332924	9714	667076	333025	9719	666975	000101	05	999899	46
15	338753	9586	661247	338856	9590	661144	000103	05	999897	45
16	344504	9460	655496	344610	9465	655390	000106	05	999894	44
17	350181	9338	649819	350289	9343	649711	000109	05	999891	43
18	355783	9219	644217	355895	9224	644105	000112	05	999888	42
19	361315	9103	638685	361430	9108	638570	000115	05	999885	41
20	366777	8990	633223	366895	8995	633105	000118	05	999882	40
21	8.372171	8880	11.627829	8.372292	8885	11.627708	10.000121	05	9.999879	39
22	377499	8772	622501	377622	8777	622378	000124	05	999876	38
23	382762	8667	617238	382889	8672	617111	000127	05	999873	37
24	387962	8564	612038	388092	8570	611908	000130	05	999870	36
25	393101	8464	606899	393234	8470	606766	000133	05	999867	35
26	398179	8366	601821	398315	8371	601685	000136	05	999864	34
27	403199	8271	596801	403338	8276	596662	000139	05	999861	33
28	408161	8177	591839	408304	8182	591696	000142	05	999858	32
29	413058	8086	586942	413213	8091	586787	000146	05	999854	31
30	417919	7996	582081	418068	8002	581932	000149	06	999851	30
31	8.422717	7909	11.577283	8.422869	7914	11.577131	10.000152	06	9.999848	29
32	427462	7823	572538	427618	7830	572382	000156	06	999844	28
33	432156	7740	567844	432315	7745	567685	000159	06	999841	27
34	436800	7657	563200	436962	7663	563038	000162	06	999838	26
35	441394	7577	558606	441560	7583	558440	000166	06	999834	25
36	445941	7499	554059	446110	7505	553890	000169	06	999831	24
37	450440	7422	549560	450613	7428	549387	000173	06	999827	23
38	454893	7346	545107	455070	7352	544930	000177	06	999823	22
39	459301	7273	540699	459481	7279	540519	000180	06	999820	21
40	463665	7200	536335	463849	7206	536151	000184	06	999816	20
41	8.467985	7129	11.532015	8.468172	7135	11.531828	10.000188	06	9.999812	19
42	472263	7060	527737	472454	7066	527546	000191	06	999809	18
43	476498	6991	523502	476693	6998	523307	000195	06	999805	17
44	480693	6924	519307	480892	6931	519108	000199	06	999801	16
45	484848	6859	515152	485050	6865	514950	000203	07	999797	15
46	488963	6794	511037	489170	6801	510830	000207	07	999793	14
47	493040	6731	506960	493250	6738	506750	000210	07	999790	13
48	497078	6669	502922	497293	6676	502707	000214	07	999786	12
49	501080	6608	498920	501298	6615	498702	000218	07	999782	11
50	505045	6548	494955	505267	6555	494733	000222	07	999778	10
51	8.508974	6489	11.491026	8.509200	6496	11.490800	10.000226	07	9.999774	9
52	512867	6431	487133	513098	6439	486902	000231	07	999769	8
53	516726	6375	483274	516961	6382	483039	000235	07	999765	7
54	520551	6319	479449	520790	6326	479210	000239	07	999761	6
55	524343	6264	475657	524586	6272	475414	000243	07	999757	5
56	528102	6211	471898	528349	6218	471651	000247	07	999753	4
57	531828	6158	468172	532080	6165	467920	000252	07	999748	3
58	535523	6106	464477	535779	6114	464221	000256	07	999744	2
59	539186	6055	460814	539447	6062	460553	000260	07	999740	1
60	542819	6004	457181	543084	6012	456916	000265	07	999735	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

88 Degrees

M	Sine	D	Cosec.	Tang	D	Cotang	Secant	D	Cosine	
0	8 542819	6004	11 457181	8 543084	6012	11 456916	10 000265	07	9 999735	60
1	546422	5955	453578	546691	5962	453309	000269	07	999731	59
2	549995	5906	450005	550268	5914	449732	000274	07	999726	58
3	553539	5858	446461	553817	5866	446183	000278	08	999722	57
4	557054	5811	442946	557336	5819	442664	000283	08	999717	56
5	560540	5765	439460	560828	5773	439172	000287	08	999713	55
6	563999	5719	436001	564291	5727	435709	000292	08	999708	54
7	567431	5674	432569	567727	5682	432273	000296	08	999704	53
8	570836	5630	429164	571137	5638	428863	000301	08	999699	52
9	574214	5587	425786	574520	5595	425480	000306	08	999694	51
10	577566	5544	422434	577877	5552	422123	000311	08	999689	50
11	8 580892	5502	11 419108	8 581208	5510	11 418792	10 000315	08	9 999685	49
12	584193	5460	415807	584514	5468	415486	000320	08	999680	48
13	587469	5419	412531	587795	5427	412205	000325	08	999675	47
14	590721	5379	409279	591051	5387	408949	000330	08	999670	46
15	593948	5339	406052	594283	5347	405717	000335	08	999665	45
16	597152	5300	402848	597492	5308	402508	000340	08	999660	44
17	600332	5261	399668	600677	5270	399323	000345	08	999655	43
18	603489	5223	396511	603839	5232	396161	000350	08	999650	42
19	606623	5186	393377	606978	5194	393022	000355	09	999645	41
20	609734	5149	390266	610094	5158	389906	000360	09	999640	40
21	8 612823	5112	11 387177	8 613189	5121	11 386811	10 000365	09	9 999635	39
22	615891	5076	384109	616262	5085	383738	000371	09	999629	38
23	618937	5041	381063	619313	5050	380687	000376	09	999621	37
24	621962	5006	378038	622343	5015	377657	000381	09	999619	36
25	624965	4972	375035	625352	4981	374648	000386	09	999614	35
26	627918	4938	372052	628340	4947	371660	000392	09	999608	34
27	630911	4904	369089	631308	4913	368692	000397	09	999603	33
28	633854	4871	366146	634256	4880	365714	000403	09	999597	32
29	636776	4839	363224	637184	4848	362816	000408	09	999592	31
30	639680	4806	360320	640093	1816	359907	000414	09	999586	30
31	8 642563	4775	11 357437	8 642982	4784	11 357018	10 000419	09	9 999581	29
32	645128	4743	354572	645853	4753	354147	000425	09	999575	28
33	648274	4712	351726	648704	4722	351296	000430	09	999570	27
34	651102	4682	348898	651537	4691	348463	000436	09	999564	26
35	653911	4652	346089	654352	4661	345648	000412	10	999558	25
36	656702	4622	343298	657149	4631	342851	000447	10	999553	24
37	659475	4592	340525	659928	4602	340072	000453	10	999547	23
38	662230	4563	337770	662689	4573	337311	000459	10	999541	22
39	664968	4535	335032	665433	4544	334567	000465	10	999535	21
40	667689	4506	332311	668160	4526	331810	000471	10	999529	20
41	8 670393	4479	11 329607	8 670870	4488	11 329130	10 000476	10	9 999524	19
42	673080	4451	326920	673563	4461	326437	000482	10	999518	18
43	675751	1424	324249	676239	4434	323761	000488	10	999512	17
44	678405	4397	321595	678900	4417	321100	000494	10	999506	16
45	681043	4370	318957	681544	4380	318456	000500	10	999500	15
46	683665	4344	316335	684172	4351	315828	000507	10	999493	14
47	686272	4318	313728	686784	4328	313216	000513	10	999487	13
48	688863	4292	311137	689381	4303	310619	000519	10	999481	12
49	691438	4267	308562	691963	4277	308037	000525	10	999175	11
50	693998	4242	306002	694529	4252	305471	000531	10	999469	10
51	8 696543	4217	11 303457	8 697081	4228	11 302919	10 000537	11	9 999463	9
52	699073	4192	300927	699617	4203	300383	000544	11	999456	8
53	701589	4168	298411	702139	4179	297861	000550	11	999450	7
54	704090	4144	295910	704646	4155	295354	000557	11	999443	6
55	706577	4121	293423	707140	4132	292860	000563	11	999437	5
56	709049	4097	290951	709618	4108	290382	000569	11	999431	4
57	711507	4074	288493	712083	4085	287917	000576	11	999424	3
58	713952	4051	286048	714534	4062	285465	000582	11	999118	2
59	716383	4029	283617	716972	4040	283028	000589	11	999411	1
60	718800	1006	281200	719396	4017	280604	000596	11	999404	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec.	Tang	D	Cotang	Secant	D	Cosine	
0	8 718800	4006	11.281200	8 719396	4017	11 280604	10 000596	11	9 999404	60
1	721204	3984	278796	721806	3995	278194	000602	11	999398	59
2	723595	3962	276405	724204	3974	275796	000609	11	999391	58
3	725972	3941	274028	726588	3952	273412	000616	11	999384	57
4	728337	3919	271663	728959	3930	271041	000622	11	999378	56
5	730688	3898	269312	731317	3909	268683	000629	11	999371	55
6	733027	3877	266973	733663	3889	266337	000636	12	999364	54
7	735354	3857	264646	735996	3868	264004	000643	12	999357	53
8	737667	3836	262333	738317	3848	261683	000650	12	999350	52
9	739969	3816	260031	740626	3827	259374	000657	12	999343	51
10	742259	3796	257741	742922	3807	257078	000664	12	999336	50
11	8 744536	3776	11.255464	8 745207	3787	11.254793	10 000671	12	9 999329	49
12	746802	3756	253198	747479	3768	252521	000678	12	999322	48
13	749055	3737	250945	749740	3749	250260	000685	12	999315	47
14	751297	3717	248703	751989	3729	248011	000692	12	999308	46
15	753528	3698	246472	754227	3710	245773	000699	12	999301	45
16	755747	3679	244253	756453	3692	243547	000706	12	999294	44
17	757955	3661	242045	758668	3673	241332	000714	12	999286	43
18	760151	3642	239849	760872	3655	239128	000721	12	999279	42
19	762337	3624	237663	763065	3636	236935	000728	12	999272	41
20	764511	3606	235489	765246	3618	234754	000735	12	999265	40
21	8 766675	3588	11 233325	8 767417	3600	11 232583	10 000743	12	9 999257	39
22	768828	3570	231172	769578	3583	230422	000750	13	999250	38
23	770970	3553	229030	771727	3565	228273	000758	13	999242	37
24	773101	3535	226899	773866	3548	226134	000765	13	999235	36
25	775223	3518	224777	775995	3531	224005	000773	13	999227	35
26	777333	3501	222667	778114	3514	221886	000780	13	999220	34
27	779434	3484	220566	780222	3497	219778	000788	13	999212	33
28	781524	3467	218476	782320	3480	217680	000795	13	999205	32
29	783605	3451	216395	784408	3464	215592	000803	13	999197	31
30	785675	3431	214325	786486	3447	213514	000811	13	999189	30
31	8 787736	3418	11 212264	8 788554	3431	11 211446	10 000819	13	9 999181	29
32	789787	3402	210213	790613	3415	209387	000826	13	999174	28
33	791828	3386	208172	792662	3399	207338	000834	13	999166	27
34	793859	3370	206141	794701	3383	205299	000842	13	999158	26
35	795881	3354	204119	796731	3368	203269	000850	13	999150	25
36	797894	3339	202106	798752	3352	201248	000858	13	999142	24
37	799897	3323	200103	800763	3337	199237	000866	13	999134	23
38	801892	3308	198108	802765	3322	197235	000874	13	999126	22
39	803876	3293	196124	804758	3307	195242	000882	13	999118	21
40	805852	3278	194148	806742	3292	193258	000890	13	999110	20
41	8 807819	3263	11 192181	8 808717	3278	11 191283	10 000898	13	9 999102	19
42	809777	3249	190223	810683	3262	189317	000906	14	999094	18
43	811726	3234	188274	812641	3248	187359	000914	14	999086	17
44	813667	3219	186333	814589	3233	185411	000923	14	999077	16
45	815599	3205	184401	816529	3219	183471	000931	14	999069	15
46	817522	3191	182478	818461	3205	181539	000939	14	999061	14
47	819436	3177	180564	820384	3191	179616	000947	14	999053	13
48	821343	3163	178657	822298	3177	177702	000956	14	999044	12
49	823240	3149	176760	824205	3163	175795	000964	14	999036	11
50	825130	3135	174870	826103	3150	173897	000973	14	999027	10
51	8 827011	3122	11 172989	8 827992	3136	11 172008	10 000981	14	9 999019	9
52	828884	3108	171116	829874	3123	170126	000990	14	999010	8
53	830749	3095	169251	831748	3110	168252	000998	14	999002	7
54	832607	3082	167393	833613	3096	166387	001007	14	998993	6
55	834456	3069	165544	835471	3083	164529	001016	14	998984	5
56	836297	3056	163703	837321	3070	162679	001024	14	998976	4
57	838130	3043	161870	839163	3057	160837	001033	15	998967	3
58	839956	3030	160044	840998	3045	159002	001042	15	998958	2
59	841774	3017	158226	842825	3032	157175	001050	15	998950	1
60	843585	3000	156415	844644	3019	155356	001059	15	998941	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec.	Tang	D	Cotang	Secant	D	Cosine	
0	8 843585	3005	11 156415	8 844644	3019	11 155356	10 001059	15	9 998941	60
1	845387	2992	154613	846455	3007	153545	001068	15	998932	59
2	847183	2980	152817	848260	2995	151740	001077	15	998923	58
3	848971	2967	151029	850057	2982	149943	001086	15	998914	57
4	850751	2955	149249	851846	2970	148154	001095	15	998905	56
5	852525	2943	147475	853628	2958	146372	001104	15	998896	55
6	854291	2931	145709	855403	2946	144597	001113	15	998887	54
7	856049	2919	143951	857171	2935	142829	001122	15	998878	53
8	857801	2907	142199	858932	2923	141068	001131	15	998869	52
9	859546	2896	140454	860686	2911	139314	001140	15	998860	51
10	861283	2884	138717	862433	2900	137567	001149	15	998851	50
11	8 863014	2873	11.136980	8 864173	2888	11 135827	10.001159	15	9 998841	49
12	864738	2861	135262	865906	2877	134094	001168	15	998832	48
13	866455	2850	133545	867632	2866	132368	001177	16	998823	47
14	868165	2839	131835	869351	2854	130649	001187	16	998813	46
15	869868	2828	130132	871064	2843	128936	001196	16	998804	45
16	871565	2817	128435	872770	2832	127230	001205	16	998795	44
17	873255	2806	126745	874469	2821	125531	001215	16	998785	43
18	874938	2795	125062	876162	2811	123838	001224	16	998776	42
19	876615	2786	123385	877849	2800	122151	001234	16	998766	41
20	878285	2773	121715	879529	2789	120471	001243	16	998757	40
21	8 879949	2763	11 120051	8 881202	2779	11 118798	10.001253	16	9 998747	39
22	881607	2752	118393	882869	2768	117131	001262	16	998738	38
23	883258	2742	116742	884530	2758	115470	001272	16	998728	37
24	884903	2731	115097	886185	2747	113815	001282	16	998718	36
25	886542	2721	113458	887833	2737	112167	001292	16	998708	35
26	888174	2711	111826	889476	2727	110524	001301	16	998699	34
27	889801	2700	110199	891112	2717	108888	001311	16	998689	33
28	891421	2690	108579	892742	2707	107258	001321	16	998679	32
29	893035	2680	106965	894366	2697	105634	001331	17	998669	31
30	894643	2670	105357	895984	2687	104016	001341	17	998659	30
31	8 896246	2660	11 103754	8 897596	2677	11 102404	10 001351	17	9 998649	29
32	897842	2651	102158	899203	2667	100797	001361	17	998639	28
33	899432	2641	100568	900803	2658	099197	001371	17	998629	27
34	901017	2631	098983	902398	2648	097602	001381	17	998619	26
35	902596	2622	097404	903987	2638	096013	001391	17	998609	25
36	904169	2612	095831	905570	2629	094430	001401	17	998599	24
37	905736	2603	094264	907147	2620	092853	001411	17	998589	23
38	907297	2593	092703	908719	2610	091281	001422	17	998578	22
39	908853	2584	091147	910285	2601	089715	001432	17	998568	21
40	910404	2575	089596	911846	2592	088154	001442	17	998558	20
41	8 911949	2566	11.088051	8 913401	2583	11 086599	10 001452	17	9 998548	19
42	913488	2556	086512	914951	2574	085049	001463	17	998537	18
43	915022	2547	084978	916495	2565	083505	001473	17	998527	17
44	916550	2538	083450	918034	2556	081966	001484	18	998516	16
45	918073	2529	081927	919568	2547	080432	001494	18	998506	15
46	919591	2520	080409	921096	2538	078904	001505	18	998495	14
47	921103	2512	078897	922619	2530	077381	001515	18	998485	13
48	922610	2503	077390	924136	2521	075864	001526	18	998474	12
49	924112	2494	075888	925649	2512	074351	001536	18	998464	11
50	925609	2486	074391	927156	2503	072844	001547	18	998453	10
51	8.927100	2477	11 072900	8 928658	2495	11 071342	10 001558	18	9 998442	9
52	928587	2469	071413	930155	2486	069845	001569	18	998431	8
53	930068	2460	069932	931647	2478	068353	001579	18	998421	7
54	931544	2452	068456	933134	2470	066866	001590	18	998410	6
55	933015	2443	066985	934616	2461	065384	001601	18	998399	5
56	934481	2435	065519	936093	2453	063907	001612	18	998388	4
57	935942	2427	064058	937565	2445	062435	001623	18	998377	3
58	937398	2419	062602	939032	2437	060968	001634	18	998366	2
59	938850	2411	061150	940494	2430	059506	001645	18	998355	1
60	940296	2403	059704	941952	2421	058048	001656	18	998344	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	8 940296	2403	11.059704	8 941952	2421	11.058048	10.001656	19	9 998344	60
1	941738	2394	058262	943404	2413	056596	001667	19	998333	59
2	943174	2387	056826	944852	2405	055148	001678	19	998322	58
3	944609	2379	055391	946295	2397	053705	001689	19	998311	57
4	946031	2371	053969	947734	2390	052266	001700	19	998300	56
5	947456	2363	052544	949168	2382	050832	001711	19	998289	55
6	948874	2355	051126	950597	2374	049403	001723	19	998277	54
7	950287	2348	049713	952021	2366	047979	001734	19	998266	53
8	951696	2340	048304	953441	2360	046559	001745	19	998255	52
9	953100	2332	046900	954856	2351	045144	001757	19	998243	51
10	954499	2325	045501	956267	2344	043733	001768	19	998232	50
11	8 955894	2317	11 044106	8.957674	2337	11 042326	10 001780	19	9 998220	49
12	957284	2310	042716	959075	2329	040925	001791	19	998209	48
13	958670	2302	041330	960473	2323	039527	001803	19	998197	47
14	960052	2295	039948	961866	2314	038134	001814	19	998186	46
15	961429	2288	038571	963255	2307	036745	001826	19	998174	45
16	962801	2280	037199	964639	2300	035361	001837	19	998163	44
17	964170	2273	035830	966019	2293	033981	001849	19	998151	43
18	965534	2266	034466	967394	2286	032606	001861	20	998139	42
19	966893	2259	033107	968766	2279	031234	001872	20	998128	41
20	968249	2252	031751	970133	2271	029867	001884	20	998116	40
21	8 969600	2244	11 030400	8 971496	2265	11 028504	10 001896	20	9 998104	39
22	970947	2238	029053	972855	2257	027145	001908	20	998092	38
23	972289	2231	027711	974209	2251	025791	001920	20	998080	37
24	973628	2224	026372	975560	2244	024440	001932	20	998068	36
25	974962	2217	025038	976906	2237	023094	001944	20	998056	35
26	976293	2210	023707	978248	2230	021752	001956	20	998044	34
27	977619	2203	022381	979586	2223	020414	001968	20	998032	33
28	978941	2197	021059	980921	2217	019079	001980	20	998020	32
29	980259	2190	019741	982251	2210	017749	001992	20	998008	31
30	981573	2183	018427	983577	2204	016423	002004	20	997996	30
31	8 982883	2177	11 017117	8 984899	2197	11 015101	10 002016	20	9 997984	29
32	984189	2170	015811	986217	2191	013783	002028	20	997972	28
33	985491	2163	014509	987532	2184	012468	002041	20	997959	27
34	986789	2157	013211	988842	2178	011158	002053	20	997947	26
35	988083	2150	011917	990149	2171	009851	002065	21	997935	25
36	989374	2144	010626	991451	2165	008549	002078	21	997922	24
37	990660	2138	009340	992750	2158	007250	002090	21	997910	23
38	991943	2131	008057	994045	2152	005955	002103	21	997897	22
39	993222	2125	006778	995337	2146	004663	002115	21	997885	21
40	994497	2119	005503	996624	2140	003376	002128	21	997872	20
41	8 995768	2112	11 004232	8 997908	2134	11 002092	10 002140	21	9 997860	19
42	997036	2106	002964	999188	2127	000812	002153	21	997847	18
43	998299	2100	001701	9 000465	2121	10 999535	002165	21	997835	17
44	999560	2094	000440	001738	2115	998262	002178	21	997822	16
45	9 000816	2087	10 999184	003007	2109	996993	002191	21	997809	15
46	002069	2082	997931	004272	2103	995728	002203	21	997797	14
47	003318	2076	996682	005534	2097	994466	002216	21	997784	13
48	004563	2070	995437	006792	2091	993208	002229	21	997771	12
49	005805	2064	994195	008047	2085	991953	002242	21	997758	11
50	007044	2058	992956	009298	2080	990702	002255	21	997745	10
51	9 008278	2052	10 991722	9 010546	2074	10 989454	10 002268	21	9 997732	9
52	009510	2046	990490	011790	2068	988210	002281	21	997719	8
53	010737	2040	989263	013031	2062	986969	002294	21	997706	7
54	011962	2034	988038	014268	2056	985732	002307	22	997693	6
55	013182	2029	986818	015502	2051	984498	002320	22	997680	5
56	014400	2023	985600	016732	2045	983268	002333	22	997667	4
57	015613	2017	984387	017959	2040	982041	002346	22	997654	3
58	016824	2012	983176	019183	2033	980817	002359	22	997641	2
59	018031	2006	981969	020403	2028	979597	002372	22	997628	1
60	019235	2000	980765	021620	2023	978380	002386	22	997614	0
	Cosine		Secant	Cotang		Tang.	Cosec		Sine	M

M	Sine	D	Cosec.	Tang	D	Cotang	Secant	D	Cosine	
0	9 019235	2000	10 980765	9 021620	2023	10 978380	10 002386	22	9 997614	60
1	020435	1995	979565	022834	2017	977166	002399	22	997601	59
2	021632	1989	978368	024044	2011	975956	002412	22	997588	58
3	022825	1984	977175	025251	2006	974749	002426	22	997574	57
4	024016	1978	975984	026455	2000	973545	002439	22	997561	56
5	025203	1973	974797	027655	1995	972345	002453	22	997547	55
6	026386	1967	973614	028852	1990	971148	002466	23	997534	54
7	027567	1962	972433	030046	1985	969954	002480	23	997520	53
8	028744	1957	971256	031237	1979	968763	002493	23	997507	52
9	029918	1951	970082	032425	1974	967575	002507	23	997493	51
10	031089	1947	968911	033609	1969	966391	002520	23	997480	50
11	9 032257	1941	10 967743	9.034791	1964	10 965209	10 002534	23	9 997466	49
12	033421	1936	966579	035969	1958	964031	002548	23	997452	48
13	034582	1930	965418	037144	1953	962856	002561	23	997439	47
14	035741	1925	964259	038316	1948	961684	002575	23	997425	46
15	036896	1920	963104	039485	1943	960515	002589	23	997411	45
16	038048	1915	961952	040651	1938	959349	002603	23	997397	44
17	039197	1910	960803	041813	1933	958187	002617	23	997383	43
18	040342	1905	959658	042973	1928	957027	002631	23	997369	42
19	041485	1899	958515	044130	1923	955870	002645	23	997355	41
20	042625	1894	957375	045284	1918	954716	002659	23	997341	40
21	9 043762	1889	10 956238	9 046434	1913	10 953566	10 002673	24	9 997327	39
22	044895	1884	955105	047582	1908	952418	002687	24	997313	38
23	046026	1879	953974	048727	1903	951273	002701	24	997299	37
24	047154	1875	952846	049869	1898	950131	002715	24	997285	36
25	048279	1870	951721	051008	1893	948992	002729	24	997271	35
26	049400	1865	950600	052144	1889	947856	002743	24	997257	34
27	050519	1860	949481	053277	1884	946723	002758	24	997242	33
28	051635	1855	948365	054407	1879	945593	002772	24	997228	32
29	052749	1850	947251	055535	1874	944465	002786	24	997214	31
30	053859	1845	946141	056659	1870	943341	002801	24	997199	30
31	9 054966	1841	10 945034	9 057781	1865	10 942219	10 002815	24	9 997185	29
32	056071	1836	943929	058900	1869	941100	002830	24	997170	28
33	057172	1831	912828	060016	1855	939984	002844	24	997156	27
34	058271	1827	911729	061130	1851	938870	002859	24	997141	26
35	059367	1822	940633	062240	1846	937760	002873	24	997127	25
36	060460	1817	939540	063348	1842	936652	002888	24	997112	24
37	061551	1813	938449	064453	1837	935547	002902	24	997098	23
38	062639	1808	937361	065556	1833	934444	002917	25	997083	22
39	063724	1804	936276	066655	1828	933345	002932	25	997068	21
40	064806	1799	935194	067752	1824	932248	002947	25	997053	20
41	9 065885	1794	10 934115	9 068846	1819	10 931154	10 002961	25	9 997039	19
42	066962	1790	933038	069938	1815	930062	002976	25	997024	18
43	068036	1786	931964	071027	1810	928973	002991	25	997009	17
44	069107	1781	930893	072113	1806	927887	003006	25	996994	16
45	070176	1777	929824	073197	1802	926803	003021	25	996979	15
46	071242	1772	928758	074278	1797	925722	003036	25	996964	14
47	072306	1768	927694	075356	1793	924644	003051	25	996949	13
48	073366	1763	926634	076432	1789	923568	003066	25	996934	12
49	074424	1759	925576	077505	1784	922495	003081	25	996919	11
50	075480	1755	924520	078576	1780	921424	003096	25	996904	10
51	9 076533	1750	10 923467	9 079644	1776	10 920356	10 003111	25	9 996889	9
52	077583	1746	922417	080710	1772	919290	003126	25	996874	8
53	078631	1742	621369	081773	1767	918227	003142	25	996858	7
54	079676	1738	920324	082833	1763	917167	003157	25	996843	6
55	080719	1733	919281	083891	1759	916109	003172	25	996828	5
56	081759	1729	918241	084947	1755	915053	003188	26	996812	4
57	082797	1725	917203	086000	1751	914000	003203	26	996797	3
58	083832	1721	916168	087050	1747	912950	003218	26	996782	2
59	084864	1717	915136	088098	1743	911902	003234	26	996766	1
60	085894	1713	914106	089144	1738	910856	003249	26	996751	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec.	Tang.	D	Cotang	Secant	D	Cosine	
0	9.085894	1713	10.914106	9.089144	1738	10.910856	10.003249	26	9.996751	60
1	086922	1709	913078	090187	1734	909813	003265	26	996735	59
2	087947	1704	912053	091228	1730	908772	003280	26	996720	58
3	088970	1700	911030	092266	1727	907734	003296	26	996704	57
4	089990	1696	910010	093302	1722	906698	003312	26	996688	56
5	091008	1692	908992	094336	1719	905664	003327	26	996673	55
6	092024	1688	907976	095367	1715	904633	003343	26	996657	54
7	093037	1684	906963	096395	1711	903605	003359	26	996641	53
8	094047	1680	905953	097422	1707	902578	003375	26	996625	52
9	095056	1676	904944	098446	1703	901554	003390	26	996610	51
10	096062	1673	903938	099468	1699	900532	003406	26	996594	50
11	9.097065	1668	10.902935	9.100487	1695	10.899513	10.003422	27	9.996578	49
12	098066	1665	901934	101504	1691	898496	003438	27	996562	48
13	099065	1661	900935	102519	1687	897481	003454	27	996546	47
14	100062	1657	899938	103532	1684	896468	003470	27	996530	46
15	101056	1653	898944	104542	1680	895458	003486	27	996514	45
16	102048	1649	897952	105550	1676	894450	003502	27	996498	44
17	103037	1645	896963	106556	1672	893444	003518	27	996482	43
18	104025	1641	895975	107559	1669	892441	003535	27	996465	42
19	105010	1638	894990	108560	1665	891440	003551	27	996449	41
20	105992	1634	894008	109559	1661	890441	003567	27	996433	40
21	9.106973	1630	10.893027	9.110556	1658	10.889444	10.003583	27	9.996417	39
22	107951	1627	892049	111551	1654	888449	003600	27	996400	38
23	108927	1623	891073	112543	1650	887457	003616	27	996384	37
24	109901	1619	890099	113533	1646	886467	003632	27	996368	36
25	110873	1616	889127	114521	1643	885479	003649	27	996351	35
26	111842	1612	888158	115507	1639	884493	003665	27	996335	34
27	112809	1608	887191	116491	1636	883509	003682	27	996318	33
28	113774	1605	886226	117472	1632	882528	003698	28	996302	32
29	114737	1601	885263	118452	1629	881548	003715	28	996285	31
30	115698	1597	884302	119429	1625	880571	003731	28	996269	30
31	9.116656	1594	10.883344	9.120404	1622	10.879596	10.003748	28	9.996252	29
32	117613	1590	882387	121377	1618	878623	003765	28	996235	28
33	118567	1587	881433	122348	1615	877652	003781	28	996219	27
34	119519	1583	880481	123317	1611	876683	003798	28	996202	26
35	120469	1580	879531	124284	1607	875716	003815	28	996185	25
36	121417	1576	878583	125249	1604	874751	003832	28	996168	24
37	122362	1573	877638	126211	1601	873789	003849	28	996151	23
38	123306	1569	876694	127172	1597	872828	003866	28	996134	22
39	124248	1566	875752	128130	1594	871870	003883	28	996117	21
40	125187	1562	874813	129087	1591	870913	003900	28	996100	20
41	9.126125	1559	10.873875	9.130041	1587	10.869959	10.003917	29	9.996083	19
42	127060	1556	872940	130994	1584	869006	003934	29	996066	18
43	127993	1552	872007	131944	1581	868056	003951	29	996049	17
44	128925	1549	871075	132893	1577	867107	003968	29	996032	16
45	129854	1545	870146	133839	1574	866161	003985	29	996015	15
46	130781	1542	869219	134784	1571	865216	004002	29	995998	14
47	131706	1539	868294	135726	1567	864274	004020	29	995980	13
48	132630	1535	867370	136667	1564	863333	004037	29	995963	12
49	133551	1532	866449	137605	1561	862395	004054	29	995946	11
50	134470	1529	865530	138542	1558	861458	004072	29	995928	10
51	9.135387	1525	10.864613	9.139476	1555	10.860524	10.004089	29	9.995911	9
52	136303	1522	863697	140409	1551	859591	004106	29	995894	8
53	137216	1519	862784	141340	1548	858660	004123	29	995876	7
54	138128	1516	861872	142269	1545	857731	004141	29	995859	6
55	139037	1512	860963	143196	1542	856804	004159	29	995841	5
56	139944	1509	860056	144121	1539	855879	004177	29	995823	4
57	140850	1506	859150	145044	1535	854956	004194	29	995806	3
58	141754	1503	858246	145966	1532	854034	004212	29	995788	2
59	142655	1500	857345	146885	1529	853115	004229	29	995771	1
60	143555	1496	856445	147803	1526	852197	004247	29	995753	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

82 Degrees.

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 143555	1496	10.856445	9 147803	1526	10 852197	10 004247	30	9 995753	60
1	144453	1493	855547	148718	1523	851282	004265	30	995735	59
2	145349	1490	854651	149632	1520	850368	004283	30	995717	58
3	146243	1487	853757	150544	1517	849456	004301	30	995699	57
4	147136	1484	852864	151454	1514	848546	004319	30	995681	56
5	148026	1481	851974	152363	1511	847637	004336	30	995664	55
6	148915	1478	851085	153269	1508	846731	004354	30	995646	54
7	149802	1475	850198	154174	1505	845826	004372	30	995628	53
8	150686	1472	849314	155077	1502	844923	004390	30	995610	52
9	151569	1469	848431	155978	1499	844022	004409	30	995591	51
10	152451	1466	847549	156877	1496	843123	004427	30	995573	50
11	9 153330	1463	10.846670	9.157775	1493	10 842225	10 004445	30	9 995555	49
12	154208	1460	845792	158671	1490	841329	004463	30	995537	48
13	155083	1457	844917	159565	1487	840435	004481	30	995519	47
14	155957	1454	844043	160457	1484	839543	004499	31	995501	46
15	156830	1451	843170	161347	1481	838653	004518	31	995482	45
16	157700	1448	842300	162236	1479	837764	004536	31	995464	44
17	158569	1445	841431	163123	1476	836877	004554	31	995446	43
18	159435	1442	840565	164008	1473	835992	004573	31	995427	42
19	160301	1439	839699	164892	1470	835108	004591	31	995409	41
20	161164	1436	838836	165774	1467	834226	004610	31	995390	40
21	9 162025	1433	10 837975	9 166654	1464	10 833346	10.004628	31	9 995372	39
22	162885	1430	837115	167532	1461	832468	004647	31	995353	38
23	163743	1427	836257	168409	1458	831591	004666	31	995334	37
24	164600	1424	835400	169284	1455	830716	004684	31	995316	36
25	165454	1422	834546	170157	1453	829843	004703	31	995297	35
26	166307	1419	833693	171029	1450	828971	004722	31	995278	34
27	167159	1416	832841	171899	1447	828101	004740	31	995260	33
28	168008	1413	831992	172767	1444	827233	004759	32	995241	32
29	168856	1410	831144	173634	1442	826366	004778	32	995222	31
30	169702	1407	830298	174499	1439	825501	004797	32	995203	30
31	9 170547	1405	10 829453	9 175362	1436	10 824638	10 004816	32	9 995184	29
32	171389	1402	828611	176224	1433	823776	004835	32	995165	28
33	172230	1399	827770	177084	1431	822916	004854	32	995146	27
34	173070	1396	826930	177942	1428	822058	004873	32	995127	26
35	173908	1394	826092	178799	1425	821201	004892	32	995108	25
36	174744	1391	825256	179655	1423	820345	004911	32	995089	24
37	175578	1388	824422	180508	1420	819492	004930	32	995070	23
38	176411	1386	823589	181360	1417	818640	004949	32	995051	22
39	177242	1383	822758	182211	1415	817789	004968	32	995032	21
40	178072	1380	821928	183059	1412	816941	004987	32	995013	20
41	9 178900	1377	10 821100	9 183907	1409	10 816093	10 005007	32	9 994993	19
42	179726	1374	820274	184752	1407	815248	005026	32	994974	18
43	180551	1372	819449	185597	1404	814403	005045	32	994955	17
44	181374	1369	818626	186439	1402	813561	005065	32	994935	16
45	182196	1366	817804	187280	1399	812720	005084	33	994916	15
46	183016	1364	816984	188120	1396	811880	005104	33	994896	14
47	183834	1361	816166	188958	1393	811042	005123	33	994877	13
48	184651	1359	815349	189794	1391	810206	005143	33	994857	12
49	185466	1356	814534	190629	1389	809371	005162	33	994838	11
50	186280	1353	813720	191462	1386	808538	005182	33	994818	10
51	9 187092	1351	10 812908	9 192294	1384	10 807706	10.005202	33	9 994798	9
52	187903	1348	812097	193124	1381	806876	005221	33	994779	8
53	188712	1346	811288	193953	1379	806047	005241	33	994759	7
54	189519	1343	810481	194780	1376	805220	005261	33	994739	6
55	190325	1341	809675	195606	1374	804394	005281	33	994719	5
56	191130	1338	808870	196430	1371	803570	005300	33	994700	4
57	191933	1336	808067	197253	1369	802747	005320	33	994680	3
58	192734	1333	807266	198074	1366	801926	005340	33	994660	2
59	193534	1330	806466	198894	1364	801106	005360	33	994640	1
60	194332	1328	805668	199713	1361	800287	005380	33	994620	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang.	Secant	D	Cosine	
0	9 194332	1328	10 805668	9 199713	1361	10.800287	10.005380	33	9 994620	60
1	195129	1326	804871	200529	1359	799471	005400	33	994600	59
2	195925	1323	804075	201345	1356	798655	005420	33	994580	58
3	196719	1321	803281	202159	1354	797841	005440	34	994560	57
4	197511	1318	802489	202971	1352	797029	005460	34	994540	56
5	198302	1316	801698	203782	1349	796218	005481	34	994519	55
6	199091	1313	800909	204592	1347	795408	005501	34	994499	54
7	199879	1311	800121	205400	1345	794600	005521	34	994479	53
8	200666	1308	799334	206207	1342	793793	005541	34	994459	52
9	201451	1306	798549	207013	1340	792987	005562	34	994438	51
10	202234	1304	797766	207817	1338	792183	005582	34	994418	50
11	9 203017	1301	10.796983	9 208619	1335	10.791381	10.005603	34	9 994397	49
12	203797	1299	796203	209420	1333	790580	005623	34	994377	48
13	204577	1296	795423	210220	1331	789780	005643	34	994357	47
14	205354	1294	794646	211018	1328	788982	005664	34	994336	46
15	206131	1292	793869	211815	1326	788185	005684	34	994316	45
16	206906	1289	793094	212611	1324	787389	005705	34	994295	44
17	207679	1287	792321	213405	1321	786595	005726	35	994274	43
18	208452	1285	791548	214198	1319	785802	005746	35	994254	42
19	209222	1282	790778	214989	1317	785011	005767	35	994233	41
20	209992	1280	790008	215780	1315	784220	005788	35	994212	40
21	9 210760	1278	10 789240	9 216568	1312	10 783432	10 005809	35	9 994191	39
22	211526	1275	788474	217356	1310	782644	005829	35	994171	38
23	212291	1273	787709	218142	1308	781858	005850	35	994150	37
24	213055	1271	786945	218926	1305	781074	005871	35	994129	36
25	213818	1268	786182	219710	1303	780290	005892	35	994108	35
26	214579	1266	785421	220492	1301	779508	005913	35	994087	34
27	215338	1264	784662	221272	1299	778728	005934	35	994066	33
28	216097	1261	783903	222052	1297	777948	005955	35	994045	32
29	216854	1259	783146	222830	1294	777170	005976	35	994024	31
30	217609	1257	782391	223606	1292	6394	005997	35	994003	30
31	9 218363	1255	10 781637	9 224382	1290	10 775618	10 006019	5	9 993981	29
32	219116	1253	780884	225156	1288	774844	006040	35	993960	28
33	219868	1250	780132	225929	1286	774071	006061	35	993939	27
34	220618	1248	779382	226700	1284	773300	006082	35	993918	26
35	221367	1246	778633	227471	1281	772529	006104	36	993896	25
36	222115	1244	777885	228239	1279	771761	006125	36	993875	24
37	222861	1242	777139	229007	1277	770993	006146	36	993854	23
38	223606	1239	776394	229773	1275	770227	006168	36	993832	22
39	224349	1237	775651	230539	273	769461	006189	36	993811	21
40	225092	1235	774908	231302	1271	768698	006211	36	993789	20
41	9 225833	1233	10 774167	9 232065	1269	10 767935	10 006232	36	9 993768	19
42	226573	1231	773427	232826	1267	767174	006254	36	993746	18
43	227311	1228	772689	233586	1265	766414	006275	36	993725	17
44	228048	1226	771952	234345	1262	765655	006297	36	993703	16
45	228784	1224	771216	235103	1260	764897	006319	36	993681	15
46	229518	1222	770482	235859	1258	764141	006340	36	993660	14
47	230252	1220	769748	236614	1256	763386	006362	36	993638	13
48	230984	1218	769016	237368	1254	762632	006384	36	993616	12
49	231714	1216	768286	238120	1252	761880	006406	37	993594	11
50	232444	1214	767556	238872	1250	761128	006428	37	993572	10
51	9 233172	1212	10 766828	9 239622	1248	10 760378	10 006450	37	9 993550	9
52	233899	1209	766101	240371	1246	759629	006472	37	993528	8
53	234625	1207	765375	241118	1244	758882	006494	37	993506	7
54	235349	1205	764651	241865	1242	758135	006516	37	993484	6
55	236073	1203	763927	242610	1240	757390	006538	37	993462	5
56	236795	1201	763205	243354	1238	756646	006560	37	993440	4
57	237515	1199	762485	244097	1236	755903	006582	37	993418	3
58	238235	1197	761765	244839	1234	755161	006604	37	993396	2
59	238953	1195	761047	245579	1232	754421	006626	37	993374	1
60	239670	1193	760330	246319	1230	753681	006649	37	993351	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 239670	1193	10 760330	9 246319	1230	10 753681	10 006649	37	9 993351	60
1	240386	1191	759614	247057	1228	752943	006671	37	993329	59
2	241101	1189	758899	247794	1226	752206	006693	37	993307	58
3	241814	1187	758186	248530	1224	751470	006715	37	993285	57
4	242526	1185	757474	249264	1222	750736	006738	37	993262	56
5	243237	1183	756763	249998	1220	750002	006760	37	993240	55
6	243947	1181	756053	250730	1218	749270	006783	38	993217	54
7	244656	1179	755344	251461	1217	748539	006805	38	993195	53
8	245363	1177	754637	252191	1215	747809	006828	38	993172	52
9	246069	1175	753931	252920	1213	747080	006851	38	993149	51
10	246775	1173	753225	253648	1211	746352	006873	38	993127	50
11	9 247478	1171	10 752522	9 254374	1209	10 745626	10.006896	38	9 993104	49
12	248181	1169	751819	255100	1207	744900	006919	38	993081	48
13	248883	1167	751117	255824	1205	744176	006941	38	993059	47
14	249583	1165	750417	256547	1203	743453	006964	38	993036	46
15	250282	1163	749718	257269	1201	742731	006987	38	993013	45
16	250980	1161	749020	257990	1200	742010	007010	38	992990	44
17	251677	1159	748323	258710	1198	741290	007033	38	992967	43
18	252373	1158	747627	259429	1196	740571	007056	38	992944	42
19	253067	1156	746933	260146	1194	739854	007079	38	992921	41
20	253761	1154	746239	260863	1192	739137	007102	38	992898	40
21	9 254453	1152	10 745547	9 261578	1190	10 738422	10 007125	38	9 992875	39
22	255144	1150	744856	262292	1189	737708	007148	38	992852	38
23	255834	1148	744166	263005	1187	736995	007171	39	992829	37
24	256523	1146	743477	263717	1185	736283	007194	39	992806	36
25	257211	1144	742789	264428	1183	735572	007217	39	992783	35
26	257898	1142	742102	265138	1181	734862	007241	39	992759	34
27	258583	1141	741417	265847	1179	734153	007264	39	992736	33
28	259268	1139	740732	266555	1178	733445	007287	39	992713	32
29	259951	1137	740049	267261	1176	732739	007310	39	992690	31
30	260633	1135	739367	267967	1174	732033	007334	39	992666	30
31	9 261314	1133	10 738686	9 268671	1172	10 731329	10 007357	39	9 992643	29
32	261994	1131	738006	269375	1170	730625	007381	39	992619	28
33	262673	1130	737327	270077	1169	729923	007404	39	992596	27
34	263351	1128	736649	270779	1167	729221	007428	39	992572	26
35	264027	1126	735973	271479	1165	728521	007451	39	992549	25
36	264703	1124	735297	272178	1164	727822	007475	39	992525	24
37	265377	1122	734623	272876	1162	727124	007499	39	992501	23
38	266051	1120	733949	273573	1160	726427	007522	40	992478	22
39	266723	1119	733277	274269	1158	725731	007546	40	992454	21
40	267395	1117	732605	274964	1157	725036	007570	40	992430	20
41	9 268065	1115	10 731935	9 275658	1155	10 724342	10 007594	40	9 992406	19
42	268734	1113	731266	276351	1153	723649	007618	40	992382	18
43	269402	1111	730598	277043	1151	722957	007641	40	992359	17
44	270069	1110	729931	277734	1150	722266	007665	40	992335	16
45	270735	1108	729265	278424	1148	721576	007689	40	992311	15
46	271400	1106	728600	279113	1147	720887	007713	40	992287	14
47	272064	1105	727936	279801	1145	720199	007737	40	992263	13
48	272726	1103	727274	280488	1143	719512	007761	40	992239	12
49	273388	1101	726612	281174	1141	718826	007786	40	992214	11
50	274049	1099	725951	281858	1140	718142	007810	40	992190	10
51	9 274708	1098	10 725292	9 282542	1138	10 717458	10 007834	40	9 992166	9
52	275367	1096	724633	283225	1136	716775	007858	40	992142	8
53	276024	1094	723976	283907	1135	716093	007883	41	992117	7
54	276681	1092	723319	284588	1133	715412	007907	41	992093	6
55	277337	1091	722663	285268	1131	714732	007931	41	992069	5
56	277991	1089	722009	285947	1130	714053	007956	41	992044	4
57	278644	1087	721356	286624	1128	713376	007980	41	992020	3
58	279297	1086	720703	287301	1126	712699	008004	41	991996	2
59	279948	1084	720052	287977	1125	712023	008029	41	991971	1
60	280599	1082	719401	288652	1123	711348	008053	41	991947	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

79 Degrees.

M	Sine	D	Cosec.	Tang	D	Cotang	Secant	D	Cosine	
0	9 280599	1082	10 719401	9 288652	1123	10 711348	10.008053	41	9 991947	60
1	281248	1081	718752	289326	1122	710674	008078	41	991922	59
2	281897	1079	718103	289999	1120	710001	008103	41	991897	58
3	282544	1077	717456	290671	1118	709329	008127	41	991873	57
4	283190	1076	716810	291342	1117	708658	008152	41	991848	56
5	283836	1074	716164	292013	1115	707987	008177	41	991823	55
6	284480	1072	715520	292682	1114	707318	008201	41	991799	54
7	285124	1071	714876	293350	1112	706650	008226	42	991774	53
8	285766	1069	714234	294017	1111	705983	008251	42	991749	52
9	286408	1067	713592	294684	1109	705316	008276	42	991724	51
10	287048	1066	712952	295349	1107	704651	008301	42	991699	50
11	9 287687	1064	10.712313	9 296013	1106	10.703987	10 008326	42	9 991674	49
12	288326	1063	711674	296677	1104	703323	008351	42	991649	48
13	288964	1061	711036	297339	1103	702661	008376	42	991624	47
14	289600	1059	710400	298001	1101	701999	008401	42	991599	46
15	290236	1058	709764	298662	1100	701338	008426	42	991574	45
16	290870	1056	709130	299322	1098	700678	008451	42	991549	44
17	291504	1054	708496	299980	1096	700020	008476	42	991524	43
18	292137	1053	707863	300638	1095	699362	008502	42	991498	42
19	292768	1051	707232	301295	1093	698705	008527	42	991473	41
20	293399	1050	706601	301951	1092	698049	008552	42	991448	40
21	9 294029	1048	10 705971	9 302607	1090	10 697393	10 008578	42	9 991422	39
22	294658	1046	705342	303261	1089	696739	008603	42	991397	38
23	295286	1045	704714	303914	1087	696086	008628	43	991372	37
24	295913	1043	704087	304567	1086	695433	008654	43	991346	36
25	296539	1042	703461	305218	1084	694782	008679	43	991321	35
26	297164	1040	702836	305869	1083	694131	008705	43	991295	34
27	297788	1039	702212	306519	1081	693481	008730	43	991270	33
28	298412	1037	701588	307168	1080	692832	008756	43	991244	32
29	299034	1036	700966	307815	1078	692185	008782	43	991218	31
30	299655	1034	700345	308463	1077	691537	008807	43	991193	30
31	9 300276	1032	10 699724	9 309109	1075	10 690891	10 008833	43	9 991167	29
32	300895	1031	699105	309754	1074	690246	008859	43	991141	28
33	301514	1029	698486	310398	1073	689602	008885	43	991115	27
34	302132	1028	697868	311042	1071	688958	008910	43	991090	26
35	302748	1026	697252	311685	1070	688315	008936	43	991064	25
36	303364	1025	696636	312327	1068	687673	008962	43	991038	24
37	303979	1023	696021	312967	1067	687033	008988	43	991012	23
38	304593	1022	695407	313608	1065	686392	009011	43	990986	22
39	305207	1020	694793	314247	1064	685753	009040	43	990960	21
40	305819	1019	694181	314885	1062	685115	009066	44	990934	20
41	9 306430	1017	10 693570	9 315523	1061	10 684477	10 009092	44	9 990908	19
42	307041	1016	692959	316159	1060	683841	009118	44	990882	18
43	307650	1014	692350	316795	1058	683205	009145	44	990855	17
44	308259	1013	691741	317430	1057	682570	009171	44	990829	16
45	308867	1011	691133	318064	1055	681936	009197	44	990803	15
46	309474	1010	690526	318697	1054	681303	009223	44	990777	14
47	310080	1008	689920	319329	1053	680671	009250	44	990750	13
48	310685	1007	689315	319961	1051	680039	009276	44	990724	12
49	311289	1005	688711	320592	1050	679408	009303	44	990697	11
50	311893	1004	688107	321222	1048	678778	009329	44	990671	10
51	9 312495	1003	10 687505	9 321851	1047	10 678149	10 009356	44	9 990644	9
52	313097	1001	686903	322479	1045	677521	009382	44	990618	8
53	313698	1000	686302	323106	1044	676894	009409	44	990591	7
54	314297	998	685703	323733	1043	676267	009435	44	990565	6
55	314897	997	685104	324358	1041	675642	009462	44	990538	5
56	315495	996	684505	324983	1040	675017	009489	45	990511	4
57	316092	994	683908	325607	1039	674393	009515	45	990485	3
58	316689	993	683311	326231	1037	673769	009542	45	990458	2
59	317284	991	682716	326853	1036	673147	009569	45	990431	1
60	317879	990	682121	327475	1035	672525	009596	45	990404	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec.	Tang	D	Cotang.	Secant	D	Cosine	
0	9 317879	990	10 682121	9.327474	1035	10 672526	10 009596	45	9 990404	60
1	318473	988	681527	328095	1033	671905	009622	45	990378	59
2	319066	987	680934	328715	1032	671285	009649	45	990351	58
3	319658	986	680342	329334	1030	670666	009676	45	990324	57
4	320249	984	679751	329953	1029	670047	009703	45	990297	56
5	320840	983	679160	330570	1028	669430	009730	45	990270	55
6	321430	982	678570	331187	1026	668813	009757	45	990243	54
7	322019	980	677981	331803	1025	668197	009785	45	990215	53
8	322607	979	677393	332418	1024	667582	009812	45	990188	52
9	323194	977	676806	333033	1023	666967	009839	45	990161	51
10	323780	976	676220	333646	1021	666354	009866	45	990134	50
11	9.324366	975	10 675634	9 334259	1020	10.665741	10 009893	46	9 990107	49
12	324950	973	675050	334871	1019	665129	009921	46	990079	48
13	325534	972	674466	335482	1017	664518	009948	46	990052	47
14	326117	970	673883	336093	1016	663907	009975	46	990025	46
15	326700	969	673300	336702	1015	663298	010003	46	989997	45
16	327281	968	672719	337311	1013	662689	010030	46	989970	44
17	327862	966	672138	337919	1012	662081	010058	46	989942	43
18	328442	965	671558	338527	1011	661473	010085	46	989915	42
19	329021	964	670979	339133	1010	660867	010113	46	989887	41
20	329599	962	670401	339739	1008	660261	010140	46	989860	40
21	9 330176	961	10 669824	9 340344	1007	10 659656	10.010168	46	9 989832	39
22	330753	960	669247	340948	1006	659052	010196	46	989804	38
23	331329	958	668671	341552	1004	658448	010223	46	989777	37
24	331903	957	668097	342155	1003	657845	010251	47	989749	36
25	332478	956	667522	342757	1002	657243	010279	47	989721	35
26	333051	954	666949	343358	1000	656642	010307	47	989693	34
27	333624	953	666376	343958	999	656042	010335	47	989665	33
28	334195	952	665805	344558	998	655442	010363	47	989657	32
29	334766	950	665234	345157	997	654843	010391	47	989609	31
30	335337	949	664663	345755	996	654245	010419	47	989582	30
31	9.335906	948	10 664094	9 346353	994	10 653647	10 010447	47	9 989553	29
32	336475	946	663525	346949	993	653051	010475	47	989525	28
33	337043	945	662957	347545	992	652455	010503	47	989497	27
34	337610	944	662390	348141	991	651859	010531	47	989469	26
35	338176	943	661824	348735	990	651265	010559	47	989441	25
36	338742	941	661258	349329	988	650671	010587	47	989413	24
37	339306	940	660694	349922	987	650078	010616	47	989384	23
38	339871	939	660129	350514	986	649486	010644	47	989356	22
39	340434	937	659566	351106	985	648894	010672	47	989328	21
40	340996	936	659004	351697	983	648303	010700	47	989300	20
41	9 341558	935	10 658442	9 352287	982	10 647713	10 010729	47	9 989271	19
42	342119	934	657881	352876	981	647124	010757	47	989243	18
43	342679	932	657321	353465	980	646535	010786	47	989214	17
44	343239	931	656761	354053	979	645947	010814	47	989186	16
45	343797	930	656203	354640	977	645360	010843	47	989157	15
46	344355	929	655645	355227	976	644773	010872	48	989128	14
47	344912	927	655088	355813	975	644187	010900	48	989100	13
48	345469	926	654531	356398	974	643602	010929	48	989071	12
49	346024	925	653976	356982	973	643018	010958	48	989042	11
50	346579	924	653421	357566	971	642434	010986	48	989014	10
51	9 347134	922	10 652866	9 358149	970	10 641851	10 011015	48	9 988985	9
52	347687	921	652313	358731	969	641269	011044	48	988956	8
53	348240	920	651760	359313	968	640687	011073	48	988927	7
54	348792	919	651208	359893	967	640107	011102	48	988898	6
55	349343	917	650657	360474	966	639526	011131	48	988869	5
56	349893	916	650107	361053	965	638947	011160	48	988840	4
57	350443	915	649557	361632	963	638368	011189	49	988811	3
58	350992	914	649008	362210	962	637790	011218	49	988782	2
59	351540	913	648460	362787	961	637213	011247	49	988753	1
60	352088	911	647912	363361	960	636636	011276	49	988724	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec.	Tang.	D	Cotang.	Secant	D	Cosine	
0	9 352088	911	10.647912	9 363964	960	10 636036	10.011276	49	9 988724	60
1	352635	910	647365	363940	959	636060	011305	49	988695	59
2	353181	909	646819	364515	958	635485	011334	49	988666	58
3	353726	908	646274	365090	957	634910	011364	49	988636	57
4	354271	907	645729	365664	955	634336	011393	49	988607	56
5	354815	905	645185	366237	954	633763	011422	49	988578	55
6	355358	904	644642	366810	953	633190	011452	49	988548	54
7	355901	903	644099	367382	952	632618	011481	49	988519	53
8	356443	902	643557	367953	951	632047	011511	49	988489	52
9	356984	901	643016	368524	950	631476	011540	49	988460	51
10	357524	899	642476	369094	949	630906	011570	49	988430	50
11	9 358064	898	10 641936	9 369663	948	10 630337	10 011599	49	9 988401	49
12	358603	897	641397	370232	946	629768	011629	49	988371	48
13	359141	896	640859	370799	945	629201	011658	49	988342	47
14	359678	895	640322	371367	944	628633	011688	50	988312	46
15	360215	893	639785	371933	943	628067	011718	50	988282	45
16	360752	892	639248	372499	942	627501	011748	50	988252	44
17	361287	891	638713	373064	941	626936	011777	50	988223	43
18	361822	890	638178	373629	940	626371	011807	50	988193	42
19	362356	889	637644	374193	939	625807	011837	50	988163	41
20	362889	888	637111	374756	938	625244	011867	50	988133	40
21	9 363422	887	10 636578	9 375319	937	10 624681	10 011897	50	9 988103	39
22	363954	885	636046	375881	935	624119	011927	50	988073	38
23	364485	884	635515	376442	934	623558	011957	50	988043	37
24	365016	883	634984	377003	933	622997	011987	50	988013	36
25	365546	882	634454	377563	932	622437	012017	50	987983	35
26	366075	881	633925	378122	931	621878	012048	50	987953	34
27	366604	880	633396	378681	930	621319	012078	50	987922	33
28	367131	879	632869	379239	929	620761	012108	50	987892	32
29	367659	877	632341	379797	928	620203	012138	50	987862	31
30	368185	876	631815	380354	927	619646	012168	51	987832	30
31	9 368711	875	10 631289	9 380910	926	10 619090	10 012199	51	9 987801	29
32	369236	874	630764	381466	925	618534	012229	51	987771	28
33	369761	873	630239	382020	924	617980	012260	51	987740	27
34	370285	872	629715	382575	923	617425	012290	51	987710	26
35	370808	871	629192	383129	922	616871	012321	51	987679	25
36	371330	870	628670	383682	921	616318	012351	51	987649	24
37	371852	869	628148	384234	920	615766	012382	51	987618	23
38	372373	867	627627	384786	919	615214	012412	51	987588	22
39	372894	866	627106	385337	918	614663	012443	51	987557	21
40	373414	865	626586	385888	917	614112	012474	51	987526	20
41	9 373933	864	10 626067	9 386438	915	10 613562	10 012505	51	9 987496	19
42	374452	863	625548	386987	914	613013	012535	51	987465	18
43	374970	862	625030	387536	913	612464	012566	51	987434	17
44	375487	861	624513	388084	912	611916	012597	52	987403	16
45	376003	860	623997	388631	911	611369	012628	52	987372	15
46	376519	859	623481	389178	910	610822	012659	52	987341	14
47	377035	858	622965	389724	909	610276	012690	52	987310	13
48	377549	857	622451	390270	908	609730	012721	52	987279	12
49	378063	856	621937	390815	907	609185	012752	52	987248	11
50	378577	854	621423	391360	906	608640	012783	52	987217	10
51	9 379089	853	10 620911	9 391903	905	10 608097	10 012814	52	9 987186	9
52	379601	852	620399	392447	904	607553	012845	52	987155	8
53	380113	851	619887	392989	903	607011	012876	52	987124	7
54	380624	850	619376	393531	902	606469	012908	52	987092	6
55	381134	849	618866	394073	901	605927	012939	52	987061	5
56	381643	848	618357	394614	900	605386	012970	52	987030	4
57	382152	847	617848	395154	899	604846	013002	52	986998	3
58	382661	846	617339	395694	898	604306	013033	52	986967	2
59	383168	845	616832	396233	897	603767	013064	52	986936	1
60	383675	844	616325	396771	896	603229	013096	52	986904	0
	Cosine		Secant	Cotang		Tang	Cosec.		Sine	M

M	Sine	D	Cosec.	Tang	D	Cotang	Secant	D	Cosine	
0	9 383675	844	10.616325	9 396771	896	10 603229	10 013096	52	9 986904	60
1	384182	843	615818	397309	896	602691	013127	53	986873	59
2	384687	842	615313	397846	895	602154	013159	53	986841	58
3	385192	841	614808	398383	894	601617	013191	53	986809	57
4	385697	840	614303	398919	893	601081	013222	53	986778	56
5	386201	839	613799	399455	892	600545	013254	53	986746	55
6	386704	838	613296	399990	891	600010	013286	53	986714	54
7	387207	837	612793	400524	890	599476	013317	53	986683	53
8	387709	836	612291	401058	889	598942	013349	53	986651	52
9	388210	835	611790	401591	888	598409	013381	53	986619	51
10	388711	834	611289	402124	887	597876	013413	53	986587	50
11	9 389211	833	10 610789	9 402656	886	10.597344	10 013445	53	9 986555	49
12	389711	832	610289	403187	885	596813	013477	53	986523	48
13	390210	831	609790	403718	884	596282	013509	53	986491	47
14	390708	830	609292	404249	883	595751	013541	53	986459	46
15	391206	828	608794	404778	882	595222	013573	53	986427	45
16	391703	827	608297	405308	881	594692	013605	53	986395	44
17	392199	826	607801	405836	880	594164	013637	54	986363	43
18	392695	825	607305	406364	879	593636	013669	54	986331	42
19	393191	824	606809	406892	878	593108	013701	54	986299	41
20	393685	823	606315	407419	877	592581	013734	54	986266	40
21	9 394179	822	10 605821	9 407915	876	10 592055	10 013766	54	9 986234	39
22	394675	821	605327	408471	875	591529	013798	54	986202	38
23	395166	820	604834	408997	874	591003	013831	54	986169	37
24	395658	819	604342	409521	874	590479	013863	54	986137	36
25	396150	818	603850	410045	873	589955	013896	54	986104	35
26	396641	817	603359	410569	872	589431	013928	54	986072	34
27	397132	817	602868	411092	871	588908	013961	54	986039	33
28	397621	816	602379	411615	870	588385	013993	54	986007	32
29	398111	815	601889	412137	869	587863	014026	54	985974	31
30	398600	814	601400	412658	868	587342	014058	54	985942	30
31	9 399088	813	10 600912	9 413179	867	10 586821	10 014091	55	9 985909	29
32	399575	812	600425	413699	866	586301	014124	55	985876	28
33	400062	811	599938	414219	865	585781	014157	55	985843	27
34	400549	810	599451	414738	864	585262	014189	55	985811	26
35	401035	809	598965	415257	864	584743	014222	55	985778	25
36	401520	808	598480	415775	863	584225	014255	55	985745	24
37	402005	807	597995	416293	862	583707	014288	55	985712	23
38	402489	806	597511	416810	861	583190	014321	55	985679	22
39	402972	805	597028	417326	860	582674	014354	55	985646	21
40	403455	804	596545	417842	859	582158	014387	55	985613	20
41	9 403938	803	10 596062	9 418358	858	10 581642	10 014420	55	9 985580	19
42	404420	802	595580	418873	857	581127	014453	55	985547	18
43	404901	801	595099	419387	856	580613	014486	55	985514	17
44	405382	800	594618	419901	855	580099	014520	55	985480	16
45	405862	799	594138	420415	855	579585	014553	55	985447	15
46	406341	798	593659	420927	854	579073	014586	56	985414	14
47	406820	797	593180	421440	853	578560	014620	56	985380	13
48	407299	796	592701	421952	852	578048	014653	56	985347	12
49	407777	795	592223	422463	851	577537	014686	56	985314	11
50	408254	794	591746	422974	850	577026	014720	56	985280	10
51	9 408731	794	10 591269	9 423484	849	10 576516	10 014753	56	9 985247	9
52	409207	793	590793	423993	848	576007	014787	56	985213	8
53	409682	792	590318	424503	848	575497	014820	56	985180	7
54	410157	791	589843	425011	847	574989	014854	56	985146	6
55	410632	790	589368	425519	846	574481	014887	56	985113	5
56	411106	789	588894	426027	845	573973	014921	56	985079	4
57	411579	788	588421	426534	844	573466	014955	56	985045	3
58	412052	787	587948	427041	843	572959	014989	56	985011	2
59	412524	786	587476	427547	843	572453	015022	56	984978	1
60	412996	785	587004	428052	842	571948	015056	56	984944	0
	Cosine		Secant	Cotang		Tang	Cosec.		Sine	M

75 Degrees.

M	Sine	D	Cosec.	Tang	D.	Cotang	Secant	D	Cosine	
0	9 412996	785	10 587004	9 428052	842	10.571948	10 015056	57	9 984944	60
1	413467	784	586533	428557	841	571443	015090	57	984910	59
2	413938	783	586062	429062	840	570938	015124	57	984876	58
3	414408	783	585592	429566	839	570434	015158	57	984842	57
4	414878	782	585122	430070	838	569930	015192	57	984808	56
5	415347	781	584653	430573	838	569427	015226	57	984774	55
6	415815	780	584185	431075	837	568925	015260	57	984740	54
7	416283	779	583717	431577	836	568423	015294	57	984706	53
8	416751	778	583249	432079	835	567921	015328	57	984672	52
9	•417217	777	582783	432580	834	567420	015363	57	984637	51
10	417684	776	582316	433080	833	566920	015397	57	984603	50
11	9 418150	775	10.581850	9.433580	832	10.566420	10.015431	57	9 984569	49
12	418615	774	581385	434080	832	565920	015465	57	984535	48
13	419079	773	580921	434579	831	565421	015500	57	984500	47
14	419544	773	580456	435078	830	564922	015534	57	984466	46
15	420007	772	579993	435576	829	564424	015568	58	984432	45
16	420470	771	579530	436073	828	563927	015603	58	984397	44
17	420933	770	579067	436570	828	563430	015637	58	984363	43
18	421395	769	578605	437067	827	562933	015672	58	984328	42
19	421857	768	578143	437563	826	562437	015706	58	984294	41
20	422318	767	577682	438059	825	561941	015741	58	984259	40
21	9 422778	767	10 577222	9 438554	824	10 561446	10 015776	58	9 984224	39
22	423238	766	576762	439048	823	560952	015810	58	984190	38
23	423697	765	576303	439543	823	560457	015845	58	984155	37
24	424156	764	575844	440036	822	559964	015880	58	984120	36
25	424615	763	575385	440529	821	559471	015915	58	984085	35
26	425073	762	574927	441022	820	558978	015950	58	984050	34
27	425530	761	574470	441514	819	558486	015985	58	984015	33
28	425987	760	574013	442006	819	557994	016019	58	983981	32
29	426443	760	573557	442497	818	557503	016054	58	983946	31
30	426899	759	573101	442988	817	557012	016089	58	983911	30
31	9 427354	758	10 572646	9 443479	816	10.556521	10 016125	58	9 983875	29
32	427809	757	572191	443968	816	556032	016160	59	983840	28
33	428263	756	571737	444458	815	555542	016195	59	983805	27
34	428717	755	571283	444947	814	555053	016230	59	983770	26
35	429170	754	570830	445435	813	554565	016265	59	983735	25
36	429623	753	570377	445923	812	554077	016300	59	983700	24
37	430075	752	569925	446411	812	553589	016336	59	983664	23
38	430527	752	569473	446898	811	553102	016371	59	983629	22
39	430978	751	569022	447384	810	552616	016406	59	983594	21
40	431429	750	568571	447870	809	552130	016442	59	983558	20
41	9 431879	749	10 568121	9 448356	809	10 551644	10 016477	59	9 983523	19
42	432329	749	567671	448841	808	551159	016513	59	983487	18
43	432778	748	567222	449326	807	550674	016548	59	983452	17
44	433226	747	566774	449810	806	550190	016584	59	983416	16
45	433675	746	566325	450294	806	549706	016619	59	983381	15
46	434122	745	565878	450777	805	549223	016655	59	983345	14
47	434569	744	565431	451260	804	548740	016691	59	983309	13
48	435016	744	564984	451743	803	548257	016727	60	983273	12
49	435462	743	564538	452225	802	547775	016762	60	983238	11
50	435908	742	564092	452706	802	547294	016798	60	983202	10
51	9 436353	741	10 563647	9 453187	801	10 546813	10 016834	60	9 983166	9
52	436798	740	563202	453668	800	546332	016870	60	983130	8
53	437242	740	562758	454148	799	545852	016906	60	983094	7
54	437686	739	562314	454628	799	545372	016942	60	983058	6
55	438129	738	561871	455107	798	544893	016978	60	983022	5
56	438572	737	561428	455586	797	544414	017014	60	982986	4
57	439014	736	560986	456064	796	543936	017050	60	982950	3
58	439456	736	560544	456542	796	543458	017086	60	982914	2
59	439897	735	560103	457019	795	542981	017122	60	982878	1
60	440338	734	559662	457496	794	542504	017158	60	982842	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

74 Degrees.

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 440338	734	10 559662	9 457496	794	10 542504	10 017158	60	9 982842	60
1	440778	733	559222	457973	793	542027	017195	60	982805	59
2	441218	732	558782	458449	793	541551	017231	61	982769	58
3	441658	731	558342	458925	792	541075	017267	61	982733	57
4	442096	731	557904	459400	791	540600	017304	61	982696	56
5	442535	730	557465	459875	790	540125	017340	61	982660	55
6	442973	729	557027	460349	790	539651	017376	61	982624	54
7	443410	728	556590	460823	789	539177	017413	61	982587	53
8	443847	727	556153	461297	788	538703	017449	61	982551	52
9	444284	727	555716	461770	788	538230	017486	61	982514	51
10	444720	726	555280	462242	787	537758	017523	61	982477	50
11	9 445155	725	10 554845	9 462714	786	10 537286	10 017559	61	9 982441	49
12	445590	724	554410	463186	785	536814	017596	61	982404	48
13	446025	723	553975	463658	785	536342	017633	61	982367	47
14	446459	723	553541	464129	784	535871	017669	61	982331	46
15	446893	722	553107	464599	783	535401	017706	61	982294	45
16	447326	721	552674	465069	783	534931	017743	61	982257	44
17	447759	720	552241	465539	782	534461	017780	62	982220	43
18	448191	720	551809	466008	781	533992	017817	62	982183	42
19	448623	719	551377	466476	780	533524	017854	62	982146	41
20	449054	718	550946	466945	780	533055	017891	62	982109	40
21	9 449485	717	10 550515	9 467413	779	10 532587	10 017928	62	9 982072	39
22	449915	716	550085	467880	778	532120	017965	62	982035	38
23	450345	716	549655	468347	778	531653	018002	62	981998	37
24	450775	715	549225	468814	777	531186	018039	62	981961	36
25	451204	714	548796	469280	776	530720	018076	62	981924	35
26	451632	713	548368	469746	775	530254	018114	62	981886	34
27	452060	713	547940	470211	775	529789	018151	62	981849	33
28	452488	712	547512	470676	774	529324	018188	62	981812	32
29	452915	711	547085	471141	773	528859	018226	62	981774	31
30	453342	710	546658	471605	773	528395	018263	62	981737	30
31	9 453768	710	10 546232	9 472068	772	10 527932	10 018301	63	9 981699	29
32	454194	709	545806	472532	771	527468	018338	63	981662	28
33	454619	708	545381	472995	771	527005	018375	63	981625	27
34	455044	707	544956	473457	770	526543	018413	63	981587	26
35	455469	707	544531	473919	769	526081	018451	63	981549	25
36	455893	706	544107	474381	769	525619	018488	63	981512	24
37	456316	705	543684	474842	768	525158	018526	63	981474	23
38	456739	704	543261	475303	767	524697	018564	63	981436	22
39	457162	704	542838	475763	767	524237	018601	63	981399	21
40	457584	703	542416	476223	766	523777	018639	63	981361	20
41	9 458006	702	10 541994	9 476683	765	10 523317	10 018677	63	9 981323	19
42	458427	701	541573	477142	765	522858	018715	63	981285	18
43	458848	701	541152	477601	764	522399	018753	63	981247	17
44	459268	700	540732	478059	763	521941	018791	63	981209	16
45	459688	699	540312	478517	763	521483	018829	63	981171	15
46	460108	698	539892	478975	762	521025	018867	64	981133	14
47	460527	698	539473	479432	761	520568	018905	64	981095	13
48	460946	697	539054	479889	761	520111	018943	64	981057	12
49	461364	696	538636	480345	760	519655	018981	64	981019	11
50	461782	695	538218	480801	759	519199	019019	64	980981	10
51	9 462199	695	10 537801	9 481257	759	10 518743	10 019058	64	9 980942	9
52	462616	694	537384	481712	758	518288	019096	64	980904	8
53	463032	693	536968	482167	757	517833	019134	64	980866	7
54	463448	693	536552	482621	757	517379	019173	64	980827	6
55	463864	692	536136	483075	756	516925	019211	64	980789	5
56	464279	691	535721	483529	755	516471	019250	64	980750	4
57	464694	690	535306	483982	755	516018	019288	64	980712	3
58	465108	690	534892	484435	754	515565	019327	64	980673	2
59	465522	689	534478	484887	753	515113	019365	64	980635	1
60	465935	688	534065	485339	753	514661	019404	64	980596	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 465935	688	10.534065	9 485339	755	10.514661	10.019404	64	9 980596	60
1	466348	688	533652	485791	752	514209	019442	64	980558	59
2	466761	687	533239	486242	751	513758	019481	65	980519	58
3	467173	686	532827	486693	751	513307	019520	65	980480	57
4	467585	685	532415	487143	750	512857	019558	65	980442	56
5	467996	685	532004	487593	749	512407	019597	65	980403	55
6	468407	684	531593	488043	749	511957	019636	65	980364	54
7	468817	683	531183	488492	748	511508	019675	65	980325	53
8	469227	683	530773	488941	747	511059	019714	65	980286	52
9	469637	682	530363	489390	747	510610	019753	65	980247	51
10	470046	681	529954	489838	746	510162	019792	65	980208	50
11	9 470455	680	10 529545	9 490286	746	10 509714	10 019831	65	9 980169	49
12	470863	680	529137	490733	745	509267	019870	65	980130	48
13	471271	679	528729	491180	744	508820	019909	65	980091	47
14	471679	678	528321	491627	744	508373	019948	65	980052	46
15	472086	678	527914	492073	743	507927	019988	65	980012	45
16	472492	677	527508	492519	743	507481	020027	65	979973	44
17	472898	676	527102	492965	742	507035	020066	66	979934	43
18	473304	676	526696	493410	741	506590	020105	66	979895	42
19	473710	675	526290	493854	740	506146	020145	66	979855	41
20	474115	674	525885	494299	740	505701	020184	66	979816	40
21	9 474519	674	10 525481	9 494743	740	10 505257	10 020224	66	9 979776	39
22	474923	673	525077	495186	739	504814	020263	66	979737	38
23	475327	672	524673	495630	738	504370	020303	66	979697	37
24	475730	672	524270	496073	737	503927	020342	66	979658	36
25	476133	671	523867	496515	737	503485	020382	66	979618	35
26	476536	670	523464	496957	736	503043	020421	66	979579	34
27	476938	669	523062	497399	736	502601	020461	66	979539	33
28	477340	669	522660	497841	735	502159	020501	66	979499	32
29	477741	668	522259	498282	734	501718	020541	66	979459	31
30	478142	667	521858	498722	734	501278	020580	66	979420	30
31	9 478542	667	10 521458	9 499163	733	10 500837	10 020620	66	9 979380	29
32	478942	666	521058	499603	733	500397	020660	66	979340	28
33	479342	665	520658	500042	732	499958	020700	67	979300	27
34	479741	665	520259	500481	731	499519	020740	67	979260	26
35	480140	664	519860	500920	731	499080	020780	67	979220	25
36	480539	663	519461	501359	730	498641	020820	67	979180	24
37	480937	663	519063	501797	730	498203	020860	67	979140	23
38	481334	662	518666	502235	729	497765	020900	67	979100	22
39	481731	661	518269	502672	728	497328	020941	67	979059	21
40	482128	661	517872	503109	728	496891	020981	67	979019	20
41	9.482525	660	10 517475	9 503546	727	10 496454	10 021021	67	9 978979	19
42	482921	659	517079	503982	727	496018	021061	67	978939	18
43	483316	659	516684	504418	726	495582	021102	67	978898	17
44	483712	558	516288	504854	725	495146	021142	67	978858	16
45	484107	657	515893	505289	725	494711	021183	67	978817	15
46	484501	657	515499	505724	724	494276	021223	67	978777	14
47	484895	656	515105	506159	724	493841	021264	67	978736	13
48	485289	655	514711	506593	723	493407	021304	68	978696	12
49	485682	655	514318	507027	722	492973	021345	68	978655	11
50	486075	654	513925	507460	722	492540	021385	68	978615	10
51	9 486467	653	10 513533	9 507893	721	10 492107	10 021426	68	9 978574	9
52	486860	653	513140	508326	721	491674	021467	68	978533	8
53	487251	652	512749	508759	720	491241	021507	68	978493	7
54	487643	651	512357	509191	719	490809	021548	68	978452	6
55	488034	651	511966	509622	719	490378	021589	68	978411	5
56	488424	650	511576	510054	718	489946	021630	68	978370	4
57	488814	650	511186	510485	718	489515	021671	68	978329	3
58	489204	649	510796	510916	717	489084	021712	68	978288	2
59	489593	648	510407	511346	716	488654	021753	68	978247	1
60	489982	648	510018	511776	716	488224	021794	68	978206	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 489982	648	10 510018	9 511776	716	10.488224	10 021794	68	9 978206	60
1	490371	648	509629	512206	716	487794	021835	68	978165	59
2	490759	647	509241	512635	715	487365	021876	68	978124	58
3	491147	646	508853	513064	714	486936	021917	69	978083	57
4	491535	646	508465	513493	714	486507	021958	69	978042	56
5	491922	645	508078	513921	713	486079	021999	69	978001	55
6	492308	644	507692	514349	713	485651	022041	69	977959	54
7	492695	644	507305	514777	712	485223	022082	69	977918	53
8	493081	643	506919	515204	712	484796	022123	69	977877	52
9	493466	642	506534	515631	711	484369	022165	69	977835	51
10	493851	642	506149	516057	710	483943	022206	69	977794	50
11	9 494236	641	10 505764	9.516484	710	10.483516	10 022248	69	9 977752	49
12	494621	641	505379	516910	709	483090	022289	69	977711	48
13	495005	640	504995	517335	709	482665	022331	69	977669	47
14	495388	639	504612	517761	708	482239	022372	69	977628	46
15	495772	639	504228	518185	708	481815	022414	69	977586	45
16	496154	638	503846	518610	707	481390	022456	70	977544	44
17	496537	637	503463	519034	706	480966	022497	70	977503	43
18	496919	637	503081	519458	706	480542	022539	70	977461	42
19	497301	636	502699	519882	705	480118	022581	70	977419	41
20	497682	636	502318	520305	705	479695	022623	70	977377	40
21	9 498064	635	10 501936	9.520728	704	10 479272	10 022665	70	9 977335	39
22	498444	634	501556	521151	703	478849	022707	70	977293	38
23	498825	634	501175	521573	703	478427	022749	70	977251	37
24	499204	633	500796	521995	703	478005	022791	70	977209	36
25	499584	632	500416	522417	702	477583	022833	70	977167	35
26	499963	632	500037	522838	702	477162	022875	70	977125	34
27	500342	631	499658	523259	701	476741	022917	70	977083	33
28	500721	631	499279	523680	701	476320	022959	70	977041	32
29	501099	630	498901	524100	700	475900	023001	70	976999	31
30	501476	629	498524	524520	699	475480	023043	70	976957	30
31	9 501854	629	10.498146	9 524939	699	10 475061	10.023086	70	9 976914	29
32	502231	628	497769	525359	698	474641	023128	71	976872	28
33	502607	628	497393	525778	698	474222	023170	71	976830	27
34	502984	627	497016	526197	697	473803	023213	71	976787	26
35	503360	626	496640	526615	697	473385	023255	71	976745	25
36	503735	626	496265	527033	696	472967	023298	71	976702	24
37	504110	625	495890	527451	696	472549	023340	71	976660	23
38	504485	625	495515	527868	695	472132	023383	71	976617	22
39	504860	624	495140	528285	695	471715	023426	71	976574	21
40	505234	623	494766	528702	694	471298	023468	71	976532	20
41	9 505608	623	10 494392	9 529119	693	10 470881	10 023511	71	9 976489	19
42	505981	622	494019	529535	693	470465	023554	71	976446	18
43	506354	622	493646	529950	693	470050	023596	71	976404	17
44	506727	621	493273	530366	692	469634	023639	71	976361	16
45	507099	620	492901	530781	691	469219	023682	71	976318	15
46	507471	620	492529	531196	691	468804	023725	71	976275	14
47	507843	619	492157	531611	690	468389	023768	72	976232	13
48	508214	619	491786	532025	690	467975	023811	72	976189	12
49	508585	618	491415	532439	689	467561	023854	72	976146	11
50	508956	618	491044	532853	689	467147	023897	72	976103	10
51	9 509326	617	10 490674	9 533266	688	10.466734	10.023940	72	9 976060	9
52	509696	616	490304	533679	688	466321	023983	72	976017	8
53	510065	616	489935	534092	687	465908	024026	72	975974	7
54	510434	615	489566	534504	687	465496	024070	72	975930	6
55	510803	615	489197	534916	686	465084	024113	72	975887	5
56	511172	614	488828	535328	686	464672	024156	72	975844	4
57	511540	613	488460	535739	685	464261	024200	72	975800	3
58	511907	613	488093	536150	685	463850	024243	72	975757	2
59	512275	612	487725	536561	684	463439	024286	72	975714	1
60	512642	612	487358	536972	684	463028	024330	72	975670	0
	Cosine		Secant	Cotang		Tang	Cosec.		Sine	M

71 Degrees.

M.	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9.512642	612	10.487358	9.536972	684	10.463028	10.024330	73	9.975670	60
1	513009	611	486991	537382	683	462618	024373	73	975627	59
2	513375	611	486625	537792	683	462208	024417	73	975583	58
3	513741	610	486259	538202	682	461798	024461	73	975539	57
4	514107	609	485893	538611	682	461389	024504	73	975496	56
5	514472	609	485528	539020	681	460980	024548	73	975452	55
6	514837	608	485163	539429	681	460571	024592	73	975408	54
7	515202	608	484798	539837	680	460163	024635	73	975365	53
8	515566	607	484434	540245	680	459755	024679	73	975321	52
9	515930	607	484070	540653	679	459347	024723	73	975277	51
10	516294	606	483706	541061	679	458939	024767	73	975233	50
11	9.516657	605	10.483343	9.541468	678	10.458532	10.024811	73	9.975189	49
12	517020	605	482980	541875	678	458125	024855	73	975145	48
13	517382	604	482618	542281	677	457719	024899	73	975101	47
14	517745	604	482255	542688	677	457312	024943	73	975057	46
15	518107	603	481893	543094	676	456906	024987	73	975013	45
16	518468	603	481532	543499	676	456501	025031	74	974969	44
17	518829	602	481171	543905	675	456095	025075	74	974925	43
18	519190	601	480810	544310	675	455690	025120	74	974880	42
19	519551	601	480449	544715	674	455285	025164	74	974836	41
20	519911	600	480089	545119	674	454881	025208	74	974792	40
21	9.520271	600	10.479729	9.545524	673	10.454476	10.025252	74	9.974748	39
22	520631	599	479369	545928	673	454072	025297	74	974703	38
23	520990	599	479010	546331	672	453669	025341	74	974659	37
24	521349	598	478651	546735	672	453265	025386	74	974614	36
25	521707	598	478293	547138	671	452862	025430	74	974570	35
26	522066	597	477934	547540	671	452460	025475	74	974525	34
27	522424	596	477576	547943	670	452057	025519	74	974481	33
28	522781	596	477219	548345	670	451655	025564	74	974436	32
29	523138	595	476862	548747	669	451253	025609	74	974391	31
30	523495	595	476505	549149	669	450851	025653	75	974347	30
31	9.523852	594	10.476148	9.549550	668	10.450450	10.025698	75	9.974302	29
32	524208	594	475792	549951	668	450049	025743	75	974257	28
33	524564	593	475436	550352	667	449648	025788	75	974212	27
34	524920	593	475080	550752	667	449248	025833	75	974167	26
35	525275	592	474725	551152	666	448848	025878	75	974122	25
36	525630	591	474370	551552	666	448448	025923	75	974077	24
37	525984	591	474016	551952	665	448048	025968	75	974032	23
38	526339	590	473661	552351	665	447649	026013	75	973987	22
39	526693	590	473307	552750	665	447250	026058	75	973942	21
40	527046	589	472954	553149	664	446851	026103	75	973897	20
41	9.527400	589	10.472600	9.553548	664	10.446452	10.026148	75	9.973852	19
42	527753	588	472247	553946	663	446054	026193	75	973807	18
43	528105	588	471895	554344	663	445656	026239	75	973761	17
44	528458	587	471542	554741	662	445259	026284	76	973716	16
45	528810	587	471190	555139	562	444861	026329	76	973671	15
46	529161	586	470839	555536	661	444464	026375	76	973625	14
47	529513	586	470487	555933	661	444067	026420	76	973580	13
48	529864	585	470136	556329	660	443671	026465	76	973535	12
49	530215	585	469785	556725	660	443275	026511	76	973489	11
50	530565	584	469435	557121	659	442879	026556	76	973444	10
51	9.530915	584	10.469085	9.557517	659	10.442483	10.026602	76	9.973398	9
52	531265	583	468735	557913	659	442087	026648	76	973352	8
53	531614	582	468386	558308	658	441692	026693	76	973307	7
54	531963	582	468037	558702	658	441298	026739	76	973261	6
55	532312	581	467688	559097	657	440903	026785	76	973215	5
56	532661	581	467339	559491	657	440509	026831	76	973169	4
57	533009	580	466991	559885	656	440115	026876	76	973124	3
58	533357	580	466643	560279	656	439721	026922	76	973078	2
59	533704	579	466296	560673	655	439327	026968	77	973032	1
60	534052	578	465948	561066	655	438934	027014	77	972986	0
	Cosine		Secant	Cotang		Tang	Cosec.		Sine	M

70 Degrees.

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 534052	578	10.465948	9 561066	655	10 438934	10 027014	77	9 972986	60
1	534399	577	465601	561459	654	438541	027060	77	972940	59
2	534745	577	465255	561851	654	438149	027106	77	972894	58
3	535092	577	464908	562244	653	437756	027152	77	972848	57
4	535438	576	464562	562636	653	437364	027198	77	972802	56
5	535783	576	464217	563028	653	436972	027245	77	972755	55
6	536129	575	463871	563419	652	436581	027291	77	972709	54
7	536474	574	463526	563811	652	436189	027337	77	972663	53
8	536818	574	463182	564202	651	435798	027383	77	972617	52
9	537163	573	462837	564592	651	435408	027430	77	972570	51
10	537507	573	462493	564983	650	435017	027476	77	972524	50
11	9 537851	572	10 462149	9 565373	650	10 434627	10 027522	77	9 972478	49
12	538194	572	461806	565763	649	434237	027569	78	972431	48
13	538538	571	461462	566153	649	433847	027615	78	972385	47
14	538880	571	461120	566542	649	433458	027662	78	972338	46
15	539223	570	460777	566932	648	433068	027709	78	972291	45
16	539565	570	460435	567320	648	432680	027755	78	972245	44
17	539907	569	460093	567709	647	432291	027802	78	972198	43
18	540249	569	459751	568098	647	431902	027849	78	972151	42
19	540590	568	459410	568486	646	431514	027895	78	972105	41
20	540931	568	459069	568873	646	431127	027942	78	972058	40
21	9 541272	567	10 458728	9 569261	645	10 430739	10 027989	78	9 972011	39
22	541613	567	458387	569648	645	430352	028036	78	971964	38
23	541953	566	458047	570035	645	429965	028083	78	971917	37
24	542293	566	457707	570422	644	429578	028130	78	971870	36
25	542632	565	457368	570809	644	429191	028177	78	971823	35
26	542971	565	457029	571195	643	428805	028224	78	971776	34
27	543310	564	456690	571581	643	428419	028271	79	971729	33
28	543649	564	456351	571967	642	428033	028318	79	971682	32
29	543987	563	456013	572352	642	427648	028365	79	971635	31
30	544325	563	455675	572738	642	427262	028412	79	971588	30
31	9 544663	562	10 455337	9 573123	641	10 426877	10 028460	79	9 971540	29
32	545000	562	455000	573507	641	426493	028507	79	971493	28
33	545338	561	454662	573892	640	426108	028554	79	971446	27
34	545674	561	454326	574276	640	425724	028602	79	971398	26
35	546011	560	453989	574660	639	425340	028649	79	971351	25
36	546347	560	453653	575044	639	424956	028697	79	971303	24
37	546683	559	453317	575427	639	424573	028744	79	971256	23
38	547019	559	452981	575810	638	424190	028792	79	971208	22
39	547354	558	452646	576193	638	423807	028839	79	971161	21
40	547689	558	452311	576576	637	423424	028887	79	971113	20
41	9 548024	557	10 451976	9 576958	637	10.423042	10.028934	80	9 971066	19
42	548359	557	451641	577341	636	422659	028982	80	971018	18
43	548693	556	451307	577723	636	422277	029030	80	970970	17
44	549027	556	450973	578104	636	421896	029078	80	970922	16
45	549360	555	450640	578486	635	421514	029126	80	970874	15
46	549693	555	450307	578867	635	421133	029173	80	970827	14
47	550026	554	449974	579248	634	420752	029221	80	970779	13
48	550359	554	449641	579629	634	420371	029269	80	970731	12
49	550692	553	449308	580009	634	419991	029317	80	970683	11
50	551024	553	448976	580389	633	419611	029365	80	970635	10
51	9 551356	552	10 448644	9 580769	633	10.419231	10 029414	80	9 970586	9
52	551687	552	448313	581149	632	418851	029462	80	970538	8
53	552018	552	447982	581528	632	418472	029510	80	970490	7
54	552349	551	447651	581907	632	418093	029558	80	970442	6
55	552680	551	447320	582286	631	417714	029606	80	970394	5
56	553010	550	446990	582665	631	417335	029655	81	970345	4
57	553341	550	446659	583043	630	416957	029703	81	970297	3
58	553670	549	446330	583422	630	416578	029751	81	970249	2
59	554000	549	446000	583800	629	416200	029800	81	970200	1
60	554329	548	445671	584177	629	415823	029848	81	970152	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 554329	548	10 445671	9 584177	629	10 415823	10 029848	81	9 970152	60
1	554658	548	445342	584555	629	415445	029897	81	970103	59
2	554987	547	445013	584932	628	415068	029945	81	970055	58
3	555315	547	444685	585309	628	414691	029994	81	970006	57
4	555643	546	444357	585686	627	414314	030043	81	969957	56
5	555971	546	444029	586062	627	413938	030091	81	969909	55
6	556299	545	443701	586439	627	413561	030140	81	969860	54
7	556626	545	443374	586815	626	413185	030189	81	969811	53
8	556953	544	443047	587190	626	412810	030238	81	969762	52
9	557280	544	442720	587566	625	412434	030286	81	969714	51
10	557606	543	442394	587941	625	412059	030335	81	969665	50
11	9 557932	543	10 442068	9 588316	625	10 411684	10 030384	82	9 969616	49
12	558258	543	441742	588691	624	411309	030433	82	969567	48
13	558583	542	441417	589066	624	410934	030482	82	969518	47
14	558909	542	441091	589440	623	410560	030531	82	969469	46
15	559234	541	440766	589814	623	410186	030580	82	969420	45
16	559558	541	440442	590188	623	409812	030630	82	969370	44
17	559883	540	440117	590562	622	409438	030679	82	969321	43
18	560207	540	439793	590935	622	409065	030728	82	969272	42
19	560531	539	439469	591308	622	408692	030777	82	969223	41
20	560855	539	439145	591681	621	408319	030827	82	969173	40
21	9 561178	538	10 438822	9 592054	621	10 407946	10 030876	82	9 969124	39
22	561501	538	438499	592426	620	407574	030925	82	969075	38
23	561824	537	438176	592798	620	407202	030975	82	969025	37
24	562146	537	437854	593170	619	406829	031024	82	968976	36
25	562468	536	437532	593542	619	406458	031074	83	968926	35
26	562790	536	437210	593914	618	406086	031123	83	968877	34
27	563112	536	436888	594285	618	405715	031173	83	968827	33
28	563433	535	436567	594656	618	405344	031223	83	968777	32
29	563755	535	436245	595027	617	404973	031272	83	968728	31
30	564075	534	435925	595398	617	404602	031322	83	968678	30
31	9 564396	534	10 435604	9 595768	617	10 404232	10 031372	83	9 968628	29
32	564716	533	435284	596138	616	403862	031422	83	968578	28
33	565036	533	434964	596508	616	403492	031472	83	968528	27
34	565356	532	434644	596878	616	403122	031521	83	968479	26
35	565676	532	434324	597247	615	402753	031571	83	968429	25
36	565995	531	434005	597616	615	402384	031621	83	968379	24
37	566314	531	433686	597985	615	402015	031671	83	968329	23
38	566632	531	433368	598354	614	401646	031722	83	968278	22
39	566951	530	433049	598722	614	401278	031772	84	968228	21
40	567269	530	432731	599091	613	400909	031822	84	968178	20
41	9 567587	529	10 432413	9 599459	613	10.400541	10 031872	84	9 968128	19
42	567904	529	432096	599827	613	400173	031922	84	968078	18
43	568222	528	431778	600194	612	399806	031973	84	968027	17
44	568539	528	431461	600562	612	399438	032023	84	967977	16
45	568856	528	431144	600929	611	399071	032073	84	967927	15
46	569172	527	430828	601296	611	398704	032124	84	967876	14
47	569488	527	430512	601662	611	398338	032174	84	967826	13
48	569804	526	430196	602029	610	397971	032225	84	967775	12
49	570120	526	429880	602395	610	397605	032275	84	967725	11
50	570435	525	429565	602761	610	397239	032326	84	967674	10
51	9 570751	525	10 429249	9 603127	609	10 396873	10.032376	84	9 967624	9
52	571066	524	428934	603493	609	396507	032427	84	967573	8
53	571380	524	428620	603858	609	396142	032478	85	967522	7
54	571695	523	428305	604223	608	395777	032529	85	967471	6
55	572009	523	427991	604588	608	395412	032579	85	967421	5
56	572323	523	427677	604953	607	395047	032630	85	967370	4
57	572636	522	427364	605317	607	394683	032681	85	967319	3
58	572950	522	427050	605682	607	394318	032732	85	967268	2
59	573263	521	426737	606046	606	393954	032783	85	967217	1
60	573575	521	426425	606410	606	393590	032834	85	967166	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec.	Tang	D	Cotang	Secant	D	Cosine	
0	9.573575	521	10.426425	9.606410	606	10.393590	10.032834	85	9.967166	60
1	573888	520	426112	606773	606	393227	032885	85	967115	59
2	574200	520	425800	607137	605	392863	032936	85	967064	58
3	574512	519	425488	607500	605	392500	032987	85	967013	57
4	574824	519	425176	607863	604	392137	033039	85	966961	56
5	575136	519	424864	608225	604	391775	033090	85	966910	55
6	575447	518	424553	608588	604	391412	033141	85	966859	54
7	575758	518	424242	608950	603	391050	033192	85	966808	53
8	576069	517	423931	609312	603	390688	033244	86	966756	52
9	576379	517	423621	609674	603	390326	033295	86	966705	51
10	576689	516	423311	610036	602	389964	033347	86	966653	50
11	9.576999	516	10.423001	9.610397	602	10.389603	10.033398	86	9.966602	49
12	577309	516	422691	610759	602	389241	033450	86	966550	48
13	577618	515	422382	611120	601	388880	033501	86	966499	47
14	577927	515	422073	611480	601	388520	033553	86	966447	46
15	578236	514	421764	611841	601	388159	033605	86	966395	45
16	578515	514	421455	612201	600	387799	033656	86	966344	44
17	578853	513	421147	612561	600	387439	033708	86	966292	43
18	579162	513	420838	612921	600	387079	033760	86	966240	42
19	579470	513	420530	613281	599	386719	033812	86	966188	41
20	579777	512	420223	613641	599	386359	033864	86	966136	40
21	9.580085	512	10.419915	9.614000	598	10.386000	10.033915	87	9.966085	39
22	580392	511	419608	614359	598	385641	033967	87	966033	38
23	580699	511	419301	614718	598	385282	034019	87	965961	37
24	581005	511	418995	615077	597	384923	034072	87	965928	36
25	581312	510	418688	615435	597	384565	034124	87	965876	35
26	581618	510	418382	615793	597	384207	034176	87	965824	34
27	581924	509	418076	616151	596	383849	034228	87	965772	33
28	582229	509	417771	616509	596	383491	034280	87	965720	32
29	582535	509	417165	616867	596	383133	034332	87	965668	31
30	582840	508	417160	617224	595	382776	034385	87	965615	30
31	9.583145	508	10.416855	9.617782	595	10.382118	10.034137	87	9.965563	29
32	583449	507	416551	617939	595	382061	034489	87	965511	28
33	583754	507	416246	618295	594	381705	034542	87	965458	27
34	584058	506	415942	618652	594	381348	031591	87	965406	26
35	584361	506	415639	619008	594	380992	034647	88	965353	25
36	581665	506	415335	619364	593	380636	034699	88	965301	24
37	584968	505	415032	619721	593	380279	034752	88	965248	23
38	585272	505	414728	620076	593	379924	034805	88	965195	22
39	585574	504	414426	620132	592	379568	034857	88	965143	21
40	585877	501	414123	620787	592	379213	034910	88	965090	20
41	9.586179	503	10.413821	9.621142	592	10.378858	10.034963	88	9.965037	19
42	586482	503	413518	621497	591	378503	035016	88	964984	18
43	586783	503	413217	621852	591	378148	035069	88	964931	17
44	587085	502	412915	622207	590	377793	035121	88	964879	16
45	587386	502	412614	622561	590	377439	035174	88	964826	15
46	587688	501	412312	622915	590	377085	035227	88	964773	14
47	587989	501	412011	623269	589	376731	035281	88	964719	13
48	588289	501	411711	623623	589	376377	035334	89	964666	12
49	588590	500	411410	623976	589	376024	035386	89	964613	11
50	588890	500	411110	624330	588	375670	035440	89	964560	10
51	9.589190	499	10.410810	9.624683	588	10.375317	10.035493	89	9.964507	9
52	589489	499	410511	625036	588	374964	035546	89	964454	8
53	589789	499	410211	625388	587	374612	035600	89	964400	7
54	590088	498	409912	625741	587	374259	035653	89	964347	6
55	590387	498	409613	626093	587	373907	035706	89	964294	5
56	590686	497	409314	626445	586	373555	035760	89	964240	4
57	590984	497	409016	626797	586	373203	035813	89	964187	3
58	591282	497	408718	627149	586	372851	035867	89	964133	2
59	591580	496	408420	627501	585	372499	035920	89	964080	1
60	591878	196	408122	627852	585	372148	035974	89	964026	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec.	Tang.	D	Cotang	Secant	D	Cosine	
0	9 591878	496	10 408122	9 627852	585	10.372148	10.035974	89	9 964026	60
1	592176	495	407824	628203	585	371797	036028	89	963972	59
2	592473	495	407527	628554	585	371446	036081	89	963919	58
3	592770	495	407230	628905	584	371095	036135	90	963865	57
4	593067	494	406933	629255	584	370745	036189	90	963811	56
5	593363	494	406637	629606	583	370394	036243	90	963757	55
6	593659	493	406341	629956	583	370044	036296	90	963704	54
7	593955	493	406045	630306	583	369694	036350	90	963650	53
8	594251	493	405749	630656	583	369344	036404	90	963596	52
9	594547	492	405453	631005	582	368995	036458	90	963542	51
10	594842	492	405158	631355	582	368645	036512	90	963488	50
11	9 595137	491	10.404863	9 631704	582	10.368296	10.036566	90	9 963434	49
12	595432	491	404568	632053	581	367947	036621	90	963379	48
13	595727	491	404273	632401	581	367599	036675	90	963325	47
14	596021	490	403979	632750	581	367250	036729	90	963271	46
15	596315	490	403685	633098	580	366902	036783	90	963217	45
16	596609	489	403391	633447	580	366553	036837	90	963163	44
17	596903	489	403097	633795	580	366205	036892	91	963108	43
18	597196	489	402804	634143	579	365857	036946	91	963054	42
19	597490	488	402510	634490	579	365510	037001	91	962999	41
20	597783	488	402217	634838	579	365162	037055	91	962945	40
21	9.598075	487	10 401925	9 635185	578	10.364815	10.037110	91	9 962890	39
22	598368	487	401632	635532	578	364468	037164	91	962836	38
23	598660	487	401340	635879	578	364121	037219	91	962781	37
24	598952	486	401048	636226	577	363774	037273	91	962727	36
25	599244	486	400756	636572	577	363428	037328	91	962672	35
26	599536	485	400464	636919	577	363081	037383	91	962617	34
27	599827	485	400173	637265	577	362735	037438	91	962562	33
28	600118	485	399882	637611	576	362389	037492	91	962508	32
29	600409	484	399591	637956	576	362044	037547	91	962453	31
30	600700	484	399300	638302	576	361698	037602	92	962398	30
31	9 600990	484	10 399010	9 638647	575	10 361353	10 037657	92	9.962343	29
32	601280	483	398720	638992	575	361008	037712	92	962288	28
33	601570	483	398430	639337	575	360663	037767	92	962233	27
34	601860	482	398140	639682	574	360318	037822	92	962178	26
35	602150	482	397850	640027	574	359973	037877	92	962123	25
36	602439	482	397561	640371	574	359629	037933	92	962067	24
37	602728	481	397272	640716	573	359284	037988	92	962012	23
38	603017	481	396983	641060	573	358940	038043	92	961957	22
39	603305	481	396695	641404	573	358596	038098	92	961902	21
40	603594	480	396406	641747	572	358253	038154	92	961846	20
41	9 603882	480	10.396118	9 642091	572	10 357909	10.038209	92	9 961791	19
42	604170	479	395830	642434	572	357566	038265	92	961735	18
43	604457	479	395543	642777	572	357223	038320	92	961680	17
44	604745	479	395255	643120	571	356880	038376	93	961624	16
45	605032	478	394968	643463	571	856537	038431	93	961569	15
46	605319	478	394681	643806	571	356194	038487	93	961513	14
47	605606	478	394394	644148	570	355852	038542	93	961458	13
48	605892	477	394108	644490	570	355510	038598	93	961402	12
49	606179	477	393821	644832	570	355168	038654	93	961346	11
50	606465	476	393535	645174	569	354826	038710	93	961290	10
51	9 606751	476	10 393249	9 645516	569	10 354484	10 038765	93	9 961235	9
52	607036	476	392964	645857	569	354143	038821	93	961179	8
53	607322	475	392678	646199	569	353801	038877	93	961123	7
54	607607	475	392393	646540	568	353460	038933	93	961067	6
55	607892	474	392108	546881	568	353119	038989	93	961011	5
56	608177	474	391823	647222	568	352778	039045	93	960955	4
57	608461	474	391539	647562	567	352438	039101	93	960899	3
58	608745	473	391255	647903	567	352097	039157	94	960843	2
59	609029	473	390971	648243	567	351757	039214	94	960786	1
60	609313	473	390687	648583	566	351417	039270	94	960730	0
	Cosine		Secant	Cotang.		Tang	Cosec		Sine	M.

M	Sine	D	Cosec	Tang	D	Cotang.	Secant	D	Cosine	
0	9.609313	473	10.390687	9.648583	566	10.351417	10.039270	94	9.960730	60
1	609597	472	390403	648923	566	351077	039326	94	960674	59
2	609880	472	390120	649263	566	350737	039382	94	960618	58
3	610164	472	389836	649602	566	350398	039439	94	960561	57
4	610447	471	389553	649942	565	350058	039495	94	960505	56
5	610729	471	389271	650281	565	349719	039552	94	960448	55
6	611012	470	388988	650620	565	349380	039608	94	960392	54
7	611294	470	388706	650959	564	349041	039665	94	960335	53
8	611576	470	388424	651297	564	348703	039721	94	960279	52
9	611858	469	388142	651636	564	348364	039778	94	960222	51
10	612140	469	387860	651971	563	348026	039835	94	960165	50
11	9.612421	469	10.387579	9.652312	563	10.347688	10.039891	95	9.960109	49
12	612702	468	387298	652650	563	347350	039948	95	960052	48
13	612983	468	387017	652988	563	347012	040005	95	959995	47
14	613264	467	386736	653326	562	346674	040062	95	959938	46
15	613545	467	386455	653663	562	346337	040118	95	959882	45
16	613825	467	386175	654000	562	346000	040175	95	959825	44
17	614105	466	385895	654337	561	345663	040232	95	959768	43
18	614385	466	385615	654674	561	345326	040289	95	959711	42
19	614665	466	385335	655011	561	344989	040346	95	959654	41
20	614941	465	385056	655348	561	344652	040404	95	959596	40
21	9.615223	465	10.384777	9.655684	560	10.344316	10.040461	95	9.959539	39
22	615502	465	384498	656020	560	343980	040518	95	959182	38
23	615781	464	384219	656356	560	343641	040575	95	959425	37
24	616060	464	383940	656692	559	343308	040632	95	959368	36
25	616338	464	383662	657028	559	342972	040690	96	959310	35
26	616616	463	383381	657364	559	342636	040747	96	959253	34
27	616894	463	383106	657699	559	342301	040805	96	959195	33
28	617172	462	382828	658031	558	341966	040862	96	959138	32
29	617450	462	382550	658369	558	341631	040919	96	959081	31
30	617727	462	382273	658704	558	341296	040977	96	959023	30
31	9.618004	461	10.381996	9.659039	558	10.340961	10.041035	96	9.958965	29
32	618281	461	381719	659373	557	340627	041092	96	958908	28
33	618558	461	381442	659708	557	340292	041150	96	958850	27
34	618831	460	381166	660012	557	339958	041208	96	958792	26
35	619110	460	380890	660376	557	339624	041266	96	958734	25
36	619386	460	380614	660710	556	339290	041323	96	958677	24
37	619662	459	380338	661043	556	338957	041381	96	958619	23
38	619938	459	380062	661377	556	338623	041439	96	958561	22
39	620213	459	379787	661710	555	338290	041497	97	958503	21
40	620488	458	379512	662043	555	337957	041555	97	958145	20
41	9.620763	458	10.379237	9.662376	555	10.337624	10.041613	97	9.958387	19
42	621038	457	378962	662709	554	337291	041671	97	958329	18
43	621313	457	378687	663042	554	336958	041729	97	958271	17
44	621587	457	378413	663375	554	336625	041787	97	958213	16
45	621861	456	378139	663707	554	336293	041846	97	958154	15
46	622135	456	377865	664039	553	335961	041904	97	958096	14
47	622409	456	377591	664371	553	335629	041962	97	958038	13
48	622682	455	377318	664703	553	335297	042021	97	957979	12
49	622956	455	377044	665035	553	334965	042079	97	957921	11
50	623229	455	376771	665366	552	334634	042137	97	957863	10
51	9.623502	454	10.376498	9.665697	552	10.334303	10.042196	97	9.957804	9
52	623774	454	376226	666029	552	333971	042254	98	957746	8
53	624047	454	375953	666360	551	333640	042313	98	957687	7
54	624319	453	375681	666691	551	333309	042372	98	957628	6
55	624591	453	375409	667021	551	332979	042430	98	957570	5
56	624863	453	375137	667352	551	332648	042489	98	957511	4
57	625135	452	374865	667682	550	332318	042548	98	957452	3
58	625406	452	374594	668013	550	331987	042607	98	957393	2
59	625677	452	374323	668343	550	331657	042665	98	957335	1
60	625948	451	374052	668672	550	331328	042724	98	957276	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 625948	451	10.374052	9 668673	550	10 331327	10 042724	98	9 957276	60
1	626219	451	373781	669002	549	330998	042783	98	957217	59
2	626490	451	373510	669332	549	330668	042842	98	957158	58
3	626760	450	373240	669661	549	330339	042901	98	957099	57
4	627030	450	372970	669991	548	330009	042960	98	957040	56
5	627300	450	372700	670320	548	329680	043019	98	956981	55
6	627570	449	372430	670649	548	329351	043079	99	956921	54
7	627840	449	372160	670977	548	329023	043138	99	956862	53
8	628109	449	371891	671306	547	328694	043197	99	956803	52
9	628378	448	371622	671634	547	328366	043256	99	956744	51
10	628647	448	371353	671963	547	328037	043316	99	956684	50
11	9 628916	447	10 371084	9 672291	547	10 327709	10 043375	99	9 956625	49
12	629185	447	370815	672619	546	327381	043434	99	956566	48
13	629453	447	370547	672947	546	327053	043494	99	956506	47
14	629721	446	370279	673274	546	326726	043553	99	956447	46
15	629989	446	370011	673602	546	326398	043613	99	956387	45
16	630257	446	369743	673929	545	326071	043673	99	956327	44
17	630524	446	369476	674257	545	325743	043732	99	956268	43
18	630792	445	369208	674584	545	325416	043792	100	956208	42
19	631059	445	368941	674910	544	325090	043852	100	956148	41
20	631326	445	368674	675237	544	324763	043911	100	956089	40
21	9 631593	444	10 368407	9 675564	544	10 324436	10 043971	100	9 956029	39
22	631859	444	368141	675890	544	324110	044031	100	955969	38
23	632125	444	367875	676216	543	323784	044091	100	955909	37
24	632392	443	367608	676543	543	323457	044151	100	955849	36
25	632658	443	367342	676869	543	323131	044211	100	955789	35
26	632923	443	367077	677194	543	322806	044271	100	955729	34
27	633189	442	366811	677520	542	322480	044331	100	955669	33
28	633454	442	366546	677846	542	322154	044391	100	955609	32
29	633719	442	366281	678171	542	321829	044452	100	955548	31
30	633984	441	366016	678496	542	321504	044512	100	955488	30
31	9 634249	441	10.365751	9 678821	541	10 321179	10 044572	101	9 955428	29
32	634514	440	365486	679146	541	320854	044632	101	955368	28
33	634778	440	365222	679471	541	320529	044693	101	955307	27
34	635042	440	364958	679795	541	320205	044753	101	955247	26
35	635306	439	364694	680120	540	319880	044814	101	955186	25
36	635570	439	364430	680444	540	319556	044874	101	955126	24
37	635834	439	364166	680768	540	319232	044935	101	955065	23
38	636097	438	363903	681092	540	318908	044995	101	955005	22
39	636360	438	363640	681416	539	318584	045056	101	954944	21
40	636623	438	363377	681740	539	318260	045117	101	954883	20
41	9 636886	437	10.363114	9 682063	539	10.317937	10 045177	101	9 954823	19
42	637148	437	362852	682387	539	317613	045238	101	954762	18
43	637411	437	362589	682710	538	317290	045299	101	954701	17
44	637673	437	362327	683033	538	316967	045360	101	954640	16
45	637935	436	362065	683356	538	316644	045421	101	954579	15
46	638197	436	361803	683679	538	316321	045482	102	954518	14
47	638458	436	361542	684001	537	315999	045543	102	954457	13
48	638720	435	361280	684324	537	315676	045604	102	954396	12
49	638981	435	361019	684646	537	315354	045665	102	954335	11
50	639242	435	360758	684968	537	315032	045726	102	954274	10
51	9 639503	434	10 360497	9 685290	536	10 314710	10 045787	102	9 954213	9
52	639764	434	360236	685612	536	314388	045848	102	954152	8
53	640024	434	359976	685934	536	314066	045910	102	954090	7
54	640284	433	359716	686255	536	313745	045971	102	954029	6
55	640544	433	359456	686577	535	313423	046032	102	953968	5
56	640804	433	359196	686898	535	313102	046094	102	953906	4
57	641064	432	358936	687219	535	312781	046155	102	953845	3
58	641324	432	358677	687540	535	312460	046217	102	953783	2
59	641584	432	358416	687861	534	312139	046278	103	953722	1
60	641842	431	358158	688182	534	311818	046340	103	953660	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang.	D	Cotang.	Secant	D	Cosine	
0	9.641842	431	10.358158	9.688182	534	10.311818	10.046340	103	9.953660	60
1	642101	431	357899	688502	534	311498	046401	103	953599	59
2	642360	431	357640	688823	534	311177	046463	103	953537	58
3	642618	430	357382	689143	533	310857	046525	103	953475	57
4	642877	430	357123	689463	533	310537	046587	103	953413	56
5	643135	430	356865	689783	533	310217	046648	103	953352	55
6	643393	430	356607	690103	533	309897	046710	103	953290	54
7	643650	429	356350	690423	533	309577	046772	103	953228	53
8	643908	429	356092	690742	532	309258	046834	103	953166	52
9	644165	429	355835	691062	532	308938	046896	103	953104	51
10	644423	428	355577	691381	532	308619	046958	103	953042	50
11	9.644680	428	10.355320	9.691700	531	10.308300	10.047020	104	9.952980	49
12	644936	428	355064	692019	531	307981	047082	104	952918	48
13	645193	427	354807	692338	531	307662	047145	104	952855	47
14	645450	427	354550	692656	531	307344	047207	104	952793	46
15	645706	427	354294	692975	531	307025	047269	104	952731	45
16	645962	426	354038	693293	530	306707	047331	104	952669	44
17	646218	426	353782	693612	530	306388	047394	104	952606	43
18	646474	426	353526	693930	530	306070	047456	104	952544	42
19	646729	425	353271	694248	530	305752	047519	104	952481	41
20	646984	425	353016	694566	529	305434	047581	104	952419	40
21	9.647240	425	10.352760	9.694883	529	10.305117	10.047644	104	9.952356	39
22	647494	424	352506	695201	529	304799	047706	104	952294	38
23	647749	424	352251	695518	529	304482	047769	104	952231	37
24	648004	424	351996	695836	529	304164	047832	105	952168	36
25	648258	424	351742	696153	528	303847	047894	105	952106	35
26	648512	423	351488	696470	528	303530	047957	105	952043	34
27	648766	423	351234	696787	528	303213	048020	105	951980	33
28	649020	423	350980	697103	528	302897	048083	105	951917	32
29	649274	422	350726	697420	527	302580	048146	105	951854	31
30	649527	422	350473	697736	527	302264	048209	105	951791	30
31	9.649781	422	10.350219	9.698053	527	10.301947	10.048272	105	9.951728	29
32	650034	422	349966	698369	527	301631	048335	105	951665	28
33	650287	421	349713	698685	526	301315	048398	105	951602	27
34	650539	421	349461	699001	526	300999	048461	105	951539	26
35	650792	421	349208	699316	526	300684	048524	105	951476	25
36	651044	420	348956	699632	526	300368	048588	105	951412	24
37	651297	420	348703	699947	526	300053	048651	106	951349	23
38	651549	420	348451	700263	525	299737	048714	106	951286	22
39	651800	419	348200	700578	525	299422	048778	106	951222	21
40	652052	419	347948	700893	525	299107	048841	106	951159	20
41	9.652304	419	10.347696	9.701208	524	10.298792	10.048904	106	9.951096	19
42	652555	418	347445	701523	524	298477	048968	106	951032	18
43	652806	418	347194	701837	524	298163	049032	106	950968	17
44	653057	418	346943	702152	524	297848	049095	106	950905	16
45	653308	418	346692	702466	524	297534	049159	106	950841	15
46	653558	417	346442	702780	523	297220	049222	106	950778	14
47	653808	417	346192	703095	523	296905	049286	106	950714	13
48	654059	417	345941	703409	523	296591	049350	106	950650	12
49	654309	416	345691	703723	523	296277	049414	106	950586	11
50	654558	416	345442	704036	522	295964	049478	107	950522	10
51	9.654808	416	10.345192	9.704350	522	10.295650	10.049542	107	9.950458	9
52	655058	416	344942	704663	522	295337	049606	107	950394	8
53	655307	415	344693	704977	522	295023	049670	107	950330	7
54	655556	415	344444	705290	522	294710	049734	107	950266	6
55	655805	415	344195	705603	521	294397	049798	107	950202	5
56	656054	414	343946	705916	521	294084	049862	107	950138	4
57	656302	414	343698	706228	521	293772	049926	107	950074	3
58	656551	414	343449	706541	521	293459	049990	107	950010	2
59	656799	413	343201	706854	521	293146	050055	107	949945	1
60	657047	413	342953	707166	520	292834	050119	107	949881	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec.	Tang.	D	Cotang	Secant	D	Cosine	
0	9 657047	413	10.342953	9 707166	520	10 292834	10.050119	107	9 949881	60
1	657295	413	342705	707478	520	292522	050184	107	949816	59
2	657542	412	342458	707790	520	292210	050249	107	949752	58
3	657790	412	342210	708102	520	291898	050312	108	949688	57
4	658037	412	341963	708414	519	291586	050377	108	949623	56
5	658284	412	341716	708726	519	291274	050442	108	949558	55
6	658531	411	341469	709037	519	290963	050506	108	949494	54
7	658778	411	341222	709349	519	290651	050571	108	949429	53
8	659025	411	340975	709660	519	290340	050636	108	949364	52
9	659271	410	340729	709971	518	290029	050700	108	949300	51
10	659517	410	340483	710282	518	289718	050765	108	949235	50
11	9 659763	410	10 340237	9 710593	518	10 289407	10 050830	108	9 949170	49
12	660009	409	339991	710904	518	289096	050895	108	949105	48
13	660255	409	339745	711215	518	288785	050960	108	949040	47
14	660501	409	339499	711525	517	288475	051025	108	948975	46
15	660746	409	339254	711836	517	288164	051090	108	948910	45
16	660991	408	339009	712146	517	287854	051155	108	948845	44
17	661236	408	338764	712456	517	287544	051220	109	948780	43
18	661481	408	338519	712766	516	287234	051285	109	948715	42
19	661726	407	338274	713076	516	286924	051350	109	948650	41
20	661970	407	338030	713386	516	286614	051416	109	948584	40
21	9.662214	407	10 337786	9 713696	516	10.286304	10 051481	109	9 948519	39
22	662459	407	337540	714005	516	285995	051546	109	948454	38
23	662703	406	337297	714314	515	285686	051612	109	948388	37
24	662946	406	337054	714624	515	285376	051677	109	948323	36
25	663190	406	336810	714933	515	285067	051743	109	948257	35
26	663433	405	336567	715242	515	284758	051808	109	948192	34
27	663677	405	336323	715551	514	284449	051874	109	948126	33
28	663920	405	336080	715860	514	284140	051940	109	948060	32
29	664163	405	335837	716168	514	283832	052005	110	947995	31
30	664406	404	335594	716477	514	283523	052071	110	947929	30
31	9 664648	404	10 335352	9 716785	514	10 283215	10.052137	110	9 947863	29
32	664891	404	335109	717093	513	282907	052203	110	947797	28
33	665133	403	334867	717401	513	282599	052269	110	947731	27
34	665375	403	334625	717709	513	282291	052335	110	947665	26
35	665617	403	334383	718017	513	281983	052400	110	947600	25
36	665859	402	334141	718325	513	281675	052467	110	947533	24
37	666100	402	333900	718633	512	281367	052533	110	947467	23
38	666342	402	333658	718940	512	281060	052599	110	947401	22
39	666583	402	333417	719248	512	280752	052665	110	947335	21
40	666824	401	333176	719555	512	280445	052731	110	947269	20
41	9 667065	401	10 332935	9 719862	512	10 280138	10 052797	110	9 947203	19
42	667305	401	332695	720169	511	279831	052864	111	947136	18
43	667546	401	332454	720476	511	279524	052930	111	947070	17
44	667786	400	332214	720783	511	279217	052996	111	947004	16
45	668027	400	331973	721089	511	278911	053063	111	946937	15
46	668267	400	331733	721396	511	278604	053129	111	946871	14
47	668506	399	331494	721702	510	278298	053196	111	946804	13
48	668746	399	331254	722009	510	277991	053262	111	946738	12
49	668986	399	331014	722315	510	277685	053329	111	946671	11
50	669225	399	330775	722621	510	277379	053396	111	946604	10
51	9 669464	398	10 330536	9 722927	510	10 277073	10 053462	111	9 946538	9
52	669703	398	330297	723232	509	276768	053529	111	946471	8
53	669942	398	330058	723538	509	276462	053596	111	946404	7
54	670181	397	329819	723844	509	276156	053663	111	946337	6
55	670419	397	329581	724149	509	275851	053730	112	946270	5
56	670658	397	329342	724454	509	275546	053797	112	946203	4
57	670896	397	329104	724759	508	275241	053864	112	946136	3
58	671134	396	328866	725065	508	274935	053931	112	946069	2
59	671372	396	328628	725369	508	274631	053998	112	946002	1
60	671609	396	328391	725674	508	274326	054065	112	945935	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 671609	396	10 328391	9 725674	508	10 274326	10 054065	112	9 945935	60
1	671847	395	328153	725979	508	274021	054132	112	945868	59
2	672084	395	327916	726284	507	273716	054200	112	945800	58
3	672321	395	327679	726588	507	273412	054267	112	945733	57
4	672558	395	327442	726892	507	273108	054334	112	945666	56
5	672795	394	327205	727197	507	272803	054402	112	945598	55
6	673032	394	326968	727501	507	272499	054469	112	945531	54
7	673268	394	326732	727805	506	272195	054536	113	945464	53
8	673505	394	326495	728109	506	271891	054604	113	945396	52
9	673741	393	326259	728412	506	271588	054672	113	945328	51
10	673977	393	326023	728716	506	271284	054739	113	945261	50
11	9 674213	393	10 325787	9 729020	506	10 270980	10 054807	113	9 945193	49
12	674448	392	325552	729323	505	270677	054875	113	945125	48
13	674684	392	325316	729626	505	270374	054942	113	945058	47
14	674919	392	325081	729929	505	270071	055010	113	944990	46
15	675155	392	324845	730233	505	269767	055078	113	944922	45
16	675390	391	324610	730535	505	269465	055146	113	944854	44
17	675624	391	324376	730838	504	269162	055214	113	944786	43
18	675859	391	324141	731141	504	268859	055282	113	944718	42
19	676094	391	323906	731444	504	268556	055350	113	944650	41
20	676328	390	323672	731746	504	268254	055418	114	944582	40
21	9 676562	390	10 323438	9 732048	504	10 267952	10 055486	114	9 944514	39
22	676796	390	323204	732351	503	267649	055554	114	944446	38
23	677030	390	322970	732653	503	267347	055623	114	944377	37
24	677264	389	322736	732955	503	267045	055691	114	944309	36
25	677498	389	322502	733257	503	266743	055759	114	944241	35
26	677731	389	322269	733558	503	266442	055828	114	944172	34
27	677964	388	322036	733860	502	266140	055896	114	944104	33
28	678197	388	321803	734162	502	265838	055964	114	944036	32
29	678430	388	321570	734463	502	265537	056033	114	943967	31
30	678663	388	321337	734764	502	265236	056101	114	943899	30
31	9 678895	387	10 321105	9 735066	502	10 264934	10 056170	114	9 943830	29
32	679128	387	320872	735367	502	264633	056239	114	943761	28
33	679360	387	320640	735668	501	264332	056307	115	943693	27
34	679592	387	320408	735969	501	264031	056376	115	943624	26
35	679824	386	320176	736269	501	263731	056445	115	943555	25
36	680056	386	319944	736570	501	263430	056514	115	943486	24
37	680288	386	319712	736871	501	263129	056583	115	943417	23
38	680519	385	319481	737171	500	262829	056652	115	943348	22
39	680750	385	319250	737471	500	262529	056721	115	943279	21
40	680982	385	319018	737771	500	262229	056790	115	943210	20
41	9 681213	385	10 318787	9 738071	500	10 261929	10 056859	115	9 943141	19
42	681443	384	318557	738371	500	261629	056928	115	943072	18
43	681674	384	318326	738671	499	261329	056997	115	943003	17
44	681905	384	318095	738971	499	261029	057066	115	942934	16
45	682135	384	317865	739271	499	260729	057136	115	942864	15
46	682365	383	317635	739570	499	260430	057205	116	942795	14
47	682595	383	317405	739870	499	260130	057274	116	942726	13
48	682825	383	317175	740169	499	259831	057344	116	942656	12
49	683055	383	316945	740468	498	259532	057413	116	942587	11
50	683284	382	316716	740767	498	259233	057483	116	942517	10
51	9 683514	382	10 316486	9 741066	498	10 258934	10 057552	116	9 942448	9
52	683743	382	316257	741365	498	258635	057622	116	942378	8
53	683972	382	316028	741664	498	258336	057692	116	942308	7
54	684201	381	315799	741962	497	258038	057761	116	942239	6
55	684430	381	315570	742261	497	257739	057831	116	942169	5
56	684658	381	315342	742559	497	257441	057901	116	942099	4
57	684887	380	315113	742858	497	257142	057971	116	942029	3
58	685115	380	314885	743156	497	256844	058041	116	941959	2
59	685343	380	314657	743454	497	256546	058111	117	941889	1
60	685571	380	314429	743752	496	256248	058181	117	941819	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec.	Tang	D	Cotang	Secant	D	Cosine	
0	9 685571	380	10 314429	9 743752	496	10 256248	10 058181	117	9 941819	60
1	685799	379	314201	744050	496	255950	058251	117	941749	59
2	686027	379	313973	744348	496	255652	058321	117	941679	58
3	686254	379	313746	744645	496	255355	058391	117	941609	57
4	686482	379	313518	744943	496	255057	058461	117	941539	56
5	686709	378	313291	745240	496	254760	058531	117	941469	55
6	686936	378	313064	745538	495	254462	058602	117	941398	54
7	687163	378	312837	745835	495	254165	058672	117	941328	53
8	687389	378	312611	746132	495	253868	058742	117	941258	52
9	687616	377	312384	746429	495	253571	058813	117	941187	51
10	687843	377	312157	746726	495	253274	058883	117	941117	50
11	9 688069	377	10 311931	9 747023	494	10 252977	10 058954	118	9 941046	49
12	688295	377	311705	747319	494	252681	059025	118	940975	48
13	688521	376	311479	747616	494	252384	059095	118	940905	47
14	688747	376	311253	747913	494	252087	059166	118	940834	46
15	688972	376	311028	748209	494	251791	059237	118	940763	45
16	689198	376	310802	748505	493	251495	059307	118	940693	44
17	689423	375	310577	748801	493	251199	059378	118	940622	43
18	689648	375	310352	749097	493	250903	059449	118	940551	42
19	689873	375	310127	749393	493	250607	059520	118	940480	41
20	690098	375	309902	749689	493	250311	059591	118	940409	40
21	9 690323	374	10 309677	9 749985	493	10 250015	10 059662	118	9 940338	39
22	690548	374	309452	750281	492	249719	059733	118	940267	38
23	690772	374	309228	750576	492	249424	059804	118	940196	37
24	690996	374	309004	750872	492	249128	059875	119	940125	36
25	691220	373	308780	751167	492	248833	059946	119	940054	35
26	691444	373	308556	751462	492	248538	060018	119	939982	34
27	691668	373	308332	751757	492	248243	060089	119	939911	33
28	691892	373	308108	752052	491	247948	060160	119	939840	32
29	692115	372	307885	752347	491	247653	060232	119	939768	31
30	692339	372	307661	752642	491	247358	060303	119	939697	30
31	9 692562	372	10 307438	9 752937	491	10 247063	10 060375	119	9 939625	29
32	692785	371	307215	753231	491	246769	060446	119	939554	28
33	693008	371	306992	753526	491	246474	060518	119	939482	27
34	693231	371	306769	753820	490	246180	060590	119	939410	26
35	693453	371	306547	754115	490	245885	060661	119	939339	25
36	693676	370	306324	754409	490	245591	060733	120	939267	24
37	693898	370	306102	754703	490	245297	060805	120	939195	23
38	694120	370	305880	754997	490	245003	060877	120	939123	22
39	694342	370	305658	755291	490	244709	060948	120	939052	21
40	694564	369	305436	755585	489	244415	061020	120	938980	20
41	9 694786	369	10 305214	9 755878	489	10 244122	10 061092	120	9 938908	19
42	695007	369	304993	756172	489	243828	061164	120	938836	18
43	695229	369	304771	756465	489	243535	061237	120	938763	17
44	695450	368	304550	756759	489	243241	061309	120	938691	16
45	695671	368	304329	757052	489	242948	061381	120	938619	15
46	695892	368	304108	757345	488	242655	061453	120	938547	14
47	696113	368	303887	757638	488	242362	061525	120	938475	13
48	696334	367	303666	757931	488	242069	061598	121	938402	12
49	696554	367	303446	758224	488	241776	061670	121	938330	11
50	696775	367	303225	758517	488	241483	061742	121	938258	10
51	9 696995	367	10 303005	9 758810	488	10 241190	10 061815	121	9 938185	9
52	697215	366	302785	759102	487	240898	061887	121	938113	8
53	697435	366	302565	759395	487	240605	061960	121	938040	7
54	697654	366	302346	759687	487	240313	062033	121	937967	6
55	697874	366	302126	759979	487	240021	062105	121	937895	5
56	698094	365	301906	760272	487	239728	062178	121	937822	4
57	698313	365	301687	760564	487	239436	062251	121	937749	3
58	698532	365	301468	760856	486	239144	062324	121	937676	2
59	698751	365	301249	761148	486	238852	062396	121	937604	1
60	698970	364	301030	761439	486	238561	062469	121	937531	0
	Cosine		Secant	Cotang		Tang	Cosec.		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 698970	364	10 301030	9.761439	486	10 238561	10 062469	121	9 937531	60
1	699189	364	300811	761731	486	238269	062542	122	937458	59
2	699407	364	300593	762023	486	237977	062615	122	937385	58
3	699626	364	300374	762314	486	237686	062688	122	937312	57
4	699844	363	300156	762606	485	237394	062762	122	937238	56
5	700062	363	299938	762897	485	237103	062835	122	937165	55
6	700280	363	299720	763188	485	236812	062908	122	937092	54
7	700498	363	299502	763479	485	236521	062981	122	937019	53
8	700716	363	299284	763770	485	236230	063054	122	936946	52
9	700933	362	299067	764061	485	235939	063128	122	936872	51
10	701151	362	298849	764352	484	235648	063201	122	936799	50
11	9 701368	362	10 298632	9.764643	484	10 235357	10 063275	122	9 936725	49
12	701585	362	298415	764933	484	235067	063348	123	936652	48
13	701802	361	298198	765224	484	234776	063422	123	936578	47
14	702019	361	297981	765514	484	234486	063495	123	936505	46
15	702236	361	297764	765805	484	·234195	063569	123	936431	45
16	702452	361	297548	766095	484	233905	063643	123	936357	44
17	702669	360	297331	766385	483	233615	063716	123	936284	43
18	702885	360	297115	766675	483	233325	063790	123	936210	42
19	703101	360	296899	766965	483	233035	063864	123	936136	41
20	703317	360	296683	767255	483	232745	063938	123	936062	40
21	9 703533	359	10 296467	9.767545	483	10 232455	10 064012	123	9 935988	39
22	703749	359	296251	767834	483	232166	064086	123	935914	38
23	703964	359	296036	768124	482	231876	064160	123	935840	37
24	704179	359	295821	768413	482	231587	064234	124	935766	36
25	704395	359	295605	768703	482	231297	064308	124	935692	35
26	704610	358	295390	768992	482	231008	064382	124	935618	34
27	704825	358	295175	769281	482	230719	064457	124	935543	33
28	705040	358	294960	769570	482	230430	064531	124	935469	32
29	705254	358	294746	769860	481	230140	064605	124	935395	31
30	705469	357	294531	770148	481	229852	064680	124	935320	30
31	9 705683	357	10 294317	9 770437	481	10 229563	10.064754	124	9 935246	29
32	705898	357	294102	770726	481	229274	064829	124	935171	28
33	706112	357	293888	771015	481	228985	064903	121	935097	27
34	706326	356	293674	771303	481	228697	064978	124	935022	26
35	706539	356	293461	771592	481	228408	065052	124	934948	25
36	706753	356	293247	771880	480	228120	065127	124	934873	24
37	706967	356	293033	772168	480	227832	065202	125	934798	23
38	707180	355	292820	772457	480	227543	065277	125	934723	22
39	707393	355	292607	772745	480	227255	065351	125	934649	21
40	707606	355	292394	773033	480	226967	065426	125	934574	20
41	9 707819	355	10 292181	9 773321	480	10 226679	10 065501	125	9 934499	19
42	708032	354	291968	773608	479	226392	065576	125	934424	18
43	708245	354	291755	773896	479	226104	065651	125	934349	17
44	708458	354	291542	774184	479	225816	065726	125	934274	16
45	708670	354	291330	774171	479	225529	065801	125	934199	15
46	708882	353	291118	774759	479	225241	065877	125	934123	14
47	709094	353	290906	775046	479	224954	065952	125	934048	13
48	709306	353	290694	775333	479	224667	066027	125	933973	12
49	709518	353	290482	775621	478	224379	066102	126	933898	11
50	709730	353	290270	775908	478	224092	066178	126	933822	10
51	9 709941	352	10 290059	9 776195	478	10 223805	10 066253	126	9 933747	9
52	710153	352	289847	776482	478	223518	066329	126	933671	8
53	710364	352	289636	776769	478	223231	066404	126	933596	7
54	710575	352	289425	777055	478	222945	066480	126	933520	6
55	710786	351	289214	777342	478	222658	066555	126	933445	5
56	710997	351	289003	777628	477	222372	066631	126	933369	4
57	711208	351	288792	777915	477	222085	066707	126	933293	3
58	711419	351	288581	778201	477	221799	066783	126	933217	2
59	711629	350	288371	778487	477	221512	066859	126	933141	1
60	711839	350	288161	778774	477	221226	066934	126	933066	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

59 Degrees.

M	Sine	D	Cosec.	Tang.	D	Cotang	Secant	D	Cosine	
0	9 711839	350	10.288161	9 778774	477	10 221226	10 066934	126	9 933066	60
1	712050	350	287950	779060	477	220940	067010	127	932990	59
2	712260	350	287740	779346	476	220654	067086	127	932914	58
3	712469	349	287531	779632	476	220368	067162	127	932838	57
4	712679	349	287321	779918	476	220082	067238	127	932762	56
5	712889	349	287111	780203	476	219797	067315	127	932685	55
6	713098	349	286902	780489	476	219511	067391	127	932609	54
7	713308	349	286692	780775	476	219225	067467	127	932533	53
8	713517	348	286483	781060	476	218940	067543	127	932457	52
9	713726	348	286274	781346	475	218654	067620	127	932380	51
10	713935	348	286065	781631	475	218369	067696	127	932304	50
11	9.714144	348	10.285856	9 781916	475	10 218084	10.067772	127	9.932228	49
12	714352	347	285648	782201	475	217799	067849	127	932151	48
13	714561	347	285439	782486	475	217514	067925	128	932075	47
14	714769	347	285231	782771	475	217229	068002	128	931998	46
15	714978	347	285022	783056	475	216944	068079	128	931921	45
16	715186	347	284814	783341	475	216659	068155	128	931845	44
17	715394	346	284606	783626	474	216374	068232	128	931768	43
18	715602	346	284398	783910	474	216090	068309	128	931691	42
19	715809	346	284191	784195	474	215805	068386	128	931614	41
20	716017	346	283983	784479	474	215521	068463	128	931537	40
21	9.716224	345	10 283776	9 784764	474	10 215236	10.068540	128	9.931460	39
22	716432	345	283568	785048	474	214952	068617	128	931383	38
23	716639	345	283361	785332	473	214668	068694	128	931306	37
24	716846	345	283154	785616	473	214384	068771	129	931229	36
25	717053	345	282947	785900	473	214100	068848	129	931152	35
26	717259	344	282741	786184	473	213816	068925	129	931075	34
27	717466	344	282534	786468	473	213532	069002	129	930998	33
28	717673	344	282327	786752	473	213248	069079	129	930921	32
29	717879	344	282121	787036	473	212964	069157	129	930843	31
30	718085	343	281915	787319	472	212681	069234	129	930766	30
31	9 718291	343	10 281709	9 787603	472	10 212397	10 069312	129	9 930688	29
32	718497	343	281503	787886	472	212114	069389	129	930611	28
33	718703	343	281297	788170	472	211830	069467	129	930533	27
34	718909	343	281091	788453	472	211547	069544	129	930456	26
35	719114	342	280886	788736	472	211264	069622	129	930378	25
36	719320	342	280680	789019	472	210981	069700	130	930300	24
37	719525	342	280475	789302	471	210698	069777	130	930223	23
38	719730	342	280270	789585	471	210415	069855	130	930145	22
39	719935	341	280065	789868	471	210132	069933	130	930067	21
40	720140	341	279860	790151	471	209849	070011	130	929989	20
41	9 720345	341	10 279655	9 790433	471	10 209567	10 070089	130	9.929911	19
42	720549	341	279451	790716	471	209284	070167	130	929833	18
43	720754	340	279246	790999	471	209001	070245	130	929755	17
44	720958	340	279042	791281	471	208719	070323	130	929677	16
45	721162	340	278838	791563	470	208437	070401	130	929599	15
46	721366	340	278634	791846	470	208154	070479	130	929521	14
47	721570	340	278430	792128	470	207872	070558	130	929442	13
48	721774	339	278226	792110	470	207590	070636	131	929364	12
49	721978	339	278022	792692	470	207308	070714	131	929286	11
50	722181	339	277819	792974	470	207026	070793	131	929207	10
51	9 722385	339	10 277615	9 793256	470	10 206744	10 070871	131	9 929129	9
52	722588	339	277412	793538	469	206462	071950	131	929050	8
53	722791	338	277209	793819	469	206181	071028	131	928972	7
54	722994	338	277006	794101	469	205899	071107	131	928893	6
55	723197	338	276803	794383	469	205617	071185	131	928815	5
56	723400	338	276600	794661	469	205336	071261	131	928736	4
57	723603	337	276397	794945	469	205055	071343	131	928657	3
58	723805	337	276195	795227	469	204773	071422	131	928578	2
59	724007	337	275993	795508	468	204492	071501	131	928499	1
60	724210	337	275790	795789	468	204211	071580	131	928420	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

58 Degrees.

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 724210	337	10 275790	9.795789	468	10 204211	10 071580	132	9 928420	60
1	724412	337	275588	796070	468	203930	071658	132	928342	59
2	724614	336	275386	796351	468	203649	071737	132	928263	58
3	724816	336	275184	796632	468	203368	071817	132	928183	57
4	725017	336	274983	796913	468	203087	071896	132	928104	56
5	725219	336	274781	797194	468	202806	071975	132	928025	55
6	725420	335	274580	797475	468	202525	072054	132	927946	54
7	725622	335	274378	797755	468	202245	072133	132	927867	53
8	725823	335	274177	798036	467	201964	072213	132	927787	52
9	726024	335	273976	798316	467	201684	072292	132	927708	51
10	726225	335	273775	798596	467	201404	072371	132	927629	50
11	9 726426	334	10 273574	9 798877	467	10 201123	10 072451	132	9 927549	49
12	726626	334	273374	799157	467	200843	072530	133	927470	48
13	726827	334	273173	799437	467	200563	072610	133	927390	47
14	727027	334	272973	799717	467	200283	072690	133	927310	46
15	727228	334	272772	799997	466	200003	072769	133	927231	45
16	727428	333	272572	800277	466	199723	072849	133	927151	44
17	727628	333	272372	800557	466	199443	072929	133	927071	43
18	727828	333	272172	800836	466	199164	073009	133	926991	42
19	728027	333	271973	801116	466	198884	073089	133	926911	41
20	728227	333	271773	801396	466	198604	073169	133	926831	40
21	9 728427	332	10 271573	9 801675	466	10 198325	10.073249	133	9 926751	39
22	728626	332	271374	801955	466	198045	073329	133	926671	38
23	728825	332	271175	802234	465	197766	073409	133	926591	37
24	729024	332	270976	802513	465	197487	073489	134	926511	36
25	729223	331	270777	802792	465	197208	073569	134	926431	35
26	729422	331	270578	803072	465	196928	073649	134	926351	34
27	729621	331	270379	803351	465	196649	073730	134	926270	33
28	729820	331	270180	803630	465	196370	073810	134	926190	32
29	730018	330	269982	803908	465	196092	073890	134	926110	31
30	730216	330	269784	804187	465	195813	073971	134	926029	30
31	9 730415	330	10 269585	9 804466	464	10 195534	10 074051	134	9 925949	29
32	730613	330	269387	804745	464	195255	074132	134	925868	28
33	730811	330	269189	805023	464	194977	074212	134	925788	27
34	731009	329	268991	805302	464	194698	074293	134	925707	26
35	731206	329	268794	805580	464	194420	074374	134	925626	25
36	731404	329	268596	805859	464	194141	074455	135	925545	24
37	731602	329	268398	806137	464	193863	074535	135	925465	23
38	731799	329	268201	806415	463	193585	074616	135	925384	22
39	731996	328	268004	806693	463	193307	074697	135	925303	21
40	732193	328	267807	806971	463	193029	074778	135	925222	20
41	9 732390	328	10 267610	9 807249	463	10 192751	10 074859	135	9 925141	19
42	732587	328	267413	807527	463	192473	074940	135	925060	18
43	732784	328	267216	807805	463	192195	075021	135	924979	17
44	732980	327	267020	808083	463	191917	075103	135	924897	16
45	733177	327	266823	808361	463	191639	075184	135	924816	15
46	733373	327	266627	808638	462	191362	075265	136	924735	14
47	733569	327	266431	808916	462	191084	075346	136	924654	13
48	733765	327	266235	809193	462	190807	075428	136	924572	12
49	733961	326	266039	809471	462	190529	075509	136	924491	11
50	734157	326	265843	809748	462	190252	075591	136	924409	10
51	9 734353	326	10 265647	9 810025	462	10.189975	10 075672	136	9 924328	9
52	734549	326	265451	810302	462	189698	075754	136	924246	8
53	734744	325	265256	810580	462	189420	075836	136	924164	7
54	734939	325	265061	810857	462	189143	075917	136	924083	6
55	735135	325	264865	811134	461	188866	075999	136	924001	5
56	735330	325	264670	811410	461	188590	076081	136	923919	4
57	735525	325	264475	811687	461	188313	076163	136	923837	3
58	735719	324	264281	811964	461	188036	076245	137	923755	2
59	735914	324	264086	812241	461	187759	076327	137	923673	1
60	736109	324	263891	812517	461	187483	076409	137	923591	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

57 Degrees.

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 736109	324	10 263891	9 812517	461	10 187483	10 076409	137	9 923591	60
1	736303	324	263697	812794	461	187206	076491	137	923509	59
2	736498	324	263502	813070	461	186930	076573	137	923427	58
3	736692	323	263308	813347	460	186653	076655	137	923345	57
4	736886	323	263114	813623	460	186377	076737	137	923263	56
5	737080	323	262920	813899	460	186101	076819	137	923181	55
6	737274	323	262726	814175	460	185825	076902	137	923098	54
7	737467	323	262533	814452	460	185548	076984	137	923016	53
8	737661	322	262339	814728	460	185272	077067	137	922933	52
9	737855	322	262145	815004	460	184996	077149	137	922851	51
10	738048	322	261952	815279	460	184721	077232	138	922768	50
11	9 738241	322	10.261759	9.815555	459	10 184445	10 077314	138	9 922686	49
12	738434	322	261566	815831	459	184169	077397	138	922603	48
13	738627	321	261373	816107	459	183893	077480	138	922520	47
14	738820	321	261180	816382	459	183618	077562	138	922438	46
15	739013	321	260987	816658	459	183342	077645	138	922355	45
16	739206	321	260794	816933	459	183067	077728	138	922272	44
17	739398	321	260602	817209	459	182791	077811	138	922189	43
18	739590	320	260410	817484	459	182516	077894	138	922106	42
19	739783	320	260217	817759	459	182241	077977	138	922023	41
20	739975	320	260025	818035	458	181965	078060	138	921940	40
21	9 740167	320	10 259833	9 818310	458	10 181690	10 078143	139	9 921857	39
22	740359	320	259641	818585	458	181415	078226	139	921774	38
23	740550	319	259450	818860	458	181140	078309	139	921691	37
24	740742	319	259258	819135	458	180865	078393	139	921607	36
25	740934	319	259066	819410	458	180590	078476	139	921524	35
26	741125	319	258875	819684	458	180316	078559	139	921441	34
27	741316	319	258684	819959	458	180041	078643	139	921357	33
28	741508	318	258492	820234	458	179766	078726	139	921274	32
29	741699	318	258301	820508	457	179492	078810	139	921190	31
30	741889	318	258111	820783	457	179217	078893	139	921107	30
31	9 742080	318	10 257920	9 821057	457	10 178943	10 078977	139	9 921023	29
32	742271	318	257729	821332	457	178668	079061	140	920939	28
33	742462	317	257538	821606	457	178394	079144	140	920856	27
34	742652	317	257348	821880	457	178120	079228	140	920772	26
35	742842	317	257158	822154	457	177846	079312	140	920688	25
36	743033	317	256967	822429	457	177571	079396	140	920604	24
37	743223	317	256777	822703	457	177297	079480	140	920520	23
38	743413	316	256587	822977	456	177023	079564	140	920436	22
39	743602	316	256398	823250	456	176750	079648	140	920352	21
40	743792	316	256208	823524	456	176476	079732	140	920268	20
41	9 743982	316	10 256018	9 823798	456	10 176202	10 079816	140	9 920184	19
42	744171	316	255829	824072	456	175928	079901	140	920099	18
43	744361	315	255639	824345	456	175655	079985	140	920015	17
44	744550	315	255130	824619	456	175381	080069	141	919931	16
45	744739	315	255261	82489	456	175107	080154	141	919846	15
46	744928	315	255072	825166	456	174834	080238	141	919762	14
47	745117	315	254883	825439	455	174561	080323	141	919677	13
48	745306	314	254694	825713	455	174287	080407	141	919593	12
49	745494	314	254506	825986	455	174011	080492	141	919508	11
50	745683	314	254317	826259	455	173741	080576	141	919424	10
51	9 745871	314	10 254129	9 826532	455	10 173468	10 080661	141	9 919339	9
52	746059	314	253941	826805	455	173195	080746	141	919254	8
53	746248	313	253752	827078	455	172922	080831	141	919169	7
54	746436	313	253564	827351	455	172649	080915	141	919085	6
55	746624	313	253376	827624	455	172376	081000	141	919000	5
56	746812	313	253188	827897	454	172103	081085	142	918915	4
57	746999	313	253001	828170	454	171830	081170	142	918830	3
58	747187	312	252813	828442	454	171558	081255	142	918745	2
59	747374	312	252626	828715	454	171285	081341	142	918659	1
60	747562	312	252438	828987	454	171013	081426	142	918574	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

56 Degrees.

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 747562	312	10 252438	9 828987	454	10 171013	10 081426	142	9 918574	60
1	747749	312	252251	829260	454	170740	081511	142	918489	59
2	747936	312	252064	829532	454	170468	081596	142	918404	58
3	748123	311	251877	829805	454	170195	081682	142	918318	57
4	748310	311	251690	830077	454	169923	081767	142	918233	56
5	748497	311	251503	830349	453	169651	081853	142	918147	55
6	748683	311	251317	830621	453	169379	081938	142	918062	54
7	748870	311	251130	830893	453	169107	082024	143	917976	53
8	749056	310	250944	831165	453	168835	082109	143	917891	52
9	749243	310	250757	831437	453	168563	082195	143	917805	51
10	749429	310	250571	831709	453	168291	082281	143	917719	50
11	9 749615	310	10 250385	9 831981	453	10 168019	10 082366	143	9 917634	49
12	749801	310	250199	832253	453	167747	082452	143	917548	48
13	749987	309	250013	832525	453	167475	082538	143	917462	47
14	750172	309	249828	832796	453	167204	082624	143	917376	46
15	750358	309	249642	833068	452	166932	082710	143	917290	45
16	750543	309	249457	833339	452	166661	082796	143	917204	44
17	750729	309	249271	833611	452	166389	082882	144	917118	43
18	750914	308	249086	833882	452	166118	082968	144	917032	42
19	751099	308	248901	834154	452	165846	083054	144	916946	41
20	751284	308	248716	834425	452	165575	083141	144	916859	40
21	9 751469	308	10 248531	9 834696	452	10 165304	10 083227	144	9 916773	39
22	751654	308	248346	834967	452	165033	083313	144	916687	38
23	751839	308	248161	835238	452	164762	083400	144	916600	37
24	752023	307	247977	835509	452	164491	083486	144	916514	36
25	752208	307	247792	835780	451	164220	083573	144	916427	35
26	752392	307	247608	836051	451	163949	083659	144	916341	34
27	752576	307	247424	836322	451	163678	083746	144	916254	33
28	752760	307	247240	836593	451	163407	083833	145	916167	32
29	752944	306	247056	836864	451	163136	083919	145	916081	31
30	753128	306	246872	837134	451	162866	084006	145	915994	30
31	9 753312	306	10 246688	9 837405	451	10 162595	10 084093	145	9 915907	29
32	753495	306	246505	837675	451	162325	084180	145	915820	28
33	753679	306	246321	837946	451	162054	084267	145	915733	27
34	753862	305	246138	838216	451	161784	084354	145	915646	26
35	754046	305	245954	838487	450	161513	084441	145	915559	25
36	754229	305	245771	838757	450	161243	084528	145	915472	24
37	754412	305	245588	839027	450	160973	084615	145	915385	23
38	754595	305	245405	839297	450	160703	084703	145	915297	22
39	754778	304	245222	839568	450	160432	084790	145	915210	21
40	754960	304	245040	839838	450	160162	084877	146	915123	20
41	9 755143	304	10 244857	9 840108	450	10 159892	10 084965	146	9 915035	19
42	755326	304	244674	840378	450	159622	085052	146	914948	18
43	755508	304	244492	840647	450	159353	085140	146	914860	17
44	755690	304	244310	840917	449	159083	085227	146	914773	16
45	755872	303	244128	841187	449	158813	085315	146	914685	15
46	756054	303	243946	841457	449	158543	085402	146	914598	14
47	756236	303	243764	841726	449	158274	085490	146	914510	13
48	756418	303	243582	841996	449	158004	085578	146	914422	12
49	756600	303	243400	842266	449	157734	085666	146	914334	11
50	756782	302	243218	842535	449	157465	085754	147	914246	10
51	9 756963	302	10 243037	9 842805	449	10 157195	10 085842	147	9 914158	9
52	757144	302	242856	843074	449	156926	085930	147	914070	8
53	757326	302	242674	843343	449	156657	086018	147	913982	7
54	757507	302	242493	843612	449	156388	086106	147	913894	6
55	757688	301	242312	843882	448	156118	086194	147	913806	5
56	757869	301	242131	844151	448	155849	086282	147	913718	4
57	758050	301	241950	844420	448	155580	086370	147	913630	3
58	758230	301	241770	844689	448	155311	086459	147	913541	2
59	758411	301	241589	844958	448	155042	086547	147	913453	1
60	758591	301	241409	845227	448	154773	086635	147	913365	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

55 Degrees.

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 758591	301	10 241409	9 845227	448	10 154773	10 086635	147	9 913365	60
1	758772	300	241228	845496	448	154504	086724	147	913276	59
2	758952	300	241048	845764	448	154236	086813	148	913187	58
3	759132	300	240868	846033	448	153967	086901	148	913099	57
4	759312	300	240688	846302	448	153698	086990	148	913010	56
5	759492	300	240508	846570	447	153430	087078	148	912922	55
6	759672	299	240328	846839	447	153161	087167	148	912833	54
7	759852	299	240148	847107	447	152893	087256	148	912744	53
8	760031	299	239969	847376	447	152624	087345	148	912655	52
9	760311	299	239789	847644	447	152356	087434	148	912566	51
10	760390	299	239610	847913	447	152087	087523	148	912477	50
11	9 760569	298	10 239431	9 848181	447	10 151819	10 087612	149	9 912388	49
12	760748	298	239252	848449	447	151551	087701	149	912299	48
13	760927	298	239073	848717	447	151283	087790	149	912210	47
14	761106	298	238894	848986	447	151014	087879	149	912121	46
15	761285	298	238715	849254	447	150746	087969	149	912031	45
16	761464	298	238536	849522	447	150478	088058	149	911942	44
17	761642	297	238358	849790	446	150210	088147	149	911853	43
18	761821	297	238179	850058	446	149942	088237	149	911763	42
19	761999	297	238001	850325	446	149675	088326	149	911674	41
20	762177	297	237823	850593	446	149407	088416	149	911584	40
21	9 762356	297	10 237644	9 850861	446	10 149139	10 088505	149	9 911495	39
22	762534	296	237466	851129	446	148871	088595	149	911405	38
23	762712	296	237288	851396	446	148604	088685	150	911315	37
24	762889	296	237111	851664	446	148336	088774	150	911226	36
25	763067	296	236933	851931	446	148069	088864	150	911136	35
26	763245	296	236755	852199	446	147801	088954	150	911046	34
27	763422	296	236578	852466	446	147534	089044	150	910956	33
28	763600	295	236400	852733	445	147267	089134	150	910866	32
29	763777	295	236223	853001	445	146999	089224	150	910776	31
30	763954	295	236046	853268	445	146732	089314	150	910686	30
31	9 764131	295	10 235869	9 853535	445	10 146465	10 089404	150	9 910596	29
32	764308	295	235692	853802	445	146198	089494	150	910506	28
33	764485	294	235515	854069	445	145931	089585	150	910415	27
34	764662	294	235338	854336	445	145664	089675	151	910325	26
35	764838	294	235162	854603	445	145397	089765	151	910235	25
36	765015	294	234985	854870	445	145130	089856	151	910144	24
37	765191	294	234809	855137	445	144863	089946	151	910054	23
38	765367	294	234633	855404	445	144596	090037	151	909963	22
39	765544	293	234456	855671	444	144329	090127	151	909873	21
40	765720	293	234280	855938	444	144062	090218	151	909782	20
41	9 765896	293	10 234104	9 856204	444	10 143796	10 090309	151	9 909691	19
42	766072	293	233928	856471	444	143529	090399	151	909601	18
43	766247	293	233753	856737	444	143263	090490	151	909510	17
44	766423	293	233577	857004	444	142996	090581	151	909419	16
45	766598	292	233402	857270	444	142730	090672	152	909328	15
46	766774	292	233226	857537	444	142463	090763	152	909237	14
47	766949	292	233051	857803	444	142197	090854	152	909146	13
48	767124	292	232876	858069	444	141931	090945	152	909055	12
49	767300	292	232700	858336	444	141664	091036	152	908964	11
50	767475	291	232525	858602	444	141398	091127	152	908873	10
51	9 767649	291	10 232351	9 858868	443	10 141132	10 091219	152	9 908781	9
52	767824	291	232176	859134	443	140866	091310	152	908690	8
53	767999	291	232001	859400	443	140600	091401	152	908599	7
54	768173	291	231827	859666	443	140334	091493	152	908507	6
55	768348	290	231652	859932	443	140068	091584	153	908416	5
56	768522	290	231478	860198	443	139802	091676	153	908324	4
57	768697	290	231303	860464	443	139536	091767	153	908233	3
58	768871	290	231129	860730	443	139270	091859	153	908141	2
59	769045	290	230955	860995	443	139005	091951	153	908049	1
60	769219	290	230781	861261	443	138739	092042	153	907958	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 769219	290	10 230781	9 861261	443	10 138739	10 092042	153	9 907958	60
1	769393	289	230607	861527	443	138473	092134	153	907866	59
2	769566	289	230434	861792	442	138208	092226	153	907774	58
3	769740	289	230260	862058	442	137942	092318	153	907682	57
4	769913	289	230087	862323	442	137677	092410	153	907590	56
5	770087	289	229913	862589	442	137411	092502	153	907498	55
6	770260	288	229740	862854	442	137146	092594	153	907406	54
7	770433	288	229567	863119	442	136881	092686	154	907314	53
8	770606	288	229394	863385	442	136615	092778	154	907222	52
9	770779	288	229221	863650	442	136350	092871	154	907129	51
10	770952	288	229048	863915	442	136085	092963	154	907037	50
11	9 771125	288	10 228875	9 864180	442	10 135820	10 093055	154	9 906945	49
12	771298	287	228702	864445	442	135555	093148	154	906852	48
13	771470	287	228530	864710	442	135290	093240	154	906760	47
14	771643	287	228357	864975	441	135025	093333	154	906667	46
15	771815	287	228185	865240	441	134760	093425	154	906575	45
16	771987	287	228013	865505	441	134495	093518	154	906482	44
17	772159	287	227841	865770	441	134230	093611	155	906389	43
18	772331	286	227669	866035	441	133965	093704	155	906296	42
19	772503	286	227497	866300	441	133700	093796	155	906204	41
20	772675	286	227325	866564	441	133436	093889	155	906111	40
21	9 772847	286	10 227153	9 866829	441	10 133171	10 093982	155	9 906018	39
22	773018	286	226982	867094	441	132906	094075	155	905925	38
23	773190	286	226810	867358	441	132642	094168	155	905832	37
24	773361	285	226639	867623	441	132377	094261	155	905739	36
25	773533	285	226467	867887	441	132113	094355	155	905645	35
26	773704	285	226296	868152	440	131848	094448	155	905552	34
27	773875	285	226125	868416	440	131584	094541	155	905459	33
28	774046	285	225954	868680	440	131320	094635	156	905366	32
29	774217	285	225783	868945	440	131055	094728	156	905272	31
30	774388	284	225612	869209	440	130791	094821	156	905179	30
31	9 774558	284	10 225442	9 869473	440	10 130527	10 094915	156	9 905085	29
32	774729	284	225271	869737	440	130263	095008	156	904992	28
33	774899	284	225101	870001	440	129999	095102	156	904898	27
34	775070	284	224930	870265	440	129735	095196	156	904804	26
35	775240	284	224760	870529	440	129471	095289	156	904711	25
36	775410	284	224590	870793	440	129207	095383	156	904617	24
37	775580	283	224420	871057	440	128943	095477	156	904523	23
38	775750	283	224250	871321	440	128679	095571	157	904429	22
39	775920	283	224080	871585	440	128415	095665	157	904335	21
40	776090	283	223910	871849	439	128151	095759	157	904241	20
41	9 776259	283	10 223741	9 872112	439	10 127888	10 095853	157	9 904147	19
42	776429	282	223571	872376	439	127624	095947	157	904053	18
43	776598	282	223402	872640	439	127360	096041	157	903959	17
44	776768	282	223232	872903	439	127097	096136	157	903864	16
45	776937	282	223063	873167	439	126833	096230	157	903770	15
46	777106	282	222894	873430	439	126570	096324	157	903676	14
47	777275	281	222725	873694	439	126306	096419	157	903581	13
48	777444	281	222556	873957	439	126043	096513	157	903487	12
49	777613	281	222387	874220	439	125780	096608	158	903392	11
50	777781	281	222219	874484	439	125516	096702	158	903298	10
51	9 777950	281	10 222050	9 874747	439	10 125253	10 096797	158	9 903203	9
52	778119	281	221881	875010	439	124990	096892	158	903108	8
53	778287	280	221713	875273	438	124727	096986	158	903014	7
54	778455	280	221545	875536	438	124464	097081	158	902919	6
55	778624	280	221376	875800	438	124200	097176	158	902824	5
56	778792	280	221208	876063	438	123937	097271	158	902729	4
57	778960	280	221040	876326	438	123674	097366	158	902634	3
58	779128	280	220872	876589	438	123411	097461	159	902539	2
59	779295	279	220705	876851	438	123149	097556	159	902444	1
60	779463	279	220537	877114	438	122886	097651	159	902349	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 779463	279	10 220537	9 877114	438	10 122886	10 097651	159	9 902349	60
1	779631	279	220369	877377	438	122623	097747	159	902253	59
2	779798	279	220202	877640	438	122360	097842	159	902158	58
3	779966	279	220034	877903	438	122097	097937	159	902063	57
4	780133	279	219867	878165	438	121835	098033	159	901967	56
5	780300	278	219700	878428	438	121572	098128	159	901872	55
6	780467	278	219533	878691	438	121309	098224	159	901776	54
7	780634	278	219366	878953	437	121047	098319	159	901681	53
8	780801	278	219199	879216	437	120784	098415	159	901585	52
9	780968	278	219032	879478	437	120522	098510	159	901490	51
10	781134	278	218866	879741	437	120259	098606	160	901394	50
11	9 781301	277	10 218699	9 880003	437	10 119997	10 098702	160	9 901298	49
12	781468	277	218532	880265	437	119735	098798	160	901202	48
13	781634	277	218366	880528	437	119472	098894	160	901106	47
14	781800	277	218200	880790	437	119210	098990	160	901010	46
15	781966	277	218034	881052	437	118948	099086	160	900914	45
16	782132	277	217868	881314	437	118686	099182	160	900818	44
17	782298	276	217702	881576	437	118424	099278	160	900722	43
18	782464	276	217536	881839	437	118161	099374	160	900626	42
19	782630	276	217370	882101	437	117899	099471	160	900529	41
20	782796	276	217204	882363	436	117637	099567	161	900433	40
21	9 782961	276	10 217039	9 882625	436	10 117375	10 099663	161	9 900337	39
22	783127	276	216873	882887	436	117113	099760	161	900240	38
23	783292	275	216708	883148	436	116852	099856	161	900144	37
24	783458	275	216542	883410	436	116590	099953	161	900047	36
25	783623	275	216377	883672	436	116328	100049	161	899951	35
26	783788	275	216212	883934	436	116066	100146	161	899854	34
27	783953	275	216047	884196	436	115804	100243	161	899757	33
28	784118	275	215882	884457	436	115543	100340	161	899660	32
29	784282	274	215718	884719	436	115281	100436	161	899564	31
30	784447	274	215553	884980	436	115020	100533	162	899467	30
31	9 784612	274	10 215388	9 885242	436	10 114758	10 100630	162	9 899370	29
32	784776	274	215224	885503	436	114497	100727	162	899273	28
33	784941	274	215059	885765	436	114235	100824	162	899176	27
34	785105	274	214895	886026	436	113974	100922	162	899078	26
35	785269	273	214731	886288	436	113712	101019	162	898981	25
36	785433	273	214567	886549	435	113451	101116	162	898884	24
37	785597	273	214403	886810	435	113190	101213	162	898787	23
38	785761	273	214239	887072	435	112928	101311	162	898689	22
39	785925	273	214075	887333	435	112667	101408	162	898592	21
40	786089	273	213911	887594	435	112406	101506	163	898494	20
41	9 786252	272	10 213748	9 887855	435	10 112145	10 101603	163	9 898397	19
42	786416	272	213584	888116	435	111884	101701	163	898299	18
43	786579	272	213421	888377	435	111623	101798	163	898202	17
44	786742	272	213258	888639	435	111361	101896	163	898104	16
45	786906	272	213094	888900	435	111100	101994	163	898006	15
46	787069	272	212931	889160	435	110840	102092	163	897908	14
47	787232	271	212768	889421	435	110579	102190	163	897810	13
48	787395	271	212605	889682	435	110318	102288	163	897712	12
49	787557	271	212443	889943	435	110057	102386	163	897614	11
50	787720	271	212280	890204	434	109796	102484	163	897516	10
51	9 787883	271	10 212117	9 890465	434	10 109535	10 102582	164	9 897418	9
52	788045	271	211955	890725	434	109275	102680	164	897320	8
53	788208	271	211792	890986	434	109014	102778	164	897222	7
54	788370	270	211630	891247	434	108753	102877	164	897123	6
55	788532	270	211468	891507	434	108493	102975	164	897025	5
56	788694	270	211306	891768	434	108232	103074	164	896926	4
57	788856	270	211144	892028	434	107972	103172	164	896828	3
58	789018	270	210982	892289	434	107711	103271	164	896729	2
59	789180	270	210820	892549	434	107451	103369	164	896631	1
60	789342	269	210658	892810	434	107190	103468	164	896532	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 789342	269	10.210658	9 892810	434	10.107190	10.103468	164	9 896532	60
1	789504	269	210496	893070	434	106930	103567	165	896433	59
2	789665	269	210335	893331	434	106669	103665	165	896335	58
3	789827	269	210173	893591	434	106409	103764	165	896236	57
4	789988	269	210012	893851	434	106149	103863	165	896137	56
5	790149	269	209851	894111	434	105889	103962	165	896038	55
6	790310	268	209690	894371	434	105629	104061	165	895939	54
7	790471	268	209529	894632	433	105368	104160	165	895840	53
8	790632	268	209368	894892	433	105108	104259	165	895741	52
9	790793	268	209207	895152	433	104848	104359	165	895641	51
10	790954	268	209046	895412	433	104588	104458	165	895542	50
11	9 791115	268	10.208885	9 895672	433	10 104328	10.104557	166	9 895443	49
12	791275	267	208725	895932	433	104068	104657	166	895343	48
13	791436	267	208564	896192	433	103808	104756	166	895244	47
14	791596	267	208404	896452	433	103548	104855	166	895145	46
15	791757	267	208243	896712	433	103288	104955	166	895045	45
16	791917	267	208083	896971	433	103029	105055	166	894945	44
17	792077	267	207923	897231	433	102769	105154	166	894846	43
18	792237	266	207763	897491	433	102509	105254	166	894746	42
19	792397	266	207603	897751	433	102249	105354	166	894646	41
20	792557	266	207443	898010	433	101990	105454	166	894546	40
21	9 792716	266	10 207284	9 898270	433	10 101730	10 105554	167	9 894446	39
22	792876	266	207124	898530	433	101470	105654	167	894346	38
23	793035	266	206965	898789	433	101211	105754	167	894246	37
24	793195	265	206805	899049	432	100951	105854	167	894146	36
25	793354	265	206646	899308	432	100692	105954	167	894046	35
26	793514	265	206486	899568	432	100432	106054	167	893946	34
27	793673	265	206327	899827	432	100173	106154	167	893846	33
28	793832	265	206168	900086	432	099914	106255	167	893745	32
29	793991	265	206009	900346	432	099654	106355	167	893645	31
30	794150	264	205850	900605	432	099395	106456	167	893544	30
31	9 794308	264	10 205692	9 900864	432	10 099136	10 106556	168	9 893444	29
32	794467	264	205533	901124	432	098876	106657	168	893343	28
33	794626	264	205374	901383	432	098617	106757	168	893243	27
34	794784	264	205216	90164	432	098358	106858	168	893142	26
35	794942	264	205058	901901	432	098099	106959	168	893041	25
36	795101	264	204899	902160	432	097840	107060	168	892940	24
37	795259	263	204741	902419	432	097581	107161	168	892839	23
38	795417	263	204583	902679	432	097321	107261	168	892739	22
39	795575	263	204425	902938	432	097062	107362	168	892638	21
40	795733	263	204267	903197	431	096803	107464	168	892536	20
41	9 795891	263	10.204109	9 903455	431	10.096545	10 107565	169	9 892435	19
42	796049	263	203951	903714	431	096286	107666	169	892334	18
43	796206	263	203794	903973	431	096027	107767	169	892233	17
44	796364	262	203636	904232	431	095768	107868	169	892132	16
45	796521	262	203479	904491	431	095509	107970	169	892030	15
46	796679	262	203321	904750	431	095250	108071	169	891929	14
47	796836	262	203164	905008	431	094992	108173	169	891827	13
48	796993	262	203007	905267	431	094733	108274	169	891726	12
49	797150	261	202850	905526	431	094474	108376	169	891624	11
50	797307	261	202693	905784	431	094216	108477	170	891523	10
51	9 797464	261	10 202536	9 906043	431	10 093957	10 108579	170	9 891421	9
52	797621	261	202379	906302	431	093698	108681	170	891319	8
53	797777	261	202223	906560	431	093440	108783	170	891217	7
54	797934	261	202066	906819	431	093181	108885	170	891115	6
55	798091	261	201909	907077	431	092923	108987	170	891013	5
56	798247	261	201753	907336	431	092664	109089	170	890911	4
57	798403	260	201597	907594	431	092406	109191	170	890809	3
58	798560	260	201440	907852	431	092148	109293	170	890707	2
59	798716	260	201284	908111	430	091889	109395	170	890605	1
60	798872	260	201128	908369	430	091631	109497	170	890503	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang.	D	Cotang	Secant	D	Cosine	
0	9 798872	260	10.201128	9 908369	430	10 091631	10 109497	170	9 890503	60
1	799028	260	200972	908628	430	091372	109600	171	890400	59
2	799184	260	200816	908886	430	091114	109702	171	890298	58
3	799339	259	200661	909144	430	090856	109805	171	890195	57
4	799495	259	200505	909402	430	090598	109907	171	890093	56
5	799651	259	200349	909660	430	090340	110010	171	889990	55
6	799806	259	200194	909918	430	09008[illegible]	110112	171	889888	54
7	799962	259	200038	910177	430	089823	110215	171	889785	53
8	800117	259	199883	910435	430	089565	110318	171	889682	52
9	800272	258	199728	910693	430	089307	110421	171	889579	51
10	800427	258	199573	910951	430	089049	110523	171	889477	50
11	9 80058	258	10 199418	9 911209	430	10 088791	10.110626	172	9 889374	49
12	800737	258	199263	911467	430	088533	110729	172	889271	48
13	800892	258	199108	911724	430	088276	110832	172	889168	47
14	801047	258	198953	911982	430	088018	110936	172	889064	46
15	801201	258	198799	912240	430	087760	111039	172	888961	45
16	801356	257	198644	912498	430	087502	111142	172	888858	44
17	801511	257	198189	912756	430	087244	111245	172	888755	43
18	801665	257	198335	913014	429	086986	111349	172	888651	42
19	801819	257	198181	913271	429	086729	111452	172	888548	41
20	801973	257	198027	913529	429	086471	111556	173	888444	40
21	9.802128	257	10 197872	9 913787	429	10 086213	10 111659	173	9 888341	39
22	802282	256	197718	914044	429	085956	111763	173	888237	38
23	802436	256	197564	914302	429	085698	111866	173	888134	37
24	802589	256	197411	914560	429	085440	111970	173	888030	36
25	802743	256	197257	914817	429	085183	112074	173	887926	35
26	802897	256	197103	915075	429	084925	112178	173	887822	34
27	803050	256	196950	915332	429	084668	[illegible]	173	887718	33
28	803204	256	196796	915590	429	084410	112386	173	887614	32
29	803357	255	196643	915847	429	084153	112490	173	887510	31
30	803511	255	196189	916104	429	083896	112594	174	887406	30
31	9 803664	255	10 196336	9 916362	429	10 083638	10 112698	174	9 887302	29
32	803817	255	196183	916619	429	083381	112802	174	887198	28
33	803970	255	196030	916877	429	083123	112907	174	887093	27
34	804123	255	195877	917134	429	082866	113011	174	886989	26
35	804276	254	195724	917391	429	082609	113115	174	886885	25
36	804428	254	195572	917648	429	082352	113220	174	886780	24
37	804581	254	195419	917905	429	082095	113324	174	886676	23
38	804734	254	195266	918163	428	081837	113429	174	886571	22
39	804886	254	195114	918420	428	081580	113534	174	886466	21
40	805039	254	194961	918677	428	081323	113638	175	886362	20
41	9 805191	254	10 194809	9 918934	428	10 081066	10 113743	175	9 886257	19
42	805343	253	194657	919191	428	080809	113848	175	886152	18
43	805495	253	194505	919448	428	080552	113953	175	886017	17
44	805647	253	194353	919705	428	080295	114058	175	885942	16
45	805799	253	194201	919962	428	080038	114163	175	885837	15
46	805951	253	194049	920219	428	079781	114268	175	885732	14
47	806103	253	193897	920476	428	079524	114373	175	885627	13
48	806254	253	193746	920733	428	079267	114478	175	885522	12
49	806406	252	193594	920990	428	079010	114584	175	885416	11
50	806557	252	193443	921247	428	078753	114689	176	885311	10
51	9 806709	252	10 193291	9 921503	428	10 078497	10 114795	176	9 885205	9
52	806860	252	193140	921760	428	078240	114900	176	885100	8
53	807011	252	192989	922017	428	077983	115006	176	884994	7
54	807163	252	192837	922274	428	077726	115111	176	884889	6
55	807314	252	192686	922530	428	077470	115217	176	884783	5
56	807465	251	192535	922787	428	077213	115323	176	884677	4
57	807615	251	192385	923044	428	076956	115428	176	884572	3
58	807766	251	192234	923300	428	076700	115534	176	884466	2
59	807917	251	192083	923557	427	076443	115640	176	884360	1
60	808067	251	191933	923813	427	076187	115746	177	884254	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 808067	251	10 191933	9 923813	427	10.076187	10 115746	177	9 884254	60
1	808218	251	191782	924070	427	075930	115852	177	884148	59
2	808368	251	191632	924327	427	075673	115958	177	884042	58
3	808519	250	191481	924583	427	075417	116064	177	883936	57
4	808669	250	191331	924840	427	075160	116171	177	883829	56
5	808819	250	191181	925096	427	074904	116277	177	883723	55
6	808969	250	191031	925352	427	074648	116383	177	883617	54
7	809119	250	190881	925609	427	074391	116490	177	883510	53
8	809269	250	190731	925865	427	074135	116596	177	883404	52
9	809419	249	190581	926122	427	073878	116703	178	883297	51
10	809569	249	190431	926378	427	073622	116809	178	883191	50
11	9 809718	249	10 190282	9 926634	427	10 073366	10 116916	178	9 883084	49
12	809868	249	190132	926890	427	073110	117023	178	882977	48
13	810017	249	189983	927147	427	072853	117129	178	882871	47
14	810167	249	189833	927403	427	072597	117236	178	882764	46
15	810316	248	189684	927659	427	072341	117343	178	882657	45
16	810465	248	189535	927915	427	072085	117450	178	882550	44
17	810614	248	189386	928171	427	071829	117557	178	882443	43
18	810763	248	189237	928427	427	071573	117664	179	882336	42
19	810912	248	189088	928683	427	071317	117771	179	882229	41
20	811061	248	188939	928940	427	071060	117879	179	882121	40
21	9 811210	248	10 188790	9 929196	427	10 070804	10.117986	179	9 882014	39
22	811358	247	188642	929452	427	070548	118093	179	881907	38
23	811507	247	188493	929708	427	070292	118201	179	881799	37
24	811655	247	188345	929964	426	070036	118308	179	881692	36
25	811804	247	188196	930220	426	069780	118416	179	881584	35
26	811952	247	188048	930475	426	069525	118523	179	881477	34
27	812100	247	187900	930731	426	069269	118631	179	881369	33
28	812248	247	187752	930987	426	069013	118739	180	881261	32
29	812396	246	187604	931243	426	068757	118847	180	881153	31
30	812544	246	187456	931499	426	068501	118954	180	881046	30
31	9 812692	246	10 187308	9 931755	426	10 068245	10 119062	180	9 880938	29
32	812840	246	187160	932010	426	067990	119170	180	880830	28
33	812988	246	187012	932266	426	067734	119278	180	880722	27
34	813135	246	186865	932522	426	067478	119387	180	880613	26
35	813283	246	186717	932778	426	067222	119495	180	880505	25
36	813430	245	186570	933033	426	066967	119603	180	880397	24
37	813578	245	186422	933289	426	066711	119711	181	880289	23
38	813725	245	186275	933545	426	066455	119820	181	880180	22
39	813872	245	186128	933800	426	066200	119928	181	880072	21
40	814019	245	185981	934056	426	065944	120037	181	879963	20
41	9 814166	245	10 185834	9 934311	426	10 065689	10 120145	181	9 879855	19
42	814313	245	185687	934567	426	065433	120254	181	879746	18
43	814460	244	185540	934823	426	065177	120363	181	879637	17
44	814607	244	185393	935078	426	064922	120471	181	879529	16
45	814753	244	185247	935333	426	064667	120580	181	879420	15
46	814900	244	185100	935589	426	064411	120689	181	879311	14
47	815046	244	184954	935844	426	064156	120798	182	879202	13
48	815193	244	184807	936100	426	063900	120907	182	879093	12
49	815339	244	184661	936355	426	063645	121016	182	878984	11
50	815485	243	184515	936610	426	063390	121125	182	878875	10
51	9 815631	243	10 184368	9 936866	425	10 063134	10 121234	182	9 878766	9
52	815778	243	184222	937121	425	062879	121344	182	878656	8
53	815924	243	184076	937376	425	062624	121453	182	878547	7
54	816069	243	183931	937632	425	062368	121562	182	878438	6
55	816215	243	183785	937887	425	062113	121672	182	878328	5
56	816361	243	183639	938142	425	061858	121781	183	878219	4
57	816507	242	183493	938398	425	061602	121891	183	878109	3
58	816652	242	183348	938653	425	061347	122001	183	877999	2
59	816798	242	183202	938908	425	061092	122110	183	877890	1
60	816943	242	183057	939163	425	060837	122220	183	877780	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9.816943	242	10.183057	9.939163	425	10.060837	10.122220	183	9.877780	60
1	817088	242	182912	939418	425	060582	122330	183	877670	59
2	817233	242	182767	939673	425	060327	122440	183	877560	58
3	817379	242	182621	939928	425	060072	122550	183	877450	57
4	817524	241	182476	940183	425	059817	122660	183	877340	56
5	817668	241	182332	940438	425	059562	122770	184	877230	55
6	817813	241	182187	940694	425	059306	122880	184	877120	54
7	817958	241	182042	940949	425	059051	122990	184	877010	53
8	818103	241	181897	941204	425	058796	123101	184	876899	52
9	818247	241	181753	941458	425	058542	123211	184	876789	51
10	818392	241	181608	941714	425	058286	123322	184	876678	50
11	9.818536	240	10.181464	9.941968	425	10.058032	10.123432	184	9.876568	49
12	818681	240	181319	942223	425	057777	123543	184	876457	48
13	818825	240	181175	942478	425	057522	123653	184	876347	47
14	818969	240	181031	942733	425	057267	123764	185	876236	46
15	819113	240	180887	942988	425	057012	123875	185	876125	45
16	819257	240	180743	943243	425	056757	123986	185	876014	44
17	819401	240	180599	943498	425	056502	124096	185	875904	43
18	819545	239	180455	943752	425	056248	124207	185	875793	42
19	819689	239	180311	944007	425	055993	124318	185	875682	41
20	819832	239	180168	944262	425	055738	124429	185	875571	40
21	9.819976	239	10.180024	9.944517	425	10.055483	10.124541	185	9.875459	39
22	820120	239	179880	944771	424	055229	124652	185	875348	38
23	820263	239	179737	945026	424	054974	124763	185	875237	37
24	820406	239	179594	945281	424	054719	124874	186	875126	36
25	820550	238	179450	945535	424	054465	124986	186	875014	35
26	820693	238	179307	945790	424	054210	125097	186	874903	34
27	820836	238	179164	946045	424	053955	125209	186	874791	33
28	820979	238	179021	946299	424	053701	125320	186	874680	32
29	821122	238	178878	946554	424	053446	125432	186	874568	31
30	821265	238	178735	946808	424	053192	125544	186	874456	30
31	9.821407	238	10.178593	9.947063	424	10.052937	10.125656	186	9.874344	29
32	821550	238	178450	947318	424	052682	125768	187	874232	28
33	821693	237	178307	947572	424	052428	125879	187	874121	27
34	821835	237	178165	947826	424	052174	125991	187	874009	26
35	821977	237	178023	948081	424	051919	126104	187	873896	25
36	822120	237	177880	948336	424	051664	126216	187	873784	24
37	822262	237	177738	948590	424	051410	126328	187	873672	23
38	822404	237	177596	948844	424	051156	126440	187	873560	22
39	822546	237	177454	949099	424	050901	126552	187	873448	21
40	822688	236	177312	949353	424	050647	126665	187	873335	20
41	9.822830	236	10.177170	9.949607	424	10.050393	10.126777	187	9.873223	19
42	822972	236	177028	949862	424	050138	126890	188	873110	18
43	823114	236	176886	950116	424	049884	127002	188	872998	17
44	823255	236	176745	950370	424	049630	127115	188	872885	16
45	823397	236	176603	950625	424	049375	127228	188	872772	15
46	823539	236	176461	950879	424	049121	127341	188	872659	14
47	823680	235	176320	951133	424	048867	127453	188	872547	13
48	823821	235	176179	951388	424	048612	127566	188	872434	12
49	823963	235	176037	951642	424	048358	127679	188	872321	11
50	824104	235	175896	951896	424	048104	127792	188	872208	10
51	9.824245	235	10.175755	9.952150	424	10.047850	10.127905	189	9.872095	9
52	824386	235	175614	952405	424	047595	128019	189	871981	8
53	824527	235	175473	952659	424	047341	128132	189	871868	7
54	824668	234	175332	952913	424	047087	128245	189	871755	6
55	824808	234	175192	953167	423	046833	128359	189	871641	5
56	824949	234	175051	953421	423	046579	128472	189	871528	4
57	825090	234	174910	953675	423	046325	128586	189	871414	3
58	825230	234	174770	953929	423	046071	128699	189	871301	2
59	825371	234	174629	954183	423	045817	128813	189	871187	1
60	825511	234	174489	954437	423	045563	128927	190	871073	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9.825511	233	10.174489	9.954437	423	10.045563	10.128927	190	9.871073	60
1	825651	233	174349	954691	423	045309	129040	190	870960	59
2	825791	233	174209	954945	423	045055	129154	190	870846	58
3	825931	233	174069	955200	423	044800	129268	190	870732	57
4	826071	233	173929	955454	423	044546	129382	190	870618	56
5	826211	233	173789	955707	423	044293	129496	190	870504	55
6	826351	233	173649	955961	423	044039	129610	190	870390	54
7	826491	233	173509	956215	423	043785	129724	190	870276	53
8	826631	233	173369	956469	423	043531	129839	190	870161	52
9	826770	232	173230	956723	423	043277	129953	191	870047	51
10	826910	232	173090	956977	423	043023	130067	191	869933	50
11	9.827049	232	10.172951	9.957231	423	10.042769	10.130182	191	9.869818	49
12	827189	232	172811	957485	423	042515	130296	191	869704	48
13	827328	232	172672	957739	423	042261	130411	191	869589	47
14	827467	232	172533	957993	423	042007	130526	191	869474	46
15	827606	232	172394	958246	423	041754	130640	191	869360	45
16	827745	232	172255	958500	423	041500	130755	191	869245	44
17	827884	231	172116	958754	423	041246	130870	191	869130	43
18	828023	231	171977	959008	423	040992	130985	192	869015	42
19	828162	231	171838	959262	423	040738	131100	192	868900	41
20	828301	231	171699	959516	423	040484	131215	192	868785	40
21	9.828439	231	10.171561	9.959769	423	10.040231	10.131330	192	9.868670	39
22	828578	231	171422	960023	423	039977	131445	192	868555	38
23	828716	231	171284	960277	423	039723	131560	192	868440	37
24	828855	230	171145	960531	423	039469	131676	192	868324	36
25	828993	230	171007	960784	423	039216	131791	192	868209	35
26	829131	230	170869	961038	423	038962	131907	192	868093	34
27	829269	230	170731	961291	423	038709	132022	192	867978	33
28	829407	230	170593	961545	423	038455	132138	193	867862	32
29	829545	230	170455	961799	423	038201	132254	193	867747	31
30	829683	230	170317	962052	423	037948	132369	193	867631	30
31	9.829821	229	10.170179	9.962306	423	10.037694	10.132485	193	9.867515	29
32	829959	229	170041	962560	423	037440	132601	193	867399	28
33	830097	229	169903	962813	423	037187	132717	193	867283	27
34	830234	229	169766	963067	423	036933	132833	193	867167	26
35	830372	229	169628	963320	423	036680	132949	193	867051	25
36	830509	229	169491	963574	423	036426	133065	194	866935	24
37	830646	229	169354	963827	423	036173	133181	194	866819	23
38	830784	229	169216	964081	423	035919	133297	194	866703	22
39	830921	228	169079	964335	423	035665	133414	194	866586	21
40	831058	228	168942	964588	422	035412	133530	194	866470	20
41	9.831195	228	10.168805	9.964842	422	10.035158	10.133647	194	9.866353	19
42	831332	228	168668	965095	422	034905	133763	194	866237	18
43	831469	228	168531	965349	422	034651	133880	194	866120	17
44	831606	228	168394	965602	422	034398	133996	195	866004	16
45	831742	228	168258	965855	422	034145	134113	195	865887	15
46	831879	228	168121	966109	422	033891	134230	195	865770	14
47	832015	227	167985	966362	422	033638	134347	195	865653	13
48	832152	227	167848	966616	422	033384	134464	195	865536	12
49	832288	227	167712	966869	422	033131	134581	195	865419	11
50	832425	227	167575	967123	422	032877	134698	195	865302	10
51	9.832561	227	10.167439	9.967376	422	10.032624	10.134815	195	9.865185	9
52	832697	227	167303	967629	422	032371	134932	195	865068	8
53	832833	227	167167	967883	422	032117	135050	195	864950	7
54	832969	226	167031	968136	422	031864	135167	196	864833	6
55	833105	226	166895	968389	422	031611	135284	196	864716	5
56	833241	226	166759	968643	422	031357	135402	196	864598	4
57	833377	226	166623	968896	422	031104	135519	196	864481	3
58	833512	226	166488	969149	422	030851	135637	196	864363	2
59	833648	226	166352	969403	422	030597	135755	196	864245	1
60	833783	226	166217	969656	422	030344	135873	196	864127	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

47 Degrees.

M	Sine	D	Cosec.	Tang.	D	Cotang	Secant	D	Cosine	
0	9 833783	226	10.166217	9 969656	422	10 030344	10.135873	196	9 864127	60
1	833919	225	166081	969909	422	030091	135990	196	864010	59
2	834054	225	165946	970162	422	029838	136108	197	863892	58
3	834189	225	165811	970416	422	029584	136226	197	863774	57
4	834325	225	165675	970669	422	029331	136344	197	863656	56
5	834460	225	165540	970922	422	029078	136462	197	863538	55
6	834595	225	165405	971175	422	028825	136581	197	863419	54
7	834730	225	165270	971429	422	028571	136699	197	863301	53
8	834865	225	165135	971682	422	028318	136817	197	863183	52
9	834999	224	165001	971935	422	028065	136936	197	863064	51
10	835134	224	164866	972188	422	027812	137054	198	862946	50
11	9 835269	224	10.164731	9 972441	422	10 027559	10.137173	198	9 862827	49
12	835403	224	164597	972694	422	027306	137291	198	862709	48
13	835538	224	164462	972948	422	027052	137410	198	862590	47
14	835672	224	164328	973201	422	026799	137529	198	862471	46
15	835807	224	164193	973454	422	026546	137647	198	862353	45
16	835941	224	164059	973707	422	026293	137766	198	862234	44
17	836075	223	163925	973960	422	026040	137885	198	862115	43
18	836209	223	163791	974213	422	025787	138004	198	861996	42
19	836343	223	163657	974466	422	025534	138123	198	861877	41
20	836477	223	163523	974719	422	025281	138242	199	861758	40
21	9.836611	223	10 163389	9 974973	422	10 025027	10 138362	199	9 861638	39
22	836745	223	163255	975226	422	024774	138481	199	861519	38
23	836878	223	163122	975479	422	024521	138600	199	861400	37
24	837012	222	162988	975732	422	024268	138720	199	861280	36
25	837146	222	162854	975985	422	024015	138839	199	861161	35
26	837279	222	162721	976238	422	023762	138959	199	861041	34
27	837412	222	162588	976491	422	023509	139078	199	860922	33
28	837546	222	162454	976744	422	023256	139198	199	860802	32
29	837679	222	162321	976997	422	023003	139318	200	860682	31
30	837812	222	162188	977250	422	022750	139438	200	860562	30
31	9 837945	222	10 162055	9 977503	422	10 022497	10 139558	200	9 860442	29
32	838078	221	161922	977756	422	022244	139678	200	860322	28
33	838211	221	161789	978009	422	021991	139798	200	860202	27
34	838344	221	161656	978262	422	021738	139918	200	860082	26
35	838477	221	161523	978515	422	021485	140038	200	859962	25
36	838610	221	161390	978768	422	021232	140158	200	859842	24
37	838742	221	161258	979021	422	020979	140279	201	859721	23
38	838875	221	161125	979274	422	020726	140399	201	859601	22
39	839007	221	160993	979527	422	020476	140520	201	859480	21
40	839140	220	160860	979780	422	020220	140640	201	859360	20
41	9 839272	220	10 160728	9 980033	422	10.019967	10 140761	201	9 859239	19
42	839404	220	160596	980286	422	019714	140881	201	859119	18
43	839536	220	160464	980538	422	019462	141002	201	858998	17
44	839668	220	160332	980791	421	019209	141123	201	858877	16
45	839800	220	160200	981044	421	018956	141244	202	858756	15
46	839932	220	160068	981297	421	018703	141365	202	858635	14
47	840064	219	159936	981550	421	018450	141486	202	858514	13
48	840196	219	159804	981803	421	018197	141607	202	858393	12
49	840328	219	159672	982056	421	017914	141728	202	858272	11
50	840459	219	159541	982309	421	017691	141849	202	858151	10
51	9 840591	219	10 159409	9 982562	421	10 017438	10 141971	202	9 858029	9
52	840722	219	159278	982814	421	017186	142092	202	857908	8
53	840854	219	159146	983067	421	016933	142214	202	857786	7
54	840985	219	159015	983320	421	016680	142335	203	857665	6
55	841116	218	158884	983573	421	016427	142457	203	857543	5
56	841247	218	158753	983826	421	016174	142578	203	857422	4
57	841378	218	158622	984079	421	015921	142700	203	857300	3
58	841509	218	158491	984331	421	015669	142822	203	857178	2
59	841640	218	158360	984584	421	015416	142944	203	857056	1
60	841771	218	158229	984837	421	015163	143066	203	856934	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

46 Degrees.

M	Sine	D	Cosec	Tang	D	Cotang	Secant	D	Cosine	
0	9 841771	218	10 158229	9 984837	421	10 015163	10 143066	203	9 856934	60
1	841902	218	158098	985090	421	014910	143188	203	856812	59
2	842033	218	157967	985343	421	014657	143310	204	856690	58
3	842163	217	157837	985596	421	014404	143432	204	856568	57
4	842294	217	157706	985848	421	014152	143554	204	856446	56
5	842424	217	157576	986101	421	013899	143677	204	856323	55
6	842555	217	157445	986354	421	013646	143799	204	856201	54
7	842685	217	157315	986607	421	013393	143922	204	856078	53
8	842815	217	157185	986860	421	013140	144044	204	855956	52
9	842946	217	157054	987112	421	012888	144167	204	855833	51
10	843076	217	156924	987365	421	012635	144289	205	855711	50
11	9 843206	216	10 156794	9 987618	421	10 012382	10 144412	205	9 855588	49
12	843336	216	156664	987871	421	012129	144535	205	855465	48
13	843466	216	156534	988123	421	011877	144658	205	855342	47
14	843595	216	156405	988376	421	011624	144781	205	855219	46
15	843725	216	156275	988629	421	011371	144904	205	855096	45
16	843855	216	156145	988882	421	011118	145027	205	854973	44
17	843984	216	156016	989134	421	010866	145150	205	854850	43
18	844114	215	155886	989387	421	010613	145273	206	854727	42
19	844243	215	155757	989640	421	010360	145397	206	854603	41
20	844372	215	155628	989893	421	010107	145520	206	854480	40
21	9 844502	215	10 155498	9 990145	421	10 009855	10 145644	206	9 854356	39
22	844631	215	155369	990398	421	009602	145767	206	854233	38
23	844760	215	155240	990651	421	009349	145891	206	854109	37
24	844889	215	155111	990903	421	009097	146014	206	853986	36
25	845018	215	154982	991156	421	008844	146138	206	853862	35
26	845147	215	154853	991409	421	008591	146262	206	853738	34
27	845276	214	154724	991662	421	008338	146386	207	853614	33
28	845405	214	154595	991914	421	008086	146510	207	853490	32
29	845533	214	154467	992167	421	007833	146634	207	853366	31
30	845662	214	154338	992420	421	007580	146758	207	853242	30
31	9 845790	214	10 154210	9 992672	421	10 007328	10 146882	207	9 853118	29
32	845919	214	154081	992925	421	007075	147006	207	852994	28
33	846047	214	153953	993178	421	006822	147131	207	852869	27
34	846175	214	153825	993430	421	006570	147255	207	852745	26
35	846304	214	153696	993683	421	006317	147380	207	852620	25
36	846432	213	153568	993936	421	006064	147504	208	852496	24
37	846560	213	153440	994189	421	005811	147629	208	852371	23
38	846688	213	153312	994441	421	005559	147753	208	852247	22
39	846816	213	153184	994694	421	005306	147878	208	852122	21
40	846944	213	153056	994947	421	005053	148003	208	851997	20
41	9 847071	213	10 152929	9 995199	421	10 004801	10 148128	208	9 851872	19
42	847199	213	152801	995452	421	004548	148253	208	851747	18
43	847327	213	152673	995705	421	004295	148378	208	851622	17
44	847454	212	152546	995957	421	004043	148503	209	851497	16
45	847582	212	152418	996210	421	003790	148628	209	851372	15
46	847709	212	152291	996463	421	003537	148754	209	851246	14
47	847836	212	152164	996715	421	003285	148879	209	851121	13
48	847964	212	152036	996968	421	003032	149004	209	850996	12
49	848091	212	151909	997221	421	002779	149130	209	850870	11
50	848218	212	151782	997473	421	002527	149255	209	850745	10
51	9 848345	212	10 151655	9 997726	421	10 002274	10 149381	209	9 850619	9
52	848472	211	151528	997979	421	002021	149507	210	850493	8
53	848599	211	151401	998231	421	001769	149632	210	850368	7
54	848726	211	151274	998484	421	001516	149758	210	850242	6
55	848852	211	151148	998737	421	001263	149884	210	850116	5
56	848979	211	151021	998989	421	001011	150010	210	849990	4
57	849106	211	150894	999242	421	000758	150136	210	849864	3
58	849232	211	150768	999495	421	000505	150263	210	849738	2
59	849359	211	150641	999748	421	000253	150389	210	849611	1
60	849485	211	150515	10 000000	421	000000	150515	210	849485	0
	Cosine		Secant	Cotang		Tang	Cosec		Sine	M

45 Degrees.

TABLE OF LOGARITHMIC SINES, TANGENTS, AND SECANTS, TO EVERY POINT AND QUARTER POINT OF THE COMPASS.

Points.	Sine.	Cosine	Tangent.	Cotang	Secant.	Cosec	Points.
0	0 000000	10 000000	0 000000	Infinite	10 000000	Infinite	8
0 ¼	8 690796	9 999477	8 691319	11 308681	10 000523	11 309204	7 ¾
0 ½	8 991302	9 997904	9 993398	11 006602	10 002096	11 008698	7 ½
0 ¾	9 166520	9 995274	9 171247	10 828753	10 004726	10 833480	7 ¼
1	9 290236	9 991574	9 298662	10 701338	10 008426	10 709764	7
1 ¼	9 385571	9 986786	9 398785	10 601215	10 013214	10 614429	6 ¾
1 ½	9 467821	9 980885	9 481939	10 518061	10 019115	10 537176	6 ½
1 ¾	9 527488	9 973841	9 553647	10 416353	10 026159	10 472512	6 ¼
2	9 582840	9 965615	9 617224	10 382776	10 034385	10 417160	6
2 ¼	9 630992	9 956163	9 671829	10 325171	10 043837	10 369008	5 ¾
2 ½	9 673387	9 945130	9 727957	10 272043	10 054570	10 326613	5 ½
2 ¾	9 711050	9 933350	9 777700	10 222300	10 066650	10 288950	5 ¼
3	9 744739	9 919846	9 824893	10 175107	10 080154	10 255261	5
3 ¼	9 775027	9 904828	9 870199	10 129801	10 095172	10 224973	4 ¾
3 ½	9 802359	9 888185	9 914173	10 085827	10 111815	10 197641	4 ½
3 ¾	9 827084	9 869790	9 957295	10 042705	10 130210	10 172916	4 ¼
4	9 849485	9 849485	10 000000	10 000000	10 150515	10 150515	4
	Cosine	Sine	Cotang	Tangent	Cosec	Secant	

A

TABLE

OF

NATURAL SINES.

Min	0 Deg N sin	0 Deg N cos	1 Deg N sin	1 Deg N cos	2 Deg N sin	2 Deg N cos	3 Deg N sin	3 Deg N cos	4 Deg N sin	4 Deg N cos	
0	0	100000	1745	99985	3490	99939	5234	99863	6976	99756	60
1	29	100000	1774	99984	3519	99938	5263	99861	7005	99754	59
2	58	100000	1803	99984	3548	99937	5292	99860	7034	99752	58
3	87	100000	1832	99983	3577	99936	5321	99858	7063	99750	57
4	116	100000	1862	99983	3606	99935	5350	99857	7092	99748	56
5	145	100000	1891	99982	3635	99934	5379	99855	7121	99746	55
6	175	100000	1920	99982	3664	99933	5408	99854	7150	99744	54
7	204	100000	1949	99981	3693	99932	5437	99852	7179	99742	53
8	233	100000	1978	99980	3723	99931	5466	99851	7208	99740	52
9	262	100000	2007	99980	3752	99930	5495	99849	7237	99738	51
10	291	100000	2036	99979	3781	99929	5524	99847	7266	99736	50
11	320	99999	2065	99979	3810	99927	5553	99846	7295	99734	49
12	349	99999	2094	99978	3839	99926	5582	99844	7324	99731	48
13	378	99999	2123	99977	3868	99925	5611	99842	7353	99729	47
14	407	99999	2152	99977	3897	99924	5640	99841	7382	99727	46
15	436	99999	2181	99976	3926	99923	5669	99839	7411	99725	45
16	465	99999	2211	99976	3955	99922	5698	99838	7440	99723	44
17	495	99999	2240	99975	3984	99921	5727	99836	7469	99721	43
18	524	99999	2269	99974	4013	99920	5756	99834	7498	99719	42
19	553	99998	2298	99974	4042	99918	5785	99833	7527	99716	41
20	582	99998	2327	99973	4071	99917	5814	99831	7556	99714	40
21	611	99998	2356	99972	4100	99916	5844	99829	7585	99712	39
22	640	99998	2385	99972	4129	99915	5873	99827	7614	99710	38
23	669	99998	2414	99971	4158	99913	5902	99826	7643	99708	37
24	698	99998	2443	99970	4188	99912	5931	99824	7672	99705	36
25	727	99997	2472	99969	4217	99911	5960	99822	7701	99703	35
26	756	99997	2501	99969	4246	99910	5989	99821	7730	99701	34
27	785	99997	2530	99968	4275	99909	6018	99819	7759	99699	33
28	814	99997	2560	99967	4304	99907	6047	99817	7788	99696	32
29	844	99996	2589	99966	4333	99906	6076	99815	7817	99694	31
30	873	99996	2618	99966	4362	99905	6105	99813	7846	99692	30
31	902	99996	2647	99965	4391	99904	6134	99812	7875	99689	29
32	931	99996	2676	99964	4420	99902	6163	99810	7904	99687	28
33	960	99995	2705	99963	4449	99901	6192	99808	7933	99685	27
34	989	99995	2734	99963	4478	99900	6221	99806	7962	99683	26
35	1018	99995	2763	99962	4507	99898	6250	99804	7991	99680	25
36	1047	99994	2792	99961	4536	99897	6279	99803	8020	99678	24
37	1076	99994	2821	99960	4565	99896	6308	99801	8049	99676	23
38	1105	99994	2850	99959	4594	99894	6337	99799	8078	99673	22
39	1134	99994	2879	99959	4623	99893	6366	99797	8107	99671	21
40	1164	99993	2908	99958	4653	99892	6395	99795	8136	99668	20
41	1193	99993	2938	99957	4682	99890	6424	99793	8165	99666	19
42	1222	99993	2967	99956	4711	99889	6453	99792	8194	99664	18
43	1251	99992	2996	99955	4740	99888	6482	99790	8223	99661	17
44	1280	99992	3025	99954	4769	99886	6511	99788	8252	99659	16
45	1309	99991	3054	99953	4798	99885	6540	99786	8281	99657	15
46	1338	99991	3083	99952	4827	99883	6569	99784	8310	99654	14
47	1367	99991	3112	99952	4856	99882	6598	99782	8339	99652	13
48	1396	99990	3141	99951	4885	99881	6627	99780	8368	99649	12
49	1425	99990	3170	99950	4914	99879	6656	99778	8397	99647	11
50	1454	99989	3199	99949	4943	99878	6685	99776	8426	99644	10
51	1483	99989	3228	99948	4972	99876	6714	99774	8455	99642	9
52	1513	99989	3257	99947	5001	99875	6743	99772	8484	99639	8
53	1542	99988	3286	99946	5030	99873	6773	99770	8513	99637	7
54	1571	99988	3316	99945	5059	99872	6802	99768	8542	99635	6
55	1600	99987	3345	99944	5088	99870	6831	99766	8571	99632	5
56	1629	99987	3374	99943	5117	99869	6860	99764	8600	99630	4
57	1658	99986	3403	99942	5146	99867	6889	99762	8629	99627	3
58	1687	99986	3432	99941	5175	99866	6918	99760	8658	99625	2
59	1716	99985	3461	99940	5205	99864	6947	99758	8687	99622	1
60	1745	99985	3490	99939	5234	99863	6976	99756	8716	99619	0
	N cos	N sin	N cos	N sin	N cos	N sin	N cos	N sin	N cos	N sin	Min
	89 Deg		88 Deg		87 Deg		86 Deg		85 Deg		

	5 Deg		6 Deg		7 Deg		8 Deg		9 Deg		
Min	N sin	N cos.	N sin	N cos.	N sin	N cos.	N sin	N cos.	N sin	N cos.	
0	8716	99619	10453	99452	12187	99255	13917	99027	15643	98769	60
1	8745	99617	10482	99449	12216	99251	13946	99023	15672	98764	59
2	8774	99614	10511	99446	12245	99248	13975	99019	15701	98760	58
3	8803	99612	10540	99443	12274	99244	14004	99015	15730	98755	57
4	8831	99609	10569	99440	12302	99240	14033	99011	15758	98751	56
5	8860	99607	10597	99437	12331	99237	14061	99006	15787	98746	55
6	8889	99604	10626	99434	12360	99233	14090	99002	15816	98741	54
7	8918	99602	10655	99431	12389	99230	14119	98998	15845	98737	53
8	8947	99599	10684	99428	12418	99226	14148	98994	15873	98732	52
9	8976	99596	10713	99424	12447	99222	16177	98990	15902	98728	51
10	9005	99594	10742	99421	12476	99219	14205	98986	15931	98723	50
11	9034	99591	10771	99418	12504	99215	14234	98982	15959	98718	49
12	9063	99588	10800	99415	12533	99211	14263	98978	15988	98714	48
13	9092	99586	10829	99412	12562	99208	14292	98973	16017	98709	47
14	9121	99583	10858	99409	12591	99204	14320	98969	16046	98704	46
15	9150	99580	10887	99406	12620	99200	14349	98965	16074	98700	45
16	9179	99578	10916	99402	12649	99197	14378	98961	16103	98695	44
17	9208	99575	10945	99399	12678	99193	14407	98957	16132	98690	43
18	9237	99572	10973	99396	12706	99189	14436	98953	16160	98686	42
19	9266	99570	11002	99393	12735	99186	14464	98948	16189	98681	41
20	9295	99567	11031	99390	12764	99182	14493	98944	16218	98676	40
21	9324	99564	11060	99386	12793	99178	14522	98940	16246	98671	39
22	9353	99562	11089	99383	12822	99175	14551	98936	16275	98667	38
23	9382	99559	11118	99380	12851	99171	14580	98931	16304	98662	37
24	9411	99556	11147	99377	12880	99167	14608	98927	16333	98657	36
25	9440	99553	11176	99374	12908	99163	14637	98923	16361	98652	35
26	9469	99551	11205	99370	12937	99160	14666	98919	16390	98648	34
27	9498	99548	11234	99367	12966	99156	14695	98914	16419	98643	33
28	9527	99545	11263	99364	12995	99152	14723	98910	16447	98638	32
29	9556	99542	11291	99360	13024	99148	14752	98906	16476	98633	31
30	9585	99540	11320	99357	13053	99144	14781	98902	16505	98629	30
31	9614	99537	11349	99354	13081	99141	14810	98897	16533	98624	29
32	9642	99534	11378	99351	13110	99137	14838	98893	16562	98619	28
33	9671	99531	11407	99347	13139	99133	14867	98889	16591	98614	27
34	9700	99528	11436	9934	13168	99129	14896	98884	16620	98609	26
35	9729	99526	11465	99341	13197	99125	14925	98880	16648	98604	25
36	9758	99523	11494	99337	13226	99122	14954	98876	16677	98600	24
37	9787	99520	11523	99334	13254	99118	14982	98871	16706	98595	23
38	9816	99517	11552	99331	13283	99114	15011	98867	16734	98590	22
39	9845	99514	11580	99327	13312	99110	15040	98863	16763	98585	21
40	9874	99511	11609	99324	13341	99106	15069	98858	16792	98580	20
41	9903	99508	11638	99320	13370	99102	15097	98854	16820	98575	19
42	9932	99506	11667	99317	13399	99098	15126	98819	16849	98570	18
43	9961	99503	11696	99314	13427	99094	15155	98845	16878	98565	17
44	9990	99500	11725	99310	13456	99091	15184	98841	16906	98561	16
45	10019	99497	11754	99307	13485	99087	15212	98836	16935	98556	15
46	10048	99491	11783	99303	13514	99083	15241	98832	16964	98551	14
47	10077	99491	11812	99300	13543	99079	15270	98827	16992	98546	13
48	10106	99488	11840	99297	13572	99075	15299	98823	17021	98541	12
49	10135	99485	11869	99293	13600	99071	15327	98818	17050	98536	11
50	10164	99482	11898	99290	13629	99067	15356	98814	17078	98531	10
51	10192	99479	11927	99286	13658	99063	15385	98809	17107	98526	9
52	10221	99476	11956	99283	13687	99059	15414	98805	17136	98521	8
53	10250	99473	11985	99279	13716	99055	15442	98800	17164	98516	7
54	10279	99470	12014	99276	13744	99051	15471	98796	17193	98511	6
55	10308	99467	12043	99272	13773	99047	15500	98791	17222	98506	5
56	10337	99464	12071	99269	13802	99043	15529	98787	17250	98501	4
57	10366	99461	12100	99265	13831	99039	15557	98782	17279	98496	3
58	10395	99458	12129	99262	13860	99035	15586	98778	17308	98491	2
59	10424	99455	12158	99258	13889	99031	15615	98773	17336	98486	1
60	10453	99452	12187	99255	13917	99027	15643	98769	17365	98481	0
	N cos.	N sin	N cos	N sin	N cos.	N sin	N cos	N sin	N cos	N sin	Min
	84 Deg		83 Deg		82 Deg		81 Deg		80 Deg		

	10 Deg		11 Deg		12 Deg		13 Deg		14 Deg.		
Min	N sin	N cos	N sin	N cos	N sin	N cos.	N sin	N cos.	N sin.	N cos.	
0	17365	98481	19081	98163	20791	97815	22495	97437	24192	97030	60
1	17393	98476	19109	98157	20820	97809	22523	97430	24220	97023	59
2	17422	98471	19138	98152	20848	97803	22552	97424	24249	97015	58
3	17451	98466	19167	98146	20877	97797	22580	97417	24277	97008	57
4	17479	98461	19195	98140	20905	97791	22608	97411	24305	97001	56
5	17508	98455	19224	98135	20933	97784	22637	97404	24333	96994	55
6	17537	98450	19252	98129	20962	97778	22665	97398	24362	96987	54
7	17565	98445	19281	98124	20990	97772	22693	97391	24390	96980	53
8	17594	98440	19309	98118	21019	97766	22722	97384	24418	96973	52
9	17623	98435	19338	98112	21047	97760	22750	97378	24446	96966	51
10	17651	98430	19366	98107	21076	97754	22778	97371	24474	96959	50
11	17680	98425	19395	98101	21104	97748	22807	97365	24503	96952	49
12	17708	98420	19423	98096	21132	97742	22835	97358	24531	96945	48
13	17737	98414	19452	98090	21161	97735	22863	97351	24559	96937	47
14	17766	98409	19481	98084	21189	97729	22892	97345	24587	96930	46
15	17794	98404	19509	98079	21218	97723	22920	97338	24615	96923	45
16	17823	98399	19538	98073	21246	97717	22948	97331	24644	96916	44
17	17852	98394	19566	98067	21275	97711	22977	97325	24672	96909	43
18	17880	98389	19595	98061	21303	97705	23005	97318	24700	96902	42
19	17909	98383	19623	98056	21331	97698	23033	97311	24728	96894	41
20	17937	98378	19652	98050	21360	97692	23062	97304	24756	96887	40
21	17966	98373	19680	98044	21388	97686	23090	97298	24784	96880	39
22	17995	98368	19709	98039	21417	97680	23118	97291	24813	96873	38
23	18023	98362	19737	98033	21445	97673	23146	97284	24841	96866	37
24	18052	98357	19766	98027	21474	97667	23175	97278	24869	96858	36
25	18081	98352	19794	98021	21502	97661	23203	97271	24897	96851	35
26	18109	98347	19823	98016	21530	97655	23231	97264	24925	96844	34
27	18138	98341	19851	98010	21559	97648	23260	97257	24954	96837	33
28	18166	98336	19880	98004	21587	97642	23288	97251	24982	96829	32
29	18195	98331	19908	97998	21616	97636	23316	97244	25010	96822	31
30	18224	98325	19937	97992	21644	97630	23345	97237	25038	96815	30
31	18252	98320	19965	97987	21672	97623	23373	97230	25066	96807	29
32	18281	98315	19994	97981	21701	97617	23401	97223	25094	96800	28
33	18309	98310	20022	97975	21729	97611	23429	97217	25122	96793	27
34	18338	98304	20051	97969	21758	97604	23458	97210	25151	96786	26
35	18367	98299	20079	97963	21786	97598	23486	97203	25179	96778	25
36	18395	98294	20108	97958	21814	97592	23514	97196	25207	96771	24
37	18424	98288	20136	97952	21843	97585	23542	97189	25235	96764	23
38	18452	98283	20165	97946	21871	97579	23571	97182	25263	96756	22
39	18481	98277	20193	97940	21899	97573	23599	97176	25291	96749	21
40	18509	98272	20222	97934	21928	97566	23627	97169	25320	96742	20
41	18538	98267	20250	97928	21956	97560	23656	97162	25348	96734	19
42	18567	98261	20279	97922	21985	97553	23684	97155	25376	96727	18
43	18595	98256	20307	97916	22013	97547	23712	97148	25404	96719	17
44	18624	98250	20336	97910	22041	97541	23740	97141	25432	96712	16
45	18652	98245	20364	97905	22070	97534	23769	97134	25460	96705	15
46	18681	98240	20393	97899	22098	97528	23797	97127	25488	96697	14
47	18710	98234	20421	97893	22126	97521	23825	97120	25516	96690	13
48	18738	98229	20450	97887	22155	97515	23853	97113	25545	96682	12
49	18767	98223	20478	97881	22183	97508	23882	97106	25573	96675	11
50	18795	98218	20507	97875	22212	97502	23910	97100	25601	96667	10
51	18824	98212	20535	97869	22240	97496	23938	97093	25629	96660	9
52	18852	98207	20563	97863	22268	97489	23966	97086	25657	96653	8
53	18881	98201	20592	97857	22297	97483	23995	97079	25685	96645	7
54	18910	98196	20620	97851	22325	97476	24023	97072	25713	96638	6
55	18938	98190	20649	97845	22353	97470	24051	97065	25741	96630	5
56	18967	98185	20677	97839	22382	97463	24079	97058	25769	96623	4
57	18995	98179	20706	97833	22410	97457	24108	97051	25798	96615	3
58	19024	98174	20734	97827	22438	97450	24136	97044	25826	96608	2
59	19052	98168	20763	97821	22467	97444	24164	97037	25854	96600	1
60	19081	98163	20791	97815	22495	97437	24192	97030	25882	96593	0
	N cos.	N sin	N cos	N sin	N cos.	N sin	N cos.	N sin	N cos.	N sin	Min
	79 Deg		78 Deg		77 Deg.		76 Deg		75 Deg		

Min	15 Deg N sin	15 Deg N cos	16 Deg N sin	16 Deg N cos	17 Deg N sin	17 Deg N cos	18 Deg N sin	18 Deg N cos	19 Deg N sin	19 Deg N cos	
0	25882	96593	27564	96126	29237	95630	30902	95106	32557	94552	60
1	25910	96585	27592	96118	29265	95622	30929	95097	32584	94542	59
2	25938	96578	27620	96110	29293	95613	30957	95088	32612	94533	58
3	25966	96570	27648	96102	29321	95605	30985	95079	32639	94523	57
4	25994	96562	27676	96094	29348	95596	31012	95070	32667	94514	56
5	26022	96555	27704	96086	29376	95588	31040	95061	32694	94504	55
6	26050	96547	27731	96078	29404	95579	31068	95052	32722	94495	54
7	26079	96540	27759	96070	29432	95571	31095	95043	32749	94485	53
8	26107	96532	27787	96062	29460	95562	31123	95033	32777	94476	52
9	26135	96524	27815	96054	29487	95554	31151	95024	32804	94466	51
10	26163	96517	27843	96046	29515	95545	31178	95015	32832	94457	50
11	26191	96509	27871	96037	29543	95536	31206	95006	32859	94447	49
12	26219	96502	27899	96029	29571	95528	31233	94997	32887	94438	48
13	26247	96494	27927	96021	29599	95519	31261	94988	32914	94428	47
14	26275	96486	27955	96013	29626	95511	31289	94979	32942	94418	46
15	26303	96479	27983	96005	29654	95502	31316	94970	32969	94409	45
16	26331	96471	28011	95997	29682	95493	31344	94961	32997	94399	44
17	26359	96463	28039	95989	29710	95485	31372	94952	33024	94390	43
18	26387	96456	28067	95981	29737	95476	31399	94943	33051	94380	42
19	26415	96448	28095	95972	29765	95467	31427	94933	33079	94370	41
20	26443	96440	28123	95964	29793	95459	31454	94924	33106	94361	40
21	26471	96433	28150	95956	29821	95450	31482	94915	33134	94351	39
22	26500	96425	28178	95948	29849	95441	31510	94906	33161	94342	38
23	26528	96417	28206	95940	29876	95433	31537	94897	33189	94332	37
24	26556	96410	28234	95931	29904	95424	31565	94888	33216	94322	36
25	26584	96402	28262	95923	29932	95415	31593	94878	33244	94313	35
26	26612	96394	28290	95915	29960	95407	31620	94869	33271	94303	34
27	26640	96386	28318	95907	29987	95398	31648	94860	33298	94293	33
28	26668	96379	28346	95898	30015	95389	31675	94851	33326	94284	32
29	26696	96371	28374	95890	30043	95380	31703	94842	33353	94274	31
30	26724	96363	28402	95882	30071	95372	31730	94832	33381	94264	30
31	26752	96355	28429	95874	30098	95363	31758	94823	33408	94254	29
32	26780	96347	28457	95865	30126	95354	31786	94814	33436	94245	28
33	26808	96340	28485	95857	30154	95345	31813	94805	33463	94235	27
34	26836	96332	28513	95849	30182	95337	31841	94795	33490	94225	26
35	26864	96324	28541	95841	30209	95328	31868	94786	33518	94215	25
36	26892	96316	28569	95832	30237	95319	31896	94777	33545	94206	24
37	26920	96308	28597	95824	30265	95310	31923	94768	33573	94196	23
38	26948	96301	28625	95816	30292	95301	31951	94758	33600	94186	22
39	26976	96293	28652	95807	30320	95293	31979	94749	33627	94176	21
40	27004	96285	28680	95799	30348	95284	32006	94740	33655	94167	20
41	27032	96277	28708	95791	30376	95275	32034	94730	33682	94157	19
42	27060	96269	28736	95782	30403	95266	32061	94721	33710	94147	18
43	27088	96261	28764	95774	30431	95257	32089	94712	33737	94137	17
44	27116	96253	28792	95766	30459	95248	32116	94702	33764	94127	16
45	27144	96246	28820	95757	30486	95240	32144	94693	33792	94118	15
46	27172	96238	28847	95749	30514	95231	32171	94684	33819	94108	14
47	27200	96230	28875	95740	30542	95222	32199	94674	33846	94098	13
48	27228	96222	28903	95732	30570	95213	32227	94665	33874	94088	12
49	27256	96214	28931	95724	30597	95204	32254	94656	33901	94078	11
50	27284	96206	28959	95715	30625	95195	32282	94646	33929	94068	10
51	27312	96198	28987	95707	30653	95186	32309	94637	33956	94058	9
52	27340	96190	29015	95698	30680	95177	32337	94627	33983	94049	8
53	27368	96182	29042	95690	30708	95168	32364	94618	34011	94039	7
54	27396	96174	29070	95681	30736	95159	32392	94609	34038	94029	6
55	27424	96166	29098	95673	30763	95150	32419	94599	34065	94019	5
56	27452	96158	29126	95664	30791	95142	32447	94590	34093	94009	4
57	27480	96150	29154	95656	30819	95133	32474	94580	34120	93999	3
58	27508	96142	29182	95647	30846	95124	32502	94571	34147	93989	2
59	27536	96134	29209	95639	30874	95115	32529	94561	34175	93979	1
60	27564	96126	29237	95630	30902	95106	32557	94552	34202	93969	0
	N cos	N sin	N cos	N sin	N cos	N sin	N cos	N sin	N cos	N sin	Min
	74 Deg		73 Deg		72 Deg		71 Deg		70 Deg		

	20 Deg		21 Deg		22 Deg		23 Deg		24 Deg.		
Min	N sin	N cos	N. sin	N cos.	N sin	N cos.	N sin	N. cos.	N sin.	N. cos.	
0	34202	93969	35837	93358	37461	92718	39073	92050	40674	91355	60
1	34229	93959	35864	93348	37488	92707	39100	92039	40700	91343	59
2	34257	93949	35891	93337	37515	92697	39127	92028	40727	91331	58
3	34284	93939	35918	93327	37542	92686	39153	92016	40753	91319	57
4	34311	93929	35945	93316	37569	92675	39180	92005	40780	91307	56
5	34339	93919	35973	93306	37595	92664	39207	91994	40806	91295	55
6	34366	93909	36000	93295	37622	92653	39234	91982	40833	91283	54
7	34393	93899	36027	93285	37649	92642	39260	91971	40860	91272	53
8	34421	93889	36054	93274	37676	92631	39287	91959	40886	91260	52
9	34448	93879	36081	93264	37703	92620	39314	91948	40913	91248	51
10	34475	93869	36108	93253	37730	92609	39341	91936	40939	91236	50
11	34503	93859	36135	93243	37757	92598	39367	91925	40966	91224	49
12	34530	93849	36162	93232	37784	92587	39394	91914	40992	91212	48
13	34557	93839	36190	93222	37811	92576	39421	91902	41019	91200	47
14	34584	93829	36217	93211	37838	92565	39448	91891	41045	91188	46
15	34612	93819	36244	93201	37865	92554	39474	91879	41072	91176	45
16	34639	93809	36271	93190	37892	92543	39501	91868	41098	91164	44
17	34666	93799	36298	93180	37919	92532	39528	91856	41125	91152	43
18	34694	93789	36325	93169	37946	92521	39555	91845	41151	91140	42
19	34721	93779	36352	93159	37973	92510	39581	91833	41178	91128	41
20	34748	93769	36379	93148	37999	92499	39608	91822	41204	91116	40
21	34775	93759	36406	93137	38026	92488	39635	91810	41231	91104	39
22	34803	93748	36434	93127	38053	92477	39661	91799	41257	91092	38
23	34830	93738	36461	93116	38080	92466	39688	91787	41284	91080	37
24	34857	93728	36488	93106	38107	92455	39715	91775	41310	91068	36
25	34884	93718	36515	93095	38134	92444	39741	91764	41337	91056	35
26	34912	93708	36542	93084	38161	92432	39768	91752	41363	91044	34
27	34939	93698	36569	93074	38188	92421	39795	91741	41390	91032	33
28	34966	93688	36596	93063	38215	92410	39822	91729	41416	91020	32
29	34993	93677	36623	93052	38241	92399	39848	91718	41443	91008	31
30	35021	93667	36650	93042	38268	92388	39875	91706	41469	90996	30
31	35048	93657	36677	93031	38295	92377	39902	91694	41496	90984	29
32	35075	93647	36704	93020	38322	92366	39928	91683	41522	90972	28
33	35102	93637	36731	93010	38349	92355	39955	91671	41549	90960	27
34	35130	93626	36758	92999	38376	92343	39982	91660	41575	90948	26
35	35157	93616	36785	92988	38403	92332	40008	91648	41602	90936	25
36	35184	93606	36812	92978	38430	92321	40035	91636	41628	90924	24
37	35211	93596	36839	92967	38456	92310	40062	91625	41655	90911	23
38	35239	93585	36867	92956	38483	92299	40088	91613	41681	90899	22
39	35266	93575	36894	92945	38510	92287	40115	91601	41707	90887	21
40	35293	93565	36921	92935	38537	92276	40141	91590	41734	90875	20
41	35320	93555	36948	92924	38564	92265	40168	91578	41760	90863	19
42	35347	93544	36975	92913	38591	92254	40195	91566	41787	90851	18
43	35375	93534	37002	92902	38617	92243	40221	91555	41813	90839	17
44	35402	93524	37029	92892	38644	92231	40248	91543	41840	90826	16
45	35429	93514	37056	92881	38671	92220	40275	91531	41866	90814	15
46	35456	93503	37083	92870	38698	92209	40301	91519	41892	90802	14
47	35484	93493	37110	92859	38725	92198	40328	91508	41919	90790	13
48	35511	93483	37137	92849	38752	92186	40355	91496	41945	90778	12
49	35538	93472	37164	92838	38778	92175	40381	91484	41972	90766	11
50	35565	93462	37191	92827	38805	92164	40408	91472	41998	90753	10
51	35592	93452	37218	92816	38832	92152	40434	91461	42024	90741	9
52	35619	93441	37245	92805	38859	92141	40461	91449	42051	90729	8
53	35647	93431	37272	92794	38886	92130	40488	91437	42077	90717	7
54	35674	93420	37299	92784	38912	92119	40514	91425	42104	90704	6
55	35701	93410	37326	92773	38939	92107	40541	91414	42130	90692	5
56	35728	93400	37353	92762	38966	92096	40567	91402	42156	90680	4
57	35755	93389	37380	92751	38993	92085	40594	91390	42183	90668	3
58	35782	93379	37407	92740	39020	92073	40621	91378	42209	90655	2
59	35810	93368	37434	92729	39046	92062	40647	91366	42235	90643	1
60	35837	93358	37461	92718	39073	92050	40674	91355	42262	90631	0
	N cos.	N sin	N cos	N sin	N cos.	N sin	N cos.	N sin.	N cos.	N sin	Min
	69 Deg		68 Deg		67 Deg		66 Deg		65 Deg		

	25 Deg		26 Deg		27 Deg		28 Deg		29 Deg.		
Min	N sin.	N cos	N sin	N. cos.	N sin	N cos.	N sin	N cos.	N. sin.	N. cos.	
0	42262	90631	43837	89879	45399	89101	46947	88295	48481	87462	60
1	42288	90618	43863	89867	45425	89087	46973	88281	48506	87448	59
2	42315	90606	43889	89854	45451	89074	46999	88267	48532	87434	58
3	42341	90594	43916	89841	45477	89061	47024	88254	48557	87420	57
4	42367	90582	43942	89828	45503	89048	47050	88240	48583	87406	56
5	42394	90569	43968	89816	45529	89035	47076	88226	48608	87391	55
6	42420	90557	43994	89803	45554	89021	47101	88213	48634	87377	54
7	42446	90545	44020	89790	45580	89008	47127	88199	48659	87363	53
8	42473	90532	44046	89777	45606	88995	47153	88185	48684	87349	52
9	42499	90520	44072	89764	45632	88981	47178	88172	48710	87335	51
10	42525	90507	44098	89752	45658	88968	47204	88158	48735	87321	50
11	42552	90495	44124	89739	45684	88955	47229	88144	48761	87306	49
12	42578	90483	44151	89726	45710	88942	47255	88130	48786	87292	48
13	42604	90470	44177	89713	45736	88928	47281	88117	48811	87278	47
14	42631	90458	44203	89700	45762	88915	47306	88103	48837	87264	46
15	42657	90446	44229	89687	45787	88902	47332	88089	48862	87250	45
16	42683	90433	44255	89674	45813	88888	47358	88075	48888	87235	44
17	42709	90421	44281	89662	45839	88875	47383	88062	48913	87221	43
18	42736	90408	44307	89649	45865	88862	47409	88048	48938	87207	42
19	42762	90396	44333	89636	45891	88848	47434	88034	48964	87193	41
20	42788	90383	44359	89623	45917	88835	47460	88020	48989	87178	40
21	42815	90371	44385	89610	45942	88822	47486	88006	49014	87164	39
22	42841	90358	44411	89597	45968	88808	47511	87993	49040	87150	38
23	42867	90346	44437	89584	45994	88795	47537	87979	49065	87136	37
24	42894	90334	44464	89571	46020	88782	47562	87965	49090	87121	36
25	42920	90321	44490	89558	46046	88768	47588	87951	49116	87107	35
26	42946	90309	44516	89545	46072	88755	47614	87937	49141	87093	34
27	42972	90296	44542	89532	46097	88741	47639	87923	49166	87079	33
28	42999	90284	44568	89519	46123	88728	47665	87909	49192	87064	32
29	43025	90271	44594	89506	46149	88715	47690	87896	49217	87050	31
30	43051	90259	44620	89493	46175	88701	47716	87882	49242	87036	30
31	43077	90246	44646	89480	46201	88688	47741	87868	49268	87021	29
32	43104	90233	44672	89467	46226	88674	47767	87854	49293	87007	28
33	43130	90221	44698	89454	46252	88661	47793	87840	49318	86993	27
34	43156	90208	44724	89441	46278	88647	47818	87826	49344	86978	26
35	43182	90196	44750	89428	46304	88634	47844	87812	49369	86964	25
36	43209	90183	44776	89415	46330	88620	47869	87798	49394	86949	24
37	43235	90171	44802	89402	46355	88607	47895	87784	49419	86935	23
38	43261	90158	44828	89389	46381	88593	47920	87770	49445	86921	22
39	43287	90146	44854	89376	46407	88580	47946	87756	49470	86906	21
40	43313	90133	44880	89363	46433	88566	47971	87743	49495	86892	20
41	43340	90120	44906	89350	46458	88553	47997	87729	49521	86878	19
42	43366	90108	44932	89337	46484	88539	48022	87715	49546	86863	18
43	43392	90095	44958	89324	46510	88526	48048	87701	49571	86849	17
44	43418	90082	44984	89311	46536	88512	48073	87687	49596	86834	16
45	43445	90070	45010	89298	46561	88499	48099	87673	49622	86820	15
46	43471	90057	45036	89285	46587	88485	48124	87659	49647	86805	14
47	43497	90045	45062	89272	46613	88472	48150	87645	49672	86791	13
48	43523	90032	45088	89259	46639	88458	48175	87631	49697	86777	12
49	43549	90019	45114	89245	46664	88445	48201	87617	49723	86762	11
50	43575	90007	45140	89232	46690	88431	48226	87603	49748	86748	10
51	43602	89994	45166	89219	46716	88417	48252	87589	49773	86733	9
52	43628	89981	45192	89206	46742	88404	48277	87575	49798	86719	8
53	43654	89968	45218	89193	46767	88390	48303	87561	49824	86704	7
54	43680	89956	45243	89180	46793	88377	48328	87546	49849	86690	6
55	43706	89943	45269	89167	46819	88363	48354	87532	49874	86675	5
56	43733	89930	45295	89153	46844	88349	48379	87518	49899	86661	4
57	43759	89918	45321	89140	46870	88336	48405	87504	49924	86646	3
58	43785	89905	45347	89127	46896	88322	48430	87490	49950	86632	2
59	43811	89892	45373	89114	46921	88308	48456	87476	49975	86617	1
60	43837	89879	45399	89101	46947	88295	48481	87462	50000	86603	0
	N cos.	N sin	N cos.	N sin	N cos.	N sin	N cos	N sin	N cos	N sin	Min
	64 Deg		63 Deg		62 Deg		61 Deg		60 Deg		

Min	30 Deg N sin	N cos	31 Deg N sin	N cos	32 Deg N sin	N. cos.	33 Deg. N sin	N cos.	34 Deg N sin.	N cos.	
0	50000	86603	51504	85717	52992	84805	54464	83867	55919	82904	60
1	50025	86588	51529	85702	53017	84789	54488	83851	55943	82887	59
2	50050	86573	51554	85687	53041	84774	54513	83835	55968	82871	58
3	50076	86559	51579	85672	53066	84759	54537	83819	55992	82855	57
4	50101	86544	51604	85657	53091	84743	54561	83804	56016	82839	56
5	50126	86530	51628	85642	53115	84728	54586	83788	56040	82822	55
6	50151	86515	51653	85627	53140	84712	54610	83772	56064	82806	54
7	50176	86501	51678	85612	53164	84697	54635	83756	56088	82790	53
8	50201	86486	51703	85597	53189	84681	54659	83740	56112	82773	52
9	50227	86471	51728	85582	53214	84666	54683	83724	56136	82757	51
10	50252	86457	51753	85567	53238	84650	54708	83708	56160	82741	50
11	50277	86442	51778	85551	53263	84635	54732	83692	56184	82724	49
12	50302	86427	51803	85536	53288	84619	54756	83676	56208	82708	48
13	50327	86413	51828	85521	53312	84604	54781	83660	56232	82692	47
14	50352	86398	51852	85506	53337	84588	54805	83645	56256	82675	46
15	50377	86384	51877	85491	53361	84573	54829	83629	56280	82659	45
16	50403	86369	51902	85476	53386	84557	54854	83613	56305	82643	44
17	50428	86354	51927	85461	53411	84542	54878	83597	56329	82626	43
18	50453	86340	51952	85446	53435	84526	54902	83581	56353	82610	42
19	50478	86325	51977	85431	53460	84511	54927	83565	56377	82593	41
20	50503	86310	52002	85416	53484	84495	54951	83549	56401	82577	40
21	50528	86295	52026	85401	53509	84480	54975	83533	56425	82561	39
22	50553	86281	52051	85385	53534	84464	54999	83517	56449	82544	38
23	50578	86266	52076	85370	53558	84448	55024	83501	56473	82528	37
24	50603	86251	52101	85355	53583	84433	55048	83485	56497	82511	36
25	50628	86237	52126	85340	53607	84417	55072	83469	56521	82495	35
26	50654	86222	52151	85325	53632	84402	55097	83453	56545	82478	34
27	50679	86207	52175	85310	53656	84386	55121	83437	56569	82462	33
28	50704	86192	52200	85294	53681	84370	55145	83421	56593	82446	32
29	50729	86178	52225	85279	53705	84355	55169	83405	56617	82429	31
30	50754	86163	52250	85264	53730	84339	55194	83389	56641	82413	30
31	50779	86148	52275	85249	53754	84324	55218	83373	56665	82396	29
32	50804	86133	52299	85234	53779	84308	55242	83356	56689	82380	28
33	50829	86119	52324	85218	53804	84292	55266	83340	56713	82363	27
34	50854	86104	52349	85203	53828	84277	55291	83324	56736	82347	26
35	50879	86089	52374	85188	53853	84261	55315	83308	56760	82330	25
36	50904	86074	52399	85173	53877	84245	55339	83292	56784	82314	24
37	50929	86059	52423	85157	53902	84230	55363	83276	56808	82297	23
38	50954	86045	52448	85142	53926	84214	55388	83260	56832	82281	22
39	50979	86030	52473	85127	53951	84198	55412	83244	56856	82264	21
40	51004	86015	52498	85112	53975	84182	55436	83228	56880	82248	20
41	51029	86000	52522	85096	54000	84167	55460	83212	56904	82231	19
42	51054	85985	52547	85081	54024	84151	55484	83195	56928	82214	18
43	51079	85970	52572	85066	54049	84135	55509	83179	56952	82198	17
44	51104	85956	52597	85051	54073	84120	55533	83163	56976	82181	16
45	51129	85941	52621	85035	54097	84104	55557	83147	57000	82165	15
46	51154	85926	52646	85020	54122	84088	55581	83131	57024	82148	14
47	51179	85911	52671	85005	54146	84072	55605	83115	57047	82132	13
48	51204	85896	52696	84989	54171	84057	55630	83098	57071	82115	12
49	51229	85881	52720	84974	54195	84041	55654	83082	57095	82098	11
50	51254	85866	52745	84959	54220	84025	55678	83066	57119	82082	10
51	51279	85851	52770	84943	54244	84009	55702	83050	57143	82065	9
52	51304	85836	52794	84928	54269	83994	55726	83034	57167	82048	8
53	51329	85821	52819	84913	54293	83978	55750	83017	57191	82032	7
54	51354	85806	52844	84897	54317	83962	55775	83001	57215	82015	6
55	51379	85792	52869	84882	54342	83946	55799	82985	57238	81999	5
56	51404	85777	52893	84866	54366	83930	55823	82969	57262	81982	4
57	51429	85762	52918	84851	54391	83915	55847	82953	57286	81965	3
58	51454	85747	52943	84836	54415	83899	55871	82936	57310	81949	2
59	51479	85732	52967	84820	54440	83883	55895	82920	57334	81932	1
60	51504	85717	52992	84805	54464	83867	55919	82904	57358	81915	0
	N cos.	N sin	N cos	N sin	N cos.	N sin	N cos.	N sin	N cos.	N sin	Min
	59 Deg		58 Deg		57 Deg		56 Deg		55 Deg		

	35 Deg		36 Deg		37 Deg		38 Deg.		39 Deg		
Min.	N. sin	N cos.	N sin	N. cos.	N sin	N cos.	N sin	N cos.	N. sin	N cos.	
0	57358	81915	58779	80902	60182	79864	61566	78801	62932	77715	60
1	57381	81899	58802	80885	60205	79846	61589	78783	62955	77696	59
2	57405	81882	58826	80867	60228	79829	61612	78765	62977	77678	58
3	57429	81865	58849	80850	60251	79811	61635	78747	63000	77660	57
4	57453	81848	58873	80833	60274	79793	61658	78729	63022	77641	56
5	57477	81832	58896	80816	60298	79776	61681	78711	63045	77623	55
6	57501	81815	58920	80799	60321	79758	61704	78693	63068	77605	54
7	57524	81798	58943	80782	60344	79741	61726	78676	63090	77586	53
8	57548	81782	58967	80765	60367	79723	61749	78658	63113	77568	52
9	57572	81765	58990	80748	60390	79706	61772	78640	63135	77550	51
10	57596	81748	59014	80730	60414	79688	61795	78622	63158	77531	50
11	57619	81731	59037	80713	60437	79671	61818	78604	63180	77513	49
12	57643	81714	59061	80696	60460	79653	61841	78586	63203	77494	48
13	57667	81698	59084	80679	60483	79635	61864	78568	63225	77476	47
14	57691	81681	59108	80662	60506	79618	61887	78550	63248	77458	46
15	57715	81664	59131	80644	60529	79600	61909	78532	63271	77439	45
16	57738	81647	59154	80627	60553	79583	61932	78514	63293	77421	44
17	57762	81631	59178	80610	60576	79565	61955	78496	63316	77402	43
18	57786	81614	59201	80593	60599	79547	61978	78478	63338	77384	42
19	57810	81597	59225	80576	60622	79530	62001	78460	63361	77366	41
20	57833	81580	59248	80558	60645	79512	62024	78442	63383	77347	40
21	57857	81563	59272	80541	60668	79494	62046	78424	63406	77329	39
22	57881	81546	59295	80524	60691	79477	62069	78405	63428	77310	38
23	57904	81530	59318	80507	60714	79459	62092	78387	63451	77292	37
24	57928	81513	59342	80489	60738	79441	62115	78369	63473	77273	36
25	57952	81496	59365	80472	60761	79424	62138	78351	63496	77255	35
26	57976	81479	59389	80455	60784	79406	62160	78333	63518	77236	34
27	57999	81462	59412	80438	60807	79388	62183	78315	63540	77218	33
28	58023	81445	59436	80420	60830	79371	62206	78297	63563	77199	32
29	58047	81428	59459	80403	60853	79353	62229	78279	63585	77181	31
30	58070	81412	59482	80386	60876	79335	62251	78261	63608	77162	30
31	58094	81395	59506	80368	60899	79318	62274	78243	63630	77144	29
32	58118	81378	59529	80351	60922	79300	62297	78225	63653	77125	28
33	58141	81361	59552	80334	60945	79282	62320	78206	63675	77107	27
34	58165	81344	59576	80316	60968	79264	62342	78188	63698	77088	26
35	58189	81327	59599	80299	60991	79247	62365	78170	63720	77070	25
36	58212	81310	59622	80282	61015	79229	62388	78152	63742	77051	24
37	58236	81293	59646	80264	61038	79211	62411	78134	63765	77033	23
38	58260	81276	59669	80247	61061	79193	62433	78116	63787	77014	22
39	58283	81259	59693	80230	61084	79176	62456	78098	63810	76996	21
40	58307	81242	59716	80212	61107	79158	62479	78079	63832	76977	20
41	58330	81225	59739	80195	61130	79140	62502	78061	63854	76959	19
42	58354	81208	59762	80178	61153	79122	62524	78043	63877	76940	18
43	58378	81191	59786	80160	61176	79105	62547	78025	63899	76921	17
44	58401	81174	59809	80143	61199	79087	62570	78007	63922	76903	16
45	58425	81157	59832	80125	61222	79069	62592	77988	63944	76884	15
46	58449	81140	59856	80108	61245	79051	62615	77970	63966	76866	14
47	58472	81123	59879	80091	61268	79033	62638	77952	63989	76847	13
48	58496	81106	59902	80073	61291	79016	62660	77934	64011	76828	12
49	58519	81089	59926	80056	61314	78998	62683	77916	64033	76810	11
50	58543	81072	59949	80038	61337	78980	62706	77897	64056	76791	10
51	58567	81055	59972	80021	61360	78962	62728	77879	64078	76772	9
52	58590	81038	59995	80003	61383	78944	62751	77861	64100	76754	8
53	58614	81021	60019	79986	61406	78926	62774	77843	64123	76735	7
54	58637	81004	60042	79968	61429	78908	62796	77824	64145	76717	6
55	58661	80987	60065	79951	61451	78891	62819	77806	64167	76698	5
56	58684	80970	60089	79934	61474	78873	62842	77788	64190	76679	4
57	58708	80953	60112	79916	61497	78855	62864	77769	64212	76661	3
58	58731	80936	60135	79899	61520	78837	62887	77751	64234	76642	2
59	58755	80919	60158	79881	61543	78819	62909	77733	64256	76623	1
60	58779	80902	60182	79864	61566	78801	62932	77715	64279	76604	0
	N cos.	N sin	N. cos.	N sin.	N cos.	N sin	N cos.	N sin.	N cos.	N sin	Min
	54 Deg		53 Deg		52 Deg		51 Deg		50 Deg		

Min	40 Deg		41 Deg		42 Deg		43 Deg		44 Deg		
	N sin	N cos	N sin	N cos	N sin	N cos	N sin	N cos	N sin.	N cos.	
0	64279	76604	65606	75471	66913	74314	68200	73135	69466	71934	60
1	64301	76586	65628	75452	66935	74295	68221	73116	69487	71914	59
2	64323	76567	65650	75433	66956	74276	68242	73096	69508	71894	58
3	64346	76548	65672	75414	66978	74256	68264	73076	69529	71873	57
4	64368	76530	65694	75395	66999	74237	68285	73056	69549	71853	56
5	64390	76511	65716	75375	67021	74217	68306	73036	69570	71833	55
6	64412	76492	65738	75356	67043	74198	68327	73016	69591	71813	54
7	64435	76473	65759	75337	67064	74178	68349	72996	69612	71792	53
8	64457	76455	65781	75318	67086	74159	68370	72976	69633	71772	52
9	64479	76436	65803	75299	67107	74139	68391	72957	69654	71752	51
10	64501	76417	65825	75280	67129	74120	68412	72937	69675	71732	50
11	64524	76398	65847	75261	67151	74100	68434	72917	69696	71711	49
12	64546	76380	65869	75241	67172	74080	68455	72897	69717	71691	48
13	64568	76361	65891	75222	67194	74061	68476	72877	69737	71671	47
14	64590	76342	65913	75203	67215	74041	68497	72857	69758	71650	46
15	64612	76323	65935	75184	67237	74022	68518	72837	69779	71630	45
16	64635	76304	65956	75165	67258	74002	68539	72817	69800	71610	44
17	64657	76286	65978	75146	67280	73983	68561	72797	69821	71590	43
18	64679	76267	66000	75126	67301	73963	68582	72777	69842	71569	42
19	64701	76248	66022	75107	67323	73944	68603	72757	69862	71549	41
20	64723	76229	66044	75088	67344	73924	68624	72737	69883	71529	40
21	64746	76210	66066	75069	67366	73904	68645	72717	69904	71508	39
22	64768	76192	66088	75050	67387	73885	68666	72697	69925	71488	38
23	64790	76173	66109	75030	67409	73865	68688	72677	69946	71468	37
24	64812	76154	66131	75011	67430	73846	68709	72657	69966	71447	36
25	64834	76135	66153	74992	67452	73826	68730	72637	69987	71427	35
26	64856	76116	66175	74973	67473	73806	68751	72617	70008	71407	34
27	64878	76097	66197	74953	67495	73787	68772	72597	70029	71386	33
28	64901	76078	66218	74934	67516	73767	68793	72577	70049	71366	32
29	64923	76059	66240	74915	67538	73747	68814	72557	70070	71345	31
30	61945	76041	66262	74896	67559	73728	68835	72537	70091	71325	30
31	64967	76022	66284	74876	67580	73708	68857	72517	70112	71305	29
32	64989	76003	66306	74857	67602	73688	68878	72497	70132	71284	28
33	65011	75984	66327	74838	67623	73669	68899	72477	70153	71264	27
34	65033	75965	66349	74818	67645	73649	68920	72457	70174	71243	26
35	65055	75946	66371	74799	67666	73629	68941	72437	70195	71223	25
36	65077	75927	66393	74780	67688	73610	68962	72417	70215	71203	24
37	65100	75908	66414	74760	67709	73590	68983	72397	70236	71182	23
38	65122	75889	66436	74741	67730	73570	69004	72377	70257	71162	22
39	65144	75870	66458	74722	67752	73551	69025	72357	70277	71141	21
40	65166	75851	66480	74703	67773	73531	69046	72337	70298	71121	20
41	65188	75832	66501	74683	67795	73511	69067	72317	70319	71100	19
42	65210	75813	66523	74664	67816	73491	69088	72297	70339	71080	18
43	65232	75794	66545	74644	67837	73472	69109	72277	70360	71059	17
44	65254	75775	66566	74625	67859	73452	69130	72257	70381	71039	16
45	65276	75756	66588	74606	67880	73432	69151	72236	70401	71019	15
46	65298	75738	66610	74586	67901	73413	69172	72216	70422	70998	14
47	65320	75719	66632	74567	67923	73393	69193	72196	70443	70978	13
48	65342	75700	66653	74548	67944	73373	69214	72176	70463	70957	12
49	65364	75680	66675	74528	67965	73353	69235	72156	70484	70937	11
50	65386	75661	66697	74509	67987	73333	69256	72136	70505	70916	10
51	65408	75642	66718	74489	68008	73314	69277	72116	70525	70896	9
52	65430	75623	66740	74470	68029	73294	69298	72095	70546	70875	8
53	65152	75604	66762	74451	68051	73274	69319	72075	70567	70855	7
54	65474	75585	66783	74431	68072	73254	69340	72055	70587	70834	6
55	65496	75566	66805	74412	68093	73234	69361	72035	70608	70813	5
56	65518	75547	66827	74392	68115	73215	69382	72015	70628	70793	4
57	65540	75528	66848	74373	68136	73195	69403	71995	70649	70772	3
58	65562	75509	66870	74353	68157	73175	69424	71974	70670	70752	2
59	65584	75490	66891	74334	68179	73155	69445	71954	70690	70731	1
60	65606	75471	66913	74314	68200	73135	69466	71934	70711	70711	0
	N cos	N sin	N cos.	N sin	N cos.	N sin	N cos	N sin	N cos.	N sin	Min
	49 Deg		48 Deg		47 Deg		46 Deg		45 Deg		

A

TRAVERSE TABLE,

CONTAINING

THE DIFFERENCE OF

LATITUDE AND DEPARTURE,

TO EVERY DEGREE AND QUARTER-POINT OF THE COMPASS OR HORIZON.

THIS Table expresses the sides and angles of right-angled plane triangles, the difference of latitude and departure being represented by the two legs, the distance by the hypotenuse, and the course and its complement, by the two acute angles. Any two of these being given, except the acute angles, the other parts of the triangle may be found by inspection; provided the course be in points, quarter-points, or any exact number of degrees.

If the course be given in degrees and minutes, and the number of minutes under 30′, they are to be rejected, and the given degrees are reckoned as the course, but if the minutes be above 30′, the number of degrees given are to be increased by 1° for the course.

The distances 1, 2, 3, &c at the top and bottom may be accounted 10, 20, 30, &c., or, 100, 200, 300, &c., if the difference of latitude and departure, answering to the course, be increased in the same proportion, which is done by removing the decimal point a corresponding number of places to the right. If the distance consist of two or three significant figures, the difference of latitude and departure must be sought for each figure separately, and the results added.

PROBLEM I

The Course and Distance being given, to find the Difference of Latitude and Departure

Find the course, or the nearest to it, in the right or left-hand column, and in a straight line with it under or above the given distance, you have the difference of latitude and departure.

Ex. 1. A ship sails W S W 90 miles, required her difference of latitude and departure?

Course	*Dist*	*Diff Lat*	*Depar.*
6 Points	90	34 442	83 149 *Answer*

2 A ship sails N N E ½E, until her distance by the logline is found to be 75 miles, what is her difference of latitude and departure?

Course	*Dist*	*Diff Lat*	*Depar.*
2½ Points	70	61 734	32 998
	5	4.4096	2 357
	75	66 1436	35 355 *Answer*

3 A ship sails S. 48° 15′ E. 246 miles, required the difference of latitude and departure?

Course	*Dist.*	*Diff Lat*	*Depar.*
48°	200	133 83	148 03
	40	26.765	29.726
	6	4 0148	4 4589
	246	164.6098	182.2149 *Answer.*

In this example, the number of minutes being under 30′, are omitted, and the course assumed equal to 48°, but if the minutes had been above 30′, the course would have been assumed 49°

PROBLEM II.

Difference of Latitude and Departure being given, to find the Course and Distance.

Seek in the table till you find standing together the difference of latitude and departure, which are nearest to those given, then, in the marginal columns, directly opposite, you will find the course, and at the top or bottom of the column, where the difference of latitude and departure are found, stands the distance.

If the difference of latitude and departure given cannot be found nearly, in the table, take any aliquot parts of them, these will give

the same course as the whole, but the distance found must be multiplied by the same figure, as the given terms were divided by, for the whole distance.

Ex. 1. Given the difference of latitude 58.5 miles south, and the departure 39 miles west, required the course and distance?

In this example, the difference of latitude and departure are found in the column marked 7, which gives 70 miles for the distance, and on the margin stands 34° between south and west, for the course, being nearly S W. by S.

2 Given the difference of latitude 86 2 miles S, and the departure 42 miles E, what are the course and distance?

In this example, the numbers themselves are not to be found together in the table, but $\frac{1}{12}$ of them, viz. 7.18 and 3.5, are found together, under the distance 8, which multiplied by 12, gives 96 miles, the whole distance, and on the margin is 26° between south and east, the true course

PROBLEM III.

To Work a Traverse.

Find the difference of latitude and departure for each course, as above, and place them in columns marked North or South, and East or West, respectively, the difference of the columns marked North and South will be the difference of latitude, and the difference of the columns marked East and West, will be the departure the ship has made good in the whole traverse.

Ex A ship sailed S S. W. 54 miles, W by S 39 miles, N W by N. 40 miles, N. E. by E 69 miles, and N N W 60 miles, required the difference of latitude and departure, and the course and distance made good?

Traverse Table.

Courses	Pts.	Dist.	Diff of Lat.		Departure	
			North	South	East	West
S S. W	2	54		49 89		20 66
W by S	7	39		7 60		38 25
N W by N	3	40	33 25			22 22
N E by E	5	69	38 33		57 36	
N. N. W	2	60	55 43			22 96
			127 01	57 49	57 36	101 09
			57 49			57.36
Diff of Lat. N = 69 52				Dep W = 46 73		

Hence, the distance made good is 84 miles, the course being N 34° W, or N. W. by N. nearly

A

TABLE OF RUMBS,

SHEWING

THE DEGREES, MINUTES, AND SECONDS,

THAT EVERY POINT AND QUARTER-POINT OF THE COMPASS MAKES WITH THE MERIDIAN.

NORTH		P	Q	D	M	S	P	Q	SOUTH	
		0	1	2	48	45	0	1		
		0	2	5	37	30	0	2		
		0	3	8	26	15	0	3		
N b. E	N b W	1	0	11	15	0	1	0	S b. E	S b. W
		1	1	14	3	45	1	1		
		1	2	16	52	30	1	2		
		1	3	19	41	15	1	3		
N N E	N N W	2	0	22	30	0	2	0	S S E	S. S W
		2	1	25	18	45	2	1		
		2	2	28	7	30	2	2		
		2	3	30	56	15	2	3		
N E b N	N W b N	3	0	33	45	0	3	0	S E b S	S W b S.
		3	1	36	33	45	3	1		
		3	2	39	22	30	3	2		
		3	3	42	11	15	3	3		
N E	N W.	4	0	45	0	0	4	0	S. E	S. W
		4	1	47	48	45	4	1		
		4	2	50	37	30	4	2		
		4	3	53	26	15	4	3		
N E b E	N W b W.	5	0	56	15	0	5	0	S E b E	S. W b. W
		5	1	59	3	45	5	1		
		5	2	61	52	30	5	2		
		5	3	64	41	15	5	3		
E N E	W N W	6	0	67	30	0	6	0	E. S E.	W S W
		6	1	70	18	45	6	1		
		6	2	73	7	30	6	2		
		6	3	75	56	15	6	3		
E b N	W. b N	7	0	78	45	0	7	0	E b S.	W b S
		7	1	81	33	45	7	1		
		7	2	84	22	30	7	2		
		7	3	87	11	15	7	3		
East	West	8	0	90	0	0	8	0	East	West

Course Pts.	D	Dist. 1 Lat.	Dist. 1 Dep	Dist. 2 Lat.	Dist. 2 Dep.	Dist. 3 Lat.	Dist. 3 Dep.	Dist. 4 Lat.	Dist. 4 Dep.	Dist. 5 Lat.	Dist. 5 Dep	D	Course Pts.
	1	0.9998	0.0175	1.9997	0.0349	2.9995	0.0524	3.9994	0.0698	4.9992	0.0873	89	
	2	0.9994	0.0349	1.9988	0.0698	2.9982	0.1047	3.9976	0.1396	4.9970	0.1745	88	
0 ¼		0.9988	0.0491	1.9976	0.0981	2.9964	0.1472	3.9952	0.1963	4.9940	0.2453		7 ¾
	3	0.9986	0.0523	1.9973	0.1047	2.9959	0.1570	3.9945	0.2093	4.9931	0.2[illegible]	87	
	4	0.9976	0.0698	1.9951	0.1395	2.9927	0.2093	3.9903	0.2790	4.9878	0.3488	86	
	5	0.9962	0.0872	1.9924	0.1743	2.9886	0.2615	3.9848	0.3486	4.9810	0.4358	85	
0 ½		0.9952	0.0980	1.9904	0.1960	2.9856	0.2941	3.9807	0.3921	4.9759	0.4901		7 ½
	6	0.9945	0.1045	1.9890	0.2091	2.9836	0.3136	3.9781	0.4181	4.9726	0.5226	84	
	7	0.9925	0.1219	1.9851	0.2437	2.9776	0.3656	3.9702	0.4875	4.9627	0.6093	83	
	8	0.9903	0.1392	1.9805	0.2783	2.9708	0.4175	3.9611	0.5567	4.9513	0.6959	82	
0 ¾		0.9892	0.1467	1.9784	0.2935	2.9675	0.4402	3.9567	0.5869	4.9459	0.7337		7 ¼
	9	0.9877	0.1564	1.9754	0.3129	2.9631	0.4693	3.9508	0.6257	4.9384	0.782[illegible]	[illegible]	
	10	0.9848	0.1736	1.9696	0.3473	2.9544	0.5209	3.9392	0.6946	4.9240	0.868[illegible]	[illegible]	
	11	0.9816	0.1908	1.9633	0.3816	2.9449	0.5724	3.9265	0.7632	4.9081	0.9540	79	
1		0.9808	0.1951	1.9616	0.3902	2.9424	0.5853	3.9231	0.7804	4.9039	0.9755		7
	12	0.9781	0.2079	1.9563	0.4158	2.9344	0.6237	3.9126	0.8316	4.8907	1.0396	78	
	13	0.9744	0.2250	1.9487	0.4499	2.9231	0.6749	3.8975	0.8998	4.8719	1.1248	77	
	14	0.9703	0.2419	1.9406	0.4838	2.9109	0.7258	3.8812	0.9677	4.8515	1.2096	76	
1 ¼		0.9700	0.2430	1.9401	0.4860	2.9101	0.7289	3.8801	0.9719	4.8502	1.2149		6 ¾
	15	0.9659	0.2588	1.9319	0.5176	2.8978	0.7765	3.8637	1.0353	4.8296	1.2941	75	
	16	0.9613	0.2756	1.9225	0.5513	2.8838	0.8269	3.8450	1.1025	4.8063	1.3782	74	
1 ½		0.9569	0.2903	1.9139	0.5806	2.8708	0.8709	3.8278	1.1611	4.7847	1.4514		6 ½
	17	0.9563	0.2924	1.9126	0.5847	2.8689	0.8771	3.8252	1.1695	4.7815	1.46[illegible]	73	
	18	0.9511	0.3090	1.9021	0.6180	2.8532	0.9271	3.8042	1.2361	4.7553	1.54[illegible]	72	
	19	0.9455	0.3256	1.8910	0.6511	2.8366	0.9767	3.7821	1.3023	4.7276	1.62[illegible]	71	
1 ¾		0.9415	0.3369	1.8831	0.6738	2.8246	1.0107	3.7662	1.3476	4.7077	1.68[illegible]		6 ¼
	20	0.9397	0.3420	1.8794	0.6840	2.8191	1.0261	3.7588	1.3681	4.6985	1.71[illegible]	70	
	21	0.9336	0.3584	1.8672	0.7167	2.8007	1.0751	3.7343	1.4335	4.6679	1.7918	69	
	22	0.9272	0.3746	1.8544	0.7492	2.7816	1.1238	3.7087	1.4984	4.6359	1.8730	68	
2		0.9239	0.3827	1.8478	0.7654	2.7716	1.1481	3.6955	1.5307	4.6194	1.9134		6
	23	0.9205	0.3907	1.8410	0.7815	2.7615	1.1722	3.6820	1.5629	4.6025	1.9537	67	
	24	0.9135	0.4067	1.8271	0.8135	2.7406	1.2202	3.6542	1.6269	4.5677	2.0337	66	
	25	0.9063	0.4226	1.8126	0.8452	2.7189	1.2679	3.6252	1.6905	4.5315	2.1131	65	
2 ¼		0.9040	0.4276	1.8080	0.8551	2.7120	1.2827	3.6160	1.7102	4.5199	2.1378		5 ¾
	26	0.8988	0.4384	1.7976	0.8767	2.6964	1.3151	3.5952	1.7535	4.4940	2.1919	64	
	27	0.8910	0.4540	1.7820	0.9080	2.6730	1.3620	3.5640	1.8160	4.4550	2.2700	63	
	28	0.8829	0.4695	1.7659	0.9389	2.6488	1.4084	3.5318	1.8779	4.4147	2.3474	62	
2 ½		0.8819	0.4714	1.7638	0.9428	2.6458	1.4142	3.5277	1.8856	4.4096	2.3570		5 ½
	29	0.8746	0.4848	1.7492	0.9696	2.6239	1.4544	3.4985	1.9392	4.3731	2.4240	61	
	30	0.8660	0.5000	1.7321	1.0000	2.5981	1.5000	3.4641	2.0000	4.3301	2.5000	60	
2 ¾		0.8577	0.5141	1.7155	1.0282	2.5732	1.5423	3.4309	2.0564	4.2886	2.5705		5 ¼
	31	0.8572	0.5150	1.7143	1.0301	2.5715	1.5451	3.4287	2.0602	4.2858	2.5752	59	
	32	0.8480	0.5299	1.6961	1.0598	2.5441	1.5898	3.3922	2.1197	4.2402	2.6496	58	
	33	0.8387	0.5446	1.6773	1.0893	2.5160	1.6339	3.3547	2.1786	4.1934	2.7232	57	
3		0.8315	0.5556	1.6629	1.1111	2.4944	1.6667	3.3259	2.2223	4.1573	2.7779		5
	34	0.8290	0.5592	1.6581	1.1184	2.4871	1.6776	3.3162	2.2368	4.1452	2.7960	56	
	35	0.8192	0.5736	1.6383	1.1472	2.4575	1.7207	3.2766	2.2943	4.0958	2.8679	55	
	36	0.8090	0.5878	1.6180	1.1756	2.4271	1.7634	3.2361	2.3511	4.0451	2.9389	54	
3 ¼		0.8032	0.5957	1.6064	1.1914	2.4096	1.7871	3.2128	2.3828	4.0160	2.9785		4 ¾
	37	0.7986	0.6018	1.5973	1.2036	2.3959	1.8054	3.1945	2.4073	3.9932	3.0091	53	
	38	0.7880	0.6157	1.5760	1.2313	2.3640	1.8470	3.1520	2.4626	3.9401	3.0783	52	
	39	0.7771	0.6293	1.5543	1.2586	2.3314	1.8880	3.1086	2.5173	3.8857	3.1466	51	
3 ½		0.7730	0.6344	1.5460	1.2688	2.3190	1.9032	3.0920	2.5376	3.8650	3.1720		4 ½
	40	0.7660	0.6428	1.5321	1.2856	2.2981	1.9284	3.0642	2.5712	3.8302	3.2139	50	
	41	0.7547	0.6561	1.5094	1.3121	2.2641	1.9682	3.0188	2.6242	3.7735	3.2803	49	
	42	0.7431	0.6691	1.4803	1.3383	2.2294	2.0074	2.9726	2.6765	3.7157	3.3457	48	
3 ¾		0.7410	0.6716	1.4819	1.3431	2.2229	2.0147	2.9638	2.6862	3.7048	3.3578		4 ¼
	43	0.7314	0.6820	1.4627	1.3640	2.1941	2.0460	2.9254	2.7280	3.6568	3.4100	47	
	44	0.7193	0.6947	1.4387	1.3894	2.1580	2.0840	2.8774	2.7786	3.5967	3.4733	46	
4	45	0.7071	0.7071	1.4142	1.4142	3.1213	2.1213	2.8284	2.8284	3.5355	3.5355	45	4
Pts.	Deg	Dist. 1 Dep	Dist. 1 Lat	Dist. 2 Dep	Dist. 2 Lat.	Dist. 3 Dep	Dist. 3 Lat.	Dist. 4 Dep	Dist. 4 Lat	Dist. 5 Dep.	Dist. 5 Lat	Deg	Pts.

Course		Dist. 6		Dist. 7		Dist. 8		Dist. 9		Dist. 10		Course	
Pts.	D	Lat	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	Lat.	Dep.	D	Pts.
	1	5 9991	0 1047	6 9989	0 1222	7 9988	0 1396	8 9986	0 1571	9 9985	0 1745	89	
	2	5 9963	0 2094	6.9957	0.2443	7 9951	0.2792	8 9945	0 3141	9 9939	0 3490	88	
0 ¼		5 9928	0.2944	6.9916	0 3435	7 9904	0 3925	8 9892	0 4416	9 9880	0 4907		7 ¾
	3	5 9918	0 3140	6 9904	0.3664	7 9890	0 4187	8 9877	0.4710	9 9863	0 5234	87	
	4	5 9854	0 4185	6 9829	0 4883	7 9805	0 5581	8 9781	0 6278	9 9756	0 6976	86	
	5	5 9772	0.5229	6 9734	0.6101	7 9696	0 6972	8 9658	0 7844	9 9619	0 8716	85	
0 ½		5 9711	0.5881	6 9663	0 6861	7 9615	0 7841	8 9567	0 8822	9 9518	0 9802		7 ½
	6	5 9671	0 6272	6 9617	0 7317	7 9562	0 8362	8 9507	0 9408	9 9452	1 0453	84	
	7	5 9553	0 7312	6 9478	0 8531	7 9404	0.9750	8 9329	1.0968	9 9255	1 2187	83	
	8	5 9416	0 8350	6 9319	0 9742	7 9221	1 1134	8 9124	1 2526	9 9027	1 3917	82	
0 ¾		5 9351	0 8804	6 9242	1 0271	7 9134	1 1738	8 9026	1 3206	9 8918	1 4674		7 ¼
	[illegible]	5 9261	0 9386	6 9138	1 0950	7 9015	1.2515	8 8892	1 4079	9 8769	1 5643	81	
	10	5 9088	1 0419	6 8937	1 2155	7 8785	1 3892	8 8633	1 5628	9 8481	1 7365	80	
	11	5 8898	1 1449	6 8714	1 3357	7 8530	1.5265	8 8346	1 7173	9 8163	1 9081	79	
1		5 8847	1 1705	6 8655	1 3656	7 8463	1 5607	8 8271	1 7558	9 8079	1.9509		7
	12	5 8689	1 2475	6 8470	1 4554	7 8252	1 6633	8 8093	2 8712	9 7815	2 0791	78	
	13	5 8462	1 3497	6 8206	1 5747	7 7950	1 7996	8 7693	2 0246	9 7437	2 2495	77	
	14	5 8218	1 4515	6 7921	1 6935	7.7624	1 9354	8 7327	2 1773	9 7090	2 4192	76	
1 ¼		5 8202	1 4579	6 7902	1 7009	7 7602	1 9438	8 7303	2 1868	9 7003	2 4298		6 ¾
	15	5 7956	1 5529	6 7615	1 8117	7 7274	2 0706	8 6933	2 3294	9 6593	2 5882	75	
	16	5 7676	1 6538	6 7288	1 9295	7 6901	2 2051	8 6513	2 4807	9 6120	2 7562	74	
1 ½		5.7416	1 7417	6 6986	2 0320	7 6555	2 3223	8 6125	2 6126	9 5694	2 9028		6 ½
	17	[illegible]7378	1 7542	6 6941	2 0466	7.6504	2 3390	8 6067	2 6313	9 5630	2 9237	73	
	18	[illegible]7063	1 8541	6 6574	2 1631	7 6085	2 4721	8 5595	2 7812	9 5106	3 0902	72	
	19	[illegible].6731	1 9534	6 6186	2 2790	7 5642	2.6045	8 5097	2 9301	9 4552	3 2557	71	
1 ¾		[illegible].6493	2 0213	6 5908	2 3582	7 5324	2 6951	8 4739	3 0320	9 4154	3 3689		6 ¼
	20	5 6382	2 0521	6 5779	2 3941	7 5175	2 7362	8 4572	3 0782	9 3969	3 4202	70	
	21	5 6015	2 1502	6 5351	2 5086	7 4686	2 8669	8 4022	3 2253	9 3358	3 5837	69	
	22	5 5631	2 2476	6 4903	2 6222	7 4175	2 9969	8 3447	3 3715	9 2718	3 7461	68	
2		5 5433	2 2961	6 4672	2 6788	7 3910	3 0615	8 3149	3 4442	9 2388	3 8268		6
	23	5 5230	2.3444	6 4435	2 7351	7 3640	3 1258	8 2845	3 5166	9 2050	3 9073	67	
	24	5 4813	2 4404	6 3948	2 8472	7 3084	3 2539	8 2219	3 6606	9 1355	4 0674	66	
	25	5 4378	2 5357	6 3442	2 9583	7 2505	3 3809	8 1568	3 8036	9 0631	4 2202	65	
2 ¼		5 4239	2 5653	6 3279	2 9929	7 2319	3.4204	8 1959	3 8480	9 0399	4 2756		5 ¾
	26	5 3928	2 6302	6 2916	3 0686	7 1904	3 5070	8 0891	3 9453	8 9879	4 3837	64	
	27	5 3460	2 7239	6 2370	3 1779	7 1280	3 6319	8 0191	4 0859	8 9101	4 5399	63	
	28	5 2977	2 8168	6 1806	3 2863	7 0636	3 7558	7 9465	4 2252	8 8295	4 6947	62	
2 ½		5 2915	2 8284	6 1734	3 2998	7 0554	3 7712	7 9379	4 2426	8 8192	4 7140		5 ½
	29	5 2477	2 9089	6 1223	3 3937	6 9970	3 8785	7 8716	4 3633	8 7462	4 8481	61	
	30	5 1962	3 0000	6 0622	3 5000	6 9282	4 0000	7 7942	4 5000	8 6603	5 0000	60	
2 ¾		5 1464	3 0846	6 0041	3 5987	6 8618	4.1128	7 7196	4 6269	8 5773	5 1410		5 ¼
	31	5 1430	3 0902	6 0002	3 6053	6 8573	4 1203	7 7145	4 6353	8 5717	5 1504	59	
	32	5 0883	3 1795	5 9363	3 7094	6 7843	4 2394	7 6324	4 7093	8 4805	5 2992	58	
	33	5 0320	3 2678	5 8707	3 8125	6 7094	4 3571	7 5480	4 9018	8 3867	5 4464	57	
3		4 9888	3 3334	5 8203	3 8890	6 6518	4 4446	7 4832	5 0001	8 3147	5 5557		5
	34	4 9742	3 3552	5 8033	3 9144	6 6323	4 4735	7 4613	5 0327	8 2904	5 5919	56	
	35	4 9149	3 4415	5 7341	3 0150	6 5532	4 5886	7 3724	5 1622	8 1915	5 7358	55	
	36	4 8541	3 5267	5 6631	4 1145	6 4721	4 7023	7 2812	5 2901	8 0902	5 8779	54	
3 ¼		4 8192	3 5742	5 6225	4 1699	6 4257	4.7656	7 2289	5 3613	8 0321	5 9570		4 ¾
	37	4 7918	3 6109	5 5904	4 2127	6 3891	4 8145	7 1877	5 4163	7 9864	6.0182	53	
	38	4 7281	3 6940	5 5161	4 3096	6 3041	4 9253	7 0921	5 5409	7 8801	6 1566	5[illegible]	
	39	4 6629	3 7759	5 4400	4 4052	6 2172	5 0346	6 9943	5 6639	7 7715	6 2932	51	
3 ½		4 6381	3 8064	5 4111	4 4408	6 1841	5 0751	6 9571	5 7095	7 7301	6 3439		4 ½
	40	4 5963	3 8567	5 3623	4 4995	6 1284	5 1423	6 8944	5 7851	7 6604	6 4279	50	
	41	4 5283	3 9363	5 2830	4 5924	6 0377	5 2485	6 7924	5 9045	7 5471	6 5606	49	
	42	4 4589	4 0148	5 2020	4 6839	5 9452	5 3530	6 6883	6 0222	7 4314	6 6913	48	
3 ¾		4 4457	4 0294	5 1867	4 7009	5 9276	5 3725	6 6686	6 0440	7 4095	6 7156		4 ¼
	43	4 3881	4 0920	5 1195	4 7740	5 8508	5.4560	6 5822	6 1380	7 3135	6 8[illegible]00	47	
	44	4 3160	4 1679	5 0354	4 8626	5 7547	5 5573	6 4741	6 2519	7 1934	6 9466	46	
4	45	4 2426	4 2426	4 9497	4 9497	5 6569	5 6569	6 3640	6.3640	7 0711	7 0711	45	4
Pts.	Deg	Dep	Lat	Dep	Lat	Dep	Lat.	Dep	Lat	Dep	Lat	Deg	Pts
		Dist 6		Dist 7		Dist 8		Dist. 9		Dist 10			

TABLES

OF

COMPOUND INTEREST

AND

ANNUITIES.

Note.—The following Tables, relating to Interest and Annuities, are calculated upon the principles laid down in the Introduction, and will serve as a specimen of the larger tables of this sort, which are sometimes employed for facilitating computations in business.

AMOUNT OF L.1, *COMPOUND INTEREST*

Years.	3 per cent	4 per cent.	5 per cent.	Years.	3 per cent.	4 per cent.	5 per cent.
1	1 03000	1.04000	1 05000	26	2 15659	2 77247	3 55567
2	1 06090	1.08160	1 10250	27	2 22129	2 88337	3 73346
3	1 09273	1 12486	1 15762	28	2.28793	2 99870	3 92013
4	1 12551	1 16986	1 21551	29	2.35657	3 11865	4 11614
5	1 15927	1 21665	1 27628	30	2 42726	3 24340	4 32194
6	1 19405	1 26532	1.34010	31	2 50008	3 37313	4 53804
7	1 22987	1 31593	1 40710	32	2 57508	3.50806	4 76494
8	1 26677	1.36857	1 47746	33	2 65234	3 64838	5 00319
9	1 30477	1 42331	1 55133	34	2 73191	3 79432	5 25335
10	1 34392	1 48024	1 62889	35	2 81386	3 94609	5 51602
11	1 38423	1 53945	1 71034	36	2 89828	4 10393	5 79182
12	1 42576	1 60103	1 79586	37	2 98523	4 26809	6 08141
13	1 46853	1 66507	1 88565	38	3 07478	4 43881	6 38548
14	1.51259	1 73168	1 97993	39	3 16703	4 61637	6 70475
15	1 55797	1 80094	2 07893	40	3 26204	4 80102	7 03999
16	1 60471	1 87298	2 18287	41	3 35990	4 99306	7 39199
17	1 65285	1 94790	2 29202	42	3 46070	5 19278	7 76159
18	1 70243	2 02582	2 40662	43	3 56452	5 40050	8 14967
19	1 75351	2 10685	2 52695	44	3 67145	5 61652	8 55715
20	1 80611	2 19112	2 65330	45	3 78160	5 84118	8 98501
21	1 86029	2.27877	2.78596	46	3 89504	6 07482	9 43426
22	1 91610	2 36992	2 92526	47	4 01190	6 31782	9 90597
23	1 97359	2 46472	3.07152	48	4 13225	6 57053	10 40127
24	2 03279	2 56330	3 22510	49	4 25622	6 83335	10 92133
25	2 09378	2 66584	3 38635	50	4 38391	7 10668	11.46740

PRESENT VALUE OF L 1, *COMPOUND INTEREST.*

Years.	3 per cent.	4 per cent	5 per cent	Years.	3 per cent.	4 per cent.	5 per cent.
1	970874	.961538	952381	26	463695	360689	.281241
2	.942596	924556	907029	27	450189	346817	267848
3	915142	888996	.863838	28	.437077	.333477	255094
4	888487	854804	822702	29	.424346	320651	242946
5	862609	821927	783526	30	411987	308319	231377
6	837484	790315	746215	31	399987	296460	220359
7	813092	759918	.710681	32	388337	285058	209866
8	789409	730690	676839	33	377026	274095	199873
9	766417	702587	644609	34	366045	263552	190355
10	744094	675564	613913	35	355383	253415	181290
11	722421	649581	584679	36	345032	243669	172657
12	701380	.624597	556837	37	334983	234297	164436
13	680951	600574	530321	38	325226	225285	156605
14	.661118	577475	505068	39	315754	216621	.149148
15	.641862	.555265	481017	40	306557	.208289	142046
16	623167	533908	458112	41	.297628	200278	.135282
17	605016	513373	436297	42	.288959	192575	128840
18	587395	493628	415521	43	280543	185168	122704
19	570286	474642	395734	44	272372	178046	116864
20	.553676	456387	376889	45	.264439	171198	111297
21	537549	438834	.358942	46	256737	164614	105997
22	521893	421955	341850	47	249259	158283	100949
23	506692	405726	325571	48	241999	152195	096142
24	491934	.390121	.310068	49	.234950	146341	091564
25	.477606	375117	295303	50	228107	140713	087204

AMOUNT OF L 1 ANNUITY, *COMPOUND INTEREST*

Years.	3 per cent.	4 per cent.	5 per cent	Years.	3 per cent.	4 per cent	5 per cent.
1	1 0000	1 0000	1 0000	26	38 5530	44 3117	51 1135
2	2.0300	2.0400	2.0500	27	40 7096	47 0842	54 6691
3	3.0909	3 1216	3 1525	28	42 9309	49 9676	58 4026
4	4 1836	4 2465	4 3101	29	45.2189	52 9663	62.3227
5	5 3091	5.4163	5 5256	30	47 5754	56.0849	66.4388
6	6.4684	6 6330	6 8019	31	50 0027	59 3283	70.7608
7	7 6625	7 8983	8 1420	32	52 5028	62 7015	75 2988
8	8 8923	9 2142	9 5491	33	55 0778	66 2095	80 0638
9	10 1591	10 5828	11 0266	34	57 7302	69 8579	85 0670
10	11 4639	12 0061	12 5779	35	60 4621	73 6522	90.3203
11	12 8078	13 4864	14 2068	36	63 2759	77 5983	95 8363
12	14 1920	15 0258	15 9171	37	66 1742	81 7022	101 6281
13	15 6178	16 6268	17 7130	38	69 1594	85 9703	107 7095
14	17.0863	18 2919	19 5986	39	72.2342	90 4091	114 0950
15	18.5989	20 0236	21 5786	40	75 4013	95.0255	120 7998
16	20 1569	21 8245	23 6575	41	78 6633	99 8265	127 8398
17	21 7616	23 6975	25 8404	42	82 0232	104 8196	135 2318
18	23 4144	25.6454	28 1324	43	85 4839	110 0124	142 9933
19	25 1169	27 6712	30 5390	44	89 0484	115 4128	151.1430
20	26 8704	29 7781	33 0660	45	92 7199	121 0294	159 7002
21	28 6765	31 9692	35 7193	46	96 5014	126 8706	168 6852
22	30 5368	34 2480	38 5052	47	100 3965	132 9454	178 1194
23	32 4529	36 6179	41 4305	48	104 4084	139 2632	188 0254
24	34 4265	39 0826	44 5020	49	108 5406	145 8337	198.4267
25	36 4593	41 6459	47 7271	50	112 7969	152 6671	209 3480

PRESENT VALUE OF L 1 ANNUITY, *COMPOUND INTEREST*

Years.	3 per cent	4 per cent.	5 per cent.	Years.	3 per cent.	4 per cent.	5 per cent.
1	0 9709	0 9615	0 9524	26	17 8768	15 9828	14 3752
2	1 9135	1 8861	1 8594	27	18 3270	16 3296	14 6430
3	2 8286	2 7751	2 7232	28	18 7641	16 6631	14 8981
4	3 7171	3 6299	3 5460	29	19 1885	16 9837	15 1411
5	4 5797	4 4518	4 3295	30	19 6001	17 2920	15 3725
6	5 4172	5 2421	5 0757	31	20 0004	17 5885	15 5928
7	6 2303	6 0021	5 7864	32	20 3888	17 8736	15 8027
8	7 0197	6 7327	6 4632	33	20 7658	18 1476	16 0025
9	7 7861	7 4353	7 1078	34	21 1318	18 4112	16 1929
10	8 5302	8 1109	7 7217	35	21 4872	18 6646	16 3742
11	9 2526	8 7605	8 3064	36	21 8323	18 9083	16 5469
12	9 9540	9 3851	8 8633	37	22 1672	19 1426	16 7113
13	10 6350	9 9856	9 3936	38	22 4925	19 3679	16 8679
14	11 2961	10 5631	9 8986	39	22 8082	19 5845	17 0170
15	11 9379	11 1184	10 3797	40	23 1118	19 7928	17 1591
16	12 5611	11 6523	10 8378	41	23 4124	19 9931	17 2944
17	13 1661	12 1657	11 2741	42	23 7014	20 1856	17 4232
18	13 7535	12 6593	11 6896	43	23 9819	20 3708	17 5460
19	14 3238	13 1340	12 0853	44	24 2543	20 5488	17 6628
20	14 8775	13 5903	12 4622	45	24.5187	20 7200	17 7741
21	15 4150	14 0292	12 8212	46	24 7754	20 8847	17 8801
22	15.9369	14 4511	13 1630	47	25 0247	21 0429	17 9810
23	16 4436	14 8568	13 4886	48	25 2667	21.1951	18 0772
24	16 9355	15 2470	13 7986	49	25 5017	21 3415	18 1687
25	17 4131	15 6221	14 0939	50	25 7298	21 4822	18.2559

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PROBABILITIES OF LIFE, FORMED BY MR MILNE, FROM THE REGISTER AT CARLISLE.

Age	Number who complete that age	Number who die in the next interval	Age	Number who complete that year	Number who die in their next year	Age	Number who complete that year	Number who die in their next year
0	10000	774	33	5472	55	69	2525	124
¼	9226	256	34	5417	55	70	2401	124
½	8970	255	35	5362	55	71	2277	134
¾	8715	254	36	5307	56	72	2143	146
1	8461	682	37	5251	57	73	1997	156
2	7779	505	38	5194	58	74	1841	166
3	7274	276	39	5136	61	75	1675	160
4	6998	201	40	5075	66	76	1515	156
5	6797	121	41	5009	69	77	1359	146
6	6676	82	42	4940	71	78	1213	132
7	6594	58	43	4869	71	79	1081	128
8	6536	43	44	4798	71	80	953	116
9	6493	33	45	4727	70	81	837	112
10	6460	29	46	4657	69	82	725	102
11	6431	31	47	4588	67	83	623	94
12	6400	32	48	4521	63	84	529	84
13	6368	33	49	4458	61	85	445	78
14	6335	35	50	4397	59	86	367	71
15	6300	39	51	4338	62	87	296	64
16	6261	42	52	4276	65	88	232	51
17	6219	43	53	4211	68	89	181	39
18	6176	43	54	4143	70	90	142	37
19	6133	43	55	4073	73	91	105	30
20	6090	43	56	4000	76	92	75	21
21	6047	42	57	3924	82	93	54	14
22	6005	42	58	3842	93	94	40	10
23	5963	42	59	3749	106	95	30	7
24	5921	42	60	3643	122	96	23	5
25	5879	43	61	3521	126	97	18	4
26	5836	43	62	3395	127	98	14	3
27	5793	45	63	3268	125	99	11	2
28	5748	50	64	3143	125	100	9	2
29	5698	56	65	3018	124	101	7	2
30	5642	57	66	2894	123	102	5	2
31	5585	57	67	2771	123	103	3	2
32	5528	56	68	2648	123	104	1	1

PROBABILITIES OF LIFE, FORMED BY DR. PRICE, FROM THE REGISTER AT NORTHAMPTON.

Age	Number who complete that age	Number who die in the next interval	Age	Number who complete that Year	Number who die in their next Year	Age.	Number who complete that year	Number who die in their next year
0	11650	1340	31	4310	75	65	1632	80
¼	10310	554	32	4235	75	66	1552	80
½	9756	553	33	4160	75	67	1472	80
¾	9203	553	34	4085	75	68	1392	80
1	8650	1367	35	4010	75	69	1312	80
2	7283	502	36	3935	75	70	1232	80
3	6781	335	37	3860	75	71	1152	80
4	6446	197	38	3785	75	72	1072	80
5	6249	184	39	3710	75	73	992	80
6	6065	140	40	3635	76	74	912	80
7	5925	110	41	3559	77	75	832	80
8	5815	80	42	3482	78	76	752	77
9	5735	60	43	3404	78	77	675	73
10	5675	52	44	3326	78	78	602	68
11	5623	50	45	3248	78	79	534	65
12	5573	50	46	3170	78	80	469	63
13	5523	50	47	3092	78	81	406	60
14	5473	50	48	3014	78	82	346	57
15	5423	50	49	2936	79	83	289	55
16	5373	53	50	2857	81	84	234	48
17	5320	58	51	2776	82	85	186	41
18	5262	63	52	2694	82	86	145	34
19	5199	67	53	2612	82	87	111	28
20	5132	72	54	2530	82	88	83	21
21	5060	75	55	2448	82	89	62	16
22	4985	75	56	2366	82	90	46	12
23	4910	75	57	2284	82	91	34	10
24	4835	75	58	2202	82	92	24	8
25	4760	75	59	2120	82	93	16	7
26	4685	75	60	2038	82	94	9	5
27	4610	75	61	1956	82	95	4	3
28	4535	75	62	1874	81	96	1	1
29	4460	75	63	1793	81			
30	4385	75	64	1712	80			

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EXPECTATION OF LIFE ACCORDING TO THE CARLISLE TABLE OF PROBABILITIES.

Age	Expect.	Age	Expect.	Age	Expect	Age	Expect	Age	Expect.
0	38.72	21	40 75	42	26 34	63	12.81	84	4 39
1	44.68	22	40.04	43	25 71	64	12 30	85	4 12
2	47 55	23	39 31	44	25 09	65	11 79	86	3 90
3	49.82	24	38 59	45	24 46	66	11 27	87	3 71
4	50.76	25	37 86	46	23 82	67	10.75	88	3 59
5	51 25	26	37.14	47	23 17	68	10 23	89	3 47
6	51.17	27	36 41	48	22 50	69	9 70	90	3 28
7	50.80	28	35 69	49	21 81	70	9 18	91	3 26
8	50 24	29	35 00	50	21 11	71	8 65	92	3 37
9	49 57	30	34 34	51	20 39	72	8 16	93	3 48
10	48 82	31	33 68	52	19 68	73	7 72	94	3 53
11	48 04	32	33 03	53	18 97	74	7 33	95	3 53
12	47.27	33	32.36	54	18 28	75	7 01	96	3 46
13	46 51	34	31 68	55	17 58	76	6 69	97	3 28
14	45 75	35	31 00	56	16.89	77	6 40	98	3 07
15	45 00	36	30 32	57	16 21	78	6 12	99	2 77
16	44 27	37	29 64	58	15 55	79	5 80	100	2 28
17	43.57	38	28 96	59	14 92	80	5 51	101	1 79
18	42 87	39	28 28	60	14 34	81	5 21	102	1 30
19	42.17	40	27.61	61	13 82	82	4 93	103	0 83
20	41 46	41	26 97	62	13 31	83	4 65	104	0 50

EXPECTATION OF LIFE ACCORDING TO THE NORTHAMPTON TABLE OF PROBABILITIES

Age	Expect	Age.	Expect	Age	Expect	Age.	Expect	Age	Expect
0	25 18	20	33.43	40	23 08	60	13 21	80	4 75
1	32 74	21	32 90	41	22 56	61	12 75	81	4 41
2	37 79	22	32 39	42	22 04	62	12 28	82	4 09
3	39 55	23	31 88	43	21 54	63	11 81	83	3 80
4	40 58	24	31 36	44	21 03	64	11 35	84	3 58
5	40 84	25	30 85	45	20 52	65	10 88	85	3 37
6	41 07	26	30 33	46	20 02	66	10 42	86	3 19
7	41 03	27	29 82	47	19 51	67	9 96	87	3 01
8	40 79	28	29 30	48	19 00	68	9 50	88	2 86
9	40.36	29	28 79	49	18 49	69	9 05	89	2 66
10	39 78	30	28 27	50	17 99	70	8 60	90	2 41
11	39 14	31	27 76	51	17 50	71	8 17	91	2 09
12	38.49	32	27 24	52	17 02	72	7 74	92	1 75
13	37 83	33	26 72	53	16 54	73	7 33	93	1 37
14	37 17	34	26 20	54	16 06	74	6 92	94	1 05
15	36 51	35	25 68	55	15 58	75	6 54	95	0 75
16	35 85	36	25 16	56	15 10	76	6 18	96	0 50
17	35 20	37	24 64	57	14 63	77	5 83		
18	34 58	38	24 12	58	14 15	78	5 48		
19	33 99	39	23 60	59	13 68	79	5 11		

VALUE OF AN ANNUITY OF £.1 ON A SINGLE LIFE, ACCORDING TO THE CARLISLE TABLE OF PROBABILITIES.

Age	3 per cent.	4 per cent.	5 per cent	Age	3 per cent	4 per cent	5 per cent
0	17 320	14 283	12 083	52	13 558	12 258	11.154
1	20 085	16 556	13 995	53	13 180	11 945	10 892
2	21 501	17 728	14 983	54	12 798	11 627	10 624
3	22 683	18 717	15 824	55	12 408	11 300	10 347
4	23 285	19 233	16 271	56	12 014	10 966	10 063
5	23.693	19 594	16 590	57	11 614	10 625	9 771
6	23 846	19 747	16 735	58	11 218	10 286	9 478
7	23 867	19 792	16 790	59	10 841	9 968	9 199
8	23.801	19.766	16 786	60	10 491	9 663	8 940
9	23 677	19 693	16 742	61	10 180	9 398	8 712
10	23 512	19 585	16 669	62	9 875	9 137	8 487
11	23 327	19 460	16.581	63	9 567	8 872	8 258
12	23 143	19 336	16 494	64	9 246	8 593	8 016
13	22 957	19 210	16 406	65	8 917	8 307	7 765
14	22 769	19 082	16 316	66	8 578	8 010	7 503
15	22 582	18 956	16 227	67	8 228	7 700	7.227
16	22 404	18 837	16 144	68	7 869	7 380	6 941
17	22 232	18 723	16 066	69	7 499	7 049	6 643
18	22 058	18 608	15 987	70	7 123	6 709	6 336
19	21 879	18 488	15 904	71	6 737	6 358	6 015
20	21 694	18 363	15 817	72	6 373	6 026	5 711
21	21 504	18 233	15 726	73	6 044	5 725	5 435
22	21 304	18 095	15 628	74	5 752	5 458	5 190
23	21 098	17 951	15 525	75	5 512	5 239	4 989
24	20 885	17 801	15 417	76	5 277	5 024	4 792
25	20 665	17.645	15 303	77	5 059	4 825	4 609
26	20 442	17 486	15 187	78	4 838	4 622	4 422
27	20 212	17 320	15 065	79	4 592	4 394	4 210
28	19 981	17 154	14 942	80	4 365	4 183	4 015
29	19 761	16 997	14 827	81	4 119	3 953	3 799
30	19 556	16 852	14 723	82	3 898	3 746	3 606
31	19 348	16 705	14 617	83	3 672	3 534	3 406
32	19 134	16 552	14 506	84	3 454	3 329	3 211
33	18 910	16 390	14 387	85	3 229	3 115	3 009
34	18 675	16 219	14 260	86	3 033	2 928	2 830
35	18 433	16 041	14 127	87	2 873	2 776	2 685
36	18 183	15 856	13 987	88	2 776	2 683	2 597
37	17 928	15 666	13 843	89	2 665	2 577	2 495
38	17 669	15 471	13 695	90	2.499	2 416	2 339
39	17 405	15 272	13 542	91	2 481	2 398	2 321
40	17 143	15 074	13 390	92	2 577	2 492	2 412
41	16 890	14 883	13 245	93	2 687	2 600	2 518
42	16 610	14 694	13 101	94	2 736	2 650	2 569
43	16 389	14 505	12 957	95	2 757	2 674	2 596
44	16 130	14 308	12 806	96	2 704	2 628	2 555
45	15 863	14 104	12 648	97	2 559	2 492	2 428
46	15 585	13 889	12 480	98	2 388	2 332	2 278
47	15 294	13 662	12 301	99	2 131	2 087	2 045
48	14 986	13 419	12 107	100	1 683	1 653	1.624
49	14 654	13 153	11 892	101	1 228	1 210	1 192
50	14 303	12 869	11 660	102	0 771	0 762	0 753
51	13 932	12.566	11 410	103	0 324	0 321	0 317

VALUE OF AN ANNUITY OF £.1 ON TWO JOINT LIVES, ACCORDING TO THE CARLISLE TABLE OF PROBABILITIES.

Age		3 per cent	4 per cent	5 per cent.
10	10	19.963	17.049	14 803
	15	19 410	16 643	14.500
	20	18.873	16 264	14 221
	25	18 189	15.768	13 850
	30	17 411	15 190	13 416
	35	16 596	14 590	12 963
	40	15 605	13.835	12 378
	45	14.601	13.066	11 785
	50	13.310	12.034	10 593
	55	11.667	10 664	9 799
	60	9 957	9 196	8 530
	65	8 537	7 969	7 463
	70	6 874	6 484	6 131
	75	5 353	5 093	4 855
	80	4 262	4 088	3 925
	85	3 167	3 056	2 953
	90	2 454	2 374	2 299
15	15	18 908	16 272	14.215
	20	18 423	15.922	13 959
	25	17 794	15 460	13.608
	30	17 063	14 918	13 195
	35	16 295	14 347	12 765
	40	15 348	13 623	12 201
	45	14 381	12 884	11 630
	50	13 131	11 882	10 822
	55	11 528	10 543	9 692
	60	9 852	9 103	8.446
	65	8 458	7 897	7 398
	70	6 818	6 433	6 084
	75	5 315	5 057	4 821
	80	4 295	4 062	3 901
	85	3 149	3.039	2 937
	90	2 441	2 361	2 287
20	20	17 993	15 610	13 724
	25	17 421	15 182	13 398
	30	16.749	14 677	13 008
	35	16 031	14 142	12 602
	40	15 131	13 449	12 062
	45	14 207	12 741	11 511
	50	12 995	11 769	10 727
	55	11 429	10 458	9 621
	60	9 782	9 043	8 394
	65	8 411	7 856	7 361
	70	6 790	6 407	6 061
	75	5 298	5.042	4 807
	80	4 225	4 053	3 893
	85	3 143	3 034	2 932
	90	2 437	2.358	2 283
25	25	16 916	14 800	13 101
	30	16 311	14 339	12 742
	35	15 660	13 848	12 365
	40	14 824	13 202	11 856
	45	13 954	12 530	11 335
25	50	12 793	11.599	10 581
	55	11 274	10.325	9 505
	60	9.669	8 948	8 306
	65	8 329	7.783	7 295
	70	6 736	6 358	6 017
	75	5 263	5 010	4 778
	80	4 203	4 033	3 874
	85	3 130	3 022	2 921
	90	2.428	2 349	2 276
30	30	15 784	13 930	12 419
	35	15 209	13 491	12 078
	40	14 449	12 897	11 607
	45	13 650	12 278	11 121
	50	12 551	11 393	10 404
	55	11 089	10 164	9 364
	60	9 529	8.820	8 196
	65	8 224	7 688	7 210
	70	6 662	6 291	5 954
	75	5 213	4 964	4 735
	80	4 168	3.999	3 843
	85	3 107	3 000	2 900
	90	2 411	2 333	2 260
35	35	14 720	13 111	11 780
	40	14 048	12 581	11 351
	45	13 331	12 019	10 912
	50	12 314	11 196	10 238
	55	10 919	10 020	9 240
	60	9 410	8 716	8 105
	65	8 140	7 614	7 143
	70	6 608	6 242	5 910
	75	5 179	4 933	4 706
	80	4 148	3 981	3.826
	85	3 096	2 989	2 890
	90	2 403	2 325	2 253
40	40	13 481	12 125	10 984
	45	12 868	11 641	10 598
	50	11.954	10.894	9 984
	55	10 658	9 796	9 046
	60	9 224	8 553	7 961
	65	8 006	7 493	7 034
	70	6 515	6 157	5 832
	75	5 115	4 872	4 650
	80	4 102	3 937	3 784
	85	3 065	2 961	2 863
	90	2 380	2 304	2 233
45	45	12 371	11 243	10 278
	50	11.580	10 591	9 737
	55	10 400	9 583	8 870
	60	9 063	8 417	7 846
	65	7 910	7 411	6 964
	70	6 465	6 113	5 793
	75	5 089	4 850	4 630

(CONTINUED.)

Age.		3 per cent	4 per cent.	5 per cent
45	80	4.087	3 924	3.772
	85	3 056	2 952	2.854
	90	2 375	2 299	2 227
50	50	10 942	10 059	9.291
	55	9 924	9 181	8 523
	60	8.729	8 132	7.601
	65	7.691	7 221	6 799
	70	6 338	6.001	5 695
	75	5 022	4 790	4 577
	80	4 054	3 894	3 746
	85	3 040	2 938	2 842
	90	2 365	2 290	2 220
55	55	9 103	8 465	7.900
	60	8 098	7 574	7.106
	65	7 219	6 798	6 418
	70	6 019	5 712	5.431
	75	4.813	4.598	4.400
	80	3 920	3 770	3.630
	85	2 961	2 863	2.772
	90	2 307	2 236	2 168
60	60	7.295	6 854	6 456
	65	6 589	6 225	5 895
	70	5 565	5 293	5.044
	75	4 498	4 304	4.125
	80	3.695	3 558	3.430
60	85	2 812	2 722	2.637
	90	2 199	2 132	2.070
65	65	6 047	5.738	5 456
	70	5.193	4 956	4 737
	75	4 257	4 082	3 921
	80	3 542	3.416	3 297
	85	2 719	2.635	2.555
	90	2 131	2.069	2.009
70	70	4 556	4.367	4 191
	75	3.804	3 661	3.528
	80	3.229	3.121	3.020
	85	2 523	2.449	2 380
	90	1 987	1 932	1 880
75	75	3 231	3.119	3.015
	80	2 790	2 704	2.623
	85	2.217	2 157	2.100
	90	1.758	1 712	1 669
80	80	2 459	2 390	2 324
	85	1 993	1 943	1.895
	90	1 589	1 551	1 515
85	85	1 657	1 619	1 583
	90	1 335	1.307	1 279
90	90	1.088	1.066	1 045

VALUE OF AN ANNUITY OF £.1 ON A SINGLE LIFE, ACCORDING TO THE NORTHAMPTON TABLE OF PROBABILITIES.

Age	3 per cent	4 per cent	5 per cent	Age	3 per cent	4 per cent	5 per cent
0	12 270	10 327	8.863	48	12 951	11 685	10 616
1	16 021	13 465	11.563	49	12 693	11 475	10 443
2	18 599	15 633	13 420	50	12 436	11 264	10 269
3	19 575	16 462	14 135	51	12 183	11 057	10 097
4	20 210	17 010	14 613	52	11 930	10 849	9 925
5	20 473	17 248	14 827	53	11 674	10 637	9 748
6	20 727	17 482	15 041	54	11 414	10 421	9 567
7	20 853	17 611	15 166	55	11 150	10 201	9 382
8	20 885	17 662	15 226	56	10 882	9.977	9 193
9	20 812	17 625	15 210	57	10.611	9 749	8 999
10	20 663	17 523	15 139	58	10 337	9 516	8 801
11	20 480	17 393	15.043	59	10 058	9 280	8 599
12	20 283	17 251	14 937	60	9 777	9 039	8 392
13	20 081	17 103	14 826	61	9 193	8 795	8 181
14	19 872	16 950	14 710	62	9 205	8 547	7 966
15	19 657	16 791	14 588	63	8 910	8 291	7 742
16	19 435	16 625	14 460	64	8 611	8.090	7 514
17	19 218	16 462	14 334	65	8 304	7 761	7 276
18	19 013	16 309	14 217	66	7 994	7 488	7 034
19	18 820	16 167	14 108	67	7 682	7 211	6 787
20	18 638	16 033	14 007	68	7 367	6 930	6 536
21	18 470	15 912	13 917	69	7 051	6 647	6 281
22	18.311	15 797	13 843	70	6 734	6 361	6 023
23	18 148	15 680	13.746	71	6 418	6 075	5 764
24	17 983	15 560	13 658	72	6 103	5 790	5 504
25	17 814	15 438	13 567	73	5 794	5 507	5 245
26	17 642	15 312	13 473	74	5 491	5 230	4 990
27	17 467	15 181	13 377	75	5 199	4 962	4 744
28	17 289	15 053	13 278	76	4 925	4 710	4 511
29	17 107	14 918	13 177	77	4 652	4 457	4 277
30	16 922	14 781	13 072	78	4 372	4 197	4 035
31	16 732	14 639	12 965	79	4 077	3 921	3 776
32	16 540	14 495	12 854	80	3 781	3 643	3 515
33	16 343	14 347	12 710	81	3 199	3 377	3 263
34	16 142	14 195	12 623	82	3 229	3 122	3 020
35	15 938	14 039	12 502	83	2 982	2 887	2 797
36	15 729	13 880	12 377	84	2 793	2 708	2 627
37	15 515	13 716	12 249	85	2 620	2 543	2 471
38	15 298	13 548	12 116	86	2 462	2 393	2 328
39	15 075	13 375	11 979	87	2 312	2.251	2 193
40	14 848	13 197	11 837	88	2 185	2 131	2 080
41	14 620	13 018	11 695	89	2 013	1 967	1 921
42	14 391	12 834	11 551	90	1 794	1 758	1 723
43	14 162	12 657	11 407	91	1 501	1 174	1 147
44	13 929	12 472	11 258	92	1 190	1 171	1 153
45	13 692	12 283	11 105	93	0 839	0 827	0 816
46	13 450	12 089	10 947	94	0 596	0 590	0 524
47	13 208	11 890	10 784	95	0 242	0 240	0 238

VALUE OF AN ANNUITY OF £.1 ON TWO JOINT LIVES, ACCORDING TO THE NORTHAMPTON TABLE OF PROBABILITIES.

Age.		3 per cent	4 per cent.	5 per cent.
10	10	16 389	14 277	12 665
	15	15 762	13 841	12 302
	20	15 151	13 355	11 906
	25	14 683	12 998	11 627
	30	14 150	12 586	11 304
	35	13.525	12 098	10 916
	40	12 791	11.513	10 442
	45	11 976	10 851	9 900
	50	11 044	10 085	9 260
	55	10 055	9 256	8 560
	60	8 952	8 314	7 750
	65	7.718	7 236	6 803
	70	6 347	6 008	5 700
	75	4 962	4 725	4 522
	80	3 647	3 517	3 395
15	15	15 229	13 411	11 960
	20	14 660	12 961	11 585
	25	14 230	12 630	11 324
	30	13 734	12 246	11 021
	35	13 151	11 787	10 655
	40	12 459	11 234	10 205
	45	11 687	10 607	9 690
	50	10 799	9 872	9 076
	55	9 851	9 077	8 403
	60	8 790	8 170	7 622
	65	7 597	7 127	6 705
	70	6 261	5 993	5 631
	75	4 911	4 695	4 495
	80	3 621	3 492	3 372
20	20	14 133	12 535	11 232
	25	13 711	12 229	10 989
	30	13 286	11 873	10 707
	35	12 744	11 345	10 363
	40	12 096	10 924	9 937
	45	11 367	10 330	9 413
	50	10 523	9 630	8 861
	55	9 617	8 869	8 216
	60	8 597	7 995	7 463
	65	7 444	6 966	6 576
	70	6 149	5 826	5 532
	75	4 831	4 619	4 124
	80	3 569	3 443	3 325
25	25	13 383	11 941	10 764
	30	12 966	11 618	10 499
	35	12 463	11 217	10 175
	40	11 854	10 725	9 771
	45	11 164	10 160	9 304
	50	10 356	9 488	8 739
	55	9.484	8 754	8 116
	60	8 195	7 906	7 383
	65	7 370	6 920	6 515
	70	6 099	5 780	5 489
	75	4 799	4.589	4 396
	80	3 550	3 425	3 308
30	30	12 589	11 313	10 255
	35	12 131	10 948	9 954
	10	11.568	10 490	9 576
	45	10 923	9 959	9 135
	50	10 160	9 321	8 596
	55	9 329	8 619	7 999
	60	8 379	7 302	7 292
30	65	7 286	6 844	6 447
	70	6.043	5 729	5 442
	75	4 764	4 557	4 365
	80	3 530	3 406	3 290
35	35	11 722	10 612	9 680
	40	11 213	10 196	9 331
	45	10 622	9 706	8 921
	50	9 912	9 110	8 415
	55	9 131	8 448	7 849
	60	8 227	7 669	7 174
	65	7 177	6 747	6 360
	70	5 971	5 663	5 382
	75	4 720	4 516	4 327
	80	3 506	3 383	3 268
40	40	10 764	9 820	9.016
	45	10 236	9 381	8 643
	50	9 590	8 814	8 177
	55	8 870	8 221	7 651
	60	8.025	7 490	7 015
	65	7 030	6 614	6 240
	70	5 871	5 571	5 298
	75	4 656	4 457	4 272
	80	3.469	3 349	3 236
45	45	9 776	8 990	8 312
	50	9 204	8 503	7 891
	55	8 557	7 948	7 411
	60	7 781	7 274	6 822
	65	6 850	6 453	6 094
	70	5 749	5 460	5 195
	75	4 580	4 386	4 206
	80	3 426	3 908	3 197
50	50	8 711	8.081	7 522
	55	8 152	7 593	7 098
	60	7 461	6 989	6 568
	65	6 611	6 236	5 897
	70	5 582	5 306	5 054
	75	4 472	4 285	4 112
	80	3 362	3 247	3 140
55	55	7 681	7 179	6 735
	60	7 068	6 659	6 272
	65	6 334	5 956	5 671
	70	5 391	5 132	4 893
	75	4 350	4 171	4 006
	80	3 291	3 180	3 076
60	60	6 606	6 226	5 888
	65	5 970	5 658	5 372
	70	5 139	4 900	4 680
	75	4 189	4 021	3 866
	80	3 197	3 092	2 992
65	65	5 471	5 201	4 960
	70	4 783	5 573	4 378
	75	3 958	3 806	3 665
	80	3 063	2.965	2 873
70	70	4 261	4 087	3 930
	75	3 599	3 471	3 347
	80	2 813	2 757	2 675
75	75	3 114	3 015	2 917
	80	2 526	2 448	2 381
80	80	2 122	2 068	2.018

D	Arc	D	Arc	D	Arc	′	Arc.	″	Arc.	‴	Arc.
1	0174533	61	1 0646508	121	2.1118484	1	2909	1	48	1	1
2	.0349066	62	1 0821041	122	2.1293017	2	5818	2	97	2	2
3	0523599	63	1.0995574	123	2 1467550	3	8727	3	145	3	2
4	0698132	64	1 1170107	124	2.1642083	4	11636	4	194	4	3
5	0872665	65	1 1344640	125	2 1816616	5	14544	5	242	5	4
6	1047198	66	1.1519173	126	2 1991149	6	17453	6	291	6	5
7	1221730	67	1 1693706	127	2 2165682	7	20362	7	339	7	6
8	1396263	68	1 1868239	128	2 2340214	8	23271	8	388	8	6
9	1570796	69	1 2042772	129	2 2514747	9	26180	9	436	9	7
10	1745329	70	1.2217305	130	2 2689280	10	29089	10	485	10	8
11	1919862	71	1 2391838	131	2 2863813	11	31998	11	533	11	9
12	2094395	72	1 2566371	132	2 3038346	12	34907	12	582	12	10
13	2268928	73	1 2740904	133	2 3212879	13	37815	13	630	13	11
14	2443461	74	1 2915436	134	2 3387412	14	40724	14	679	14	11
15	2617994	75	1 3089969	135	2 3561945	15	43633	15	727	15	12
16	2792527	76	1 3264502	136	2 3736478	16	46542	16	776	16	13
17	2967060	77	1 3439035	137	2 3911011	17	49451	17	824	17	14
18	3141593	78	1 3613568	138	2 4085544	18	52360	18	873	18	15
19	3316126	79	1 3788101	139	2 4260077	19	55269	19	921	19	15
20	3490659	80	1 3962634	140	2 4434610	20	58178	20	970	20	16
21	3665191	81	1 4137167	141	2 4609142	21	61087	21	1018	21	17
22	3839724	82	1 4311700	142	2 4783675	22	63995	22	1067	22	18
23	4014257	83	1 4486233	143	2 4958208	23	66904	23	1115	23	19
24	.4188790	84	1 4660766	144	2 5132741	24	69813	24	1164	24	19
25	4363323	85	1 4835299	145	2 5307274	25	72722	25	1212	25	20
26	4537856	86	1 5009832	146	2 5481807	26	75631	26	1261	26	21
27	4712389	87	1 5184364	147	2 5656340	27	78540	27	1309	27	22
28	4886922	88	1 5358897	148	2 5830873	28	81449	28	1357	28	23
29	5061455	89	1 5533430	149	2 6005406	29	84358	29	1406	29	23
30	5235988	90	1 5707963	150	2 6179939	30	87266	30	1454	30	24
31	5410521	91	1 5882496	151	2 6354472	31	90175	31	1503	31	25
32	5585054	92	1 6057029	152	2 6529005	32	93084	32	1551	32	26
33	5759587	93	1.6231562	153	2 6703538	33	95993	33	1599	33	27
34	5934119	94	1 6406095	154	2 6878070	34	98902	34	1648	34	27
35	6108652	95	1 6580628	155	2 7052603	35	101811	35	1697	35	28
36	6283185	96	1 6755161	156	2 7227136	36	104720	36	1745	36	29
37	6457718	97	1 6929694	157	2 7401669	37	107629	37	1794	37	30
38	6632251	98	1 7104227	158	2 7576202	38	110538	38	1842	38	31
39	6806784	99	1 7278760	159	2 7750735	39	113446	39	1891	39	32
40	.6981317	100	1 7453293	160	2 7925268	40	116355	40	1939	40	32
41	7155850	101	1 7627825	161	2 8099801	41	119264	41	1988	41	33
42	.7330383	102	1 7802358	162	2 8274334	42	122173	42	2036	42	34
43	7504916	103	1 7976891	163	2 8448867	43	125082	43	2085	43	35
44	7679449	104	1 8151424	164	2 8623400	44	127991	44	2133	44	36
45	7853982	105	1 8325957	165	2.8797933	45	130900	45	2182	45	36
46	8028515	106	1 8500490	166	2 8972466	46	133809	46	2230	46	37
47	8203047	107	1 8675023	167	2 9146999	47	136717	47	2279	47	38
48	8377580	108	1 8849556	168	2.9321531	48	139626	48	2327	48	39
49	8552113	109	1 9024089	169	2 9496064	49	142535	49	2376	49	40
50	.8726646	110	1.9198622	170	2 9670597	50	145444	50	2424	50	40
51	8901179	111	1 9373155	171	2 9845130	51	148353	51	2473	51	41
52	9075712	112	1 9547688	172	3 0019663	52	151262	52	2521	52	42
53	9250245	113	1 9722220	173	3.0194196	53	154171	53	2570	53	43
54	9424778	114	1 9896753	174	3 0368729	54	157080	54	2618	54	44
55	9599311	115	2 0071286	175	3 0543262	55	159989	55	2666	55	44
56	9773844	116	2 0245819	176	3 0717795	56	162897	56	2715	56	45
57	9948377	117	2 0420352	177	3.0892328	57	165806	57	2763	57	46
58	1 0122910	118	2 0594885	178	3 1066861	58	168715	58	2812	58	47
59	1 0297443	119	2 0769418	179	3 1241394	59	171624	59	2860	59	48
60	1 0471976	120	2 0943951	180	3 1415927	60	174533	60	2909	60	48

CL.	HYP LO.	CL.	HYP LO.	CL.	HYP LO.	CL.	HYP LO.
.01	.02302585	.26	.59867212	.51	1.17431840	.76	1.74996467
.02	.04605170	.27	.62169792	.52	1.19734425	.77	1.77299052
.03	.06907755	.28	.64472383	.53	1.22037010	.78	1.79601637
.04	.09210340	.29	.66774968	.54	1.24339595	.79	1.81904222
.05	.11512925	.30	.69077553	.55	1.26642180	.80	1.84206807
.06	.13815511	.31	.71380138	.56	1.28944765	.81	1.86509392
.07	.16118096	.32	.73682723	.57	1.31247350	.82	1.88811978
.08	.18420681	.33	.75985308	.58	1.33549935	.83	1.91114563
.09	.20723266	.34	.78287893	.59	1.35852520	.84	1.93417148
.10	.23025851	.35	.80590478	.60	1.38155106	.85	1.95719733
.11	.25328436	.36	.82893063	.61	1.40457691	.86	1.98022318
.12	.27631021	.37	.85195648	.62	1.42760276	.87	2.00324903
.13	.29933606	.38	.87498234	.63	1.45062861	.88	2.02627488
.14	.32236191	.39	.89800819	.64	1.47365446	.89	2.04930073
.15	.34538776	.40	.92103404	.65	1.49668031	.90	2.07232658
.16	.36841361	.41	.94405989	.66	1.51970616	.91	2.09535243
.17	.39143947	.42	.96708574	.67	1.54273201	.92	2.11837829
.18	.41446532	.43	.99011159	.68	1.56575786	.93	2.14140414
.19	.43749117	.44	1.01313744	.69	1.58878371	.94	2.16442999
.20	.46051702	.45	1.03616329	.70	1.61180957	.95	2.18745584
.21	.48354287	.46	1.05918914	.71	1.63483542	.96	2.21048169
.22	.50656872	.47	1.08221499	.72	1.65786127	.97	2.23350754
.23	.52959457	.48	1.10524084	.73	1.68088712	.98	2.25653339
.24	.55262042	.49	1.12826670	.74	1.70391297	.99	2.27955924
.25	.57564627	.50	1.15129255	.75	1.72693882	1.00	2.30258509

AREAS OF THE SEGMENTS OF A CIRCLE, WHOSE DIAMETER IS UNITY, AND SUPPOSED TO BE DIVIDED INTO 1000 EQUAL PARTS.

Height.	Area Seg	Height.	Area Seg	Height	Area Seg	Height.	Area Seg
001	000042	031	.007209	061	019716	091	035585
002	000119	.032	007558	.062	020196	092	036162
003	000219	033	007913	063	020680	.093	036741
004	000337	.034	008273	064	021168	094	037323
005	000470	035	008638	065	021659	095	037909
.006	000618	.036	009008	066	022154	.096	038496
007	000779	037	009383	.067	022652	097	.039087
008	000951	038	009763	068	023154	098	039680
009	001135	039	010148	069	023659	099	040276
010	001329	.040	010537	070	024168	100	040875
.011	001533	041	010931	071	.024680	101	041476
012	001746	042	011330	072	025195	102	042080
013	001968	043	011734	.073	025714	103	042687
014	002199	044	012142	074	026236	104	.043296
015	002438	045	012554	075	026761	105	.043908
016	002685	.046	012971	076	.027289	106	044522
017	.002940	047	013392	077	027821	107	045139
018	003202	048	013818	078	028356	108	045759
019	003471	049	014247	.079	028894	109	046381
020	003748	050	014681	080	029435	110	.047005
021	004031	051	015119	081	029979	111	047632
022	.004322	052	015561	082	030526	112	048262
023	004618	053	016007	083	031076	113	048894
024	004921	054	016457	084	031629	114	049528
025	005230	055	.016911	.085	032186	.115	050165
026	005546	056	017369	086	032745	116	050804
027	005867	057	017831	087	033307	117	.051446
028	006194	058	018296	.088	033872	118	052090
029	006527	059	018766	089	034441	119	052736
030	006865	060	019239	090	035011	120	053385

Height	Area Seg	Height.	Area Seg	Height.	Area Seg	Height.	Area Seg
121	054036	181	096003	241	145799	301	199085
122	054689	182	097674	242	146655	302	200003
123	055345	183	098447	243	147512	303	200922
124	056003	184	099221	244	148371	304	201841
125	056663	185	099997	245	149230	305	202761
126	057326	186	100774	246	150091	306	203683
127	057991	187	101553	247	150953	307	204605
128	058658	188	102334	248	151816	308	205527
129	059327	189	103116	249	152680	309	206451
130	059999	190	103900	250	153546	310	207376
131	060672	191	104685	251	154412	311	208301
132	061348	192	105472	252	155280	312	209227
133	062026	193	106261	253	156149	313	210154
134	062707	194	107051	254	157019	314	211082
135	063389	195	107842	255	157890	315	212011
136	064074	196	108636	256	158762	316	212940
137	064760	197	109436	257	159636	317	213871
138	065449	198	110226	258	160510	318	214802
139	066140	199	111024	259	161386	319	215733
140	066833	200	111823	260	162263	320	216666
141	067528	201	112624	261	163140	321	217599
142	068225	202	113426	262	164019	322	218533
143	068924	203	114230	263	164899	323	219468
144	069625	204	115035	264	165780	324	220404
145	070328	205	115842	265	166663	325	221340
146	071033	206	116650	266	167546	326	222277
147	071741	207	117460	267	168430	327	223215
148	072450	208	118271	268	169315	328	224154
149	073161	209	119083	269	170202	329	225093
150	073874	210	119897	270	171089	330	226033
151	074589	211	120712	271	171978	331	226974
152	075306	212	121529	272	172867	332	227915
153	076026	213	122347	273	173758	333	228858
154	076747	214	123167	274	174649	334	229801
155	077469	215	123988	275	175542	335	230745
156	078194	216	124810	276	176435	336	231689
157	078921	217	125634	277	177330	337	232634
158	079649	218	126459	278	178225	338	233580
159	080380	219	127285	279	179122	339	234526
160	081112	220	128113	280	180019	340	235473
161	081846	221	128942	281	180918	341	236421
162	082582	222	129773	282	181817	342	237369
163	083320	223	130605	283	182718	343	238318
164	084059	224	131438	284	183619	344	239268
165	084801	225	132272	285	184521	345	240218
166	085544	226	133108	286	185425	346	241169
167	086289	227	133945	287	186329	347	242121
168	087036	228	134784	288	187234	348	243074
169	087785	229	135621	289	188140	349	244026
170	088535	230	136465	290	189047	350	244980
171	089287	231	137307	291	189955	351	245934
172	090041	232	138150	292	190864	352	246889
173	090797	233	138995	293	191775	353	247845
174	091554	234	139841	294	192684	354	248801
175	092313	235	140688	295	193596	355	249757
176	093074	236	141537	296	194509	356	250715
177	093836	237	142387	297	195422	357	251673
178	094601	238	143238	298	196337	358	252631
179	095366	239	144091	299	197252	359	253590
180	096134	240	144944	300	198168	360	254550

Height	Area Seg	Height	Area Seg	Height	Area Seg	Height	Area Seg
.361	255510	.396	289452	431	323918	466	358725
.362	.256471	397	290431	432	.324909	467	359724
363	257433	398	.291411	433	.325900	468	360721
.364	.258395	.399	292309	434	326892	469	361719
.365	.259357	400	293369	435	327882	.470	362717
366	260320	.401	.294349	436	328874	471	363715
367	261284	402	295330	437	329866	472	364713
368	262248	403	.296311	438	330858	473	365712
.369	263213	.404	.297292	439	331850	474	366710
370	264178	405	298273	440	332843	475	367709
.371	265144	406	299255	441	333836	476	368708
.372	266111	407	300238	442	334819	477	369707
373	.267078	408	301220	.443	335822	478	370706
374	268045	409	.302203	444	336816	.479	371705
375	269013	410	303187	445	.337810	.480	372704
376	269982	411	304171	.446	338804	.481	373703
377	270951	412	.305155	447	339798	482	374702
378	271920	.413	306140	448	340793	483	375702
379	272890	414	307125	449	341787	484	376702
.380	273861	415	308110	.450	342782	.485	377701
381	274832	416	309095	[illegible]	343777	.486	378701
382	275803	417	310081	452	344772	487	379700
383	276775	418	311068	453	345768	488	380700
384	277748	419	312054	454	346764	489	381699
.385	278721	420	313041	455	347759	490	382699
386	279694	.421	.314029	.456	348755	.491	383699
387	280668	422	315016	457	349752	492	384699
.388	281642	423	.316004	458	350748	493	385699
389	282617	424	316992	459	351745	494	386699
390	283592	425	317981	460	352742	495	387699
.391	284568	426	.318970	461	353739	496	388699
392	.285544	427	319959	462	354736	497	389699
.393	286521	428	320948	463	355732	498	390699
394	287498	429	321938	464	356730	499	391699
395	288476	430	322928	465	357727	500	392699

TABLE FOR FINDING THE DIFFERENCE BETWEEN THE TRUE AND APPARENT LEVEL

Dist. Yards	Dif of Level Inches.	Dist. Miles	Dif of Level Feet.	Inches.
100	0 026	$\frac{1}{4}$	0	0½
200	0 103	$\frac{1}{2}$	0	2
300	0 231	$\frac{3}{4}$	0	4½
400	0 411	1	0	8
500	0 643	2	2	8
600	0 925	3	6	0
700	1 260	4	10	7
800	1 645	5	16	7
900	2 081	6	23	11
1000	2 570	7	32	6
1100	3 110	8	42	6
1200	3 701	9	53	9
1300	4 344	10	66	4
1400	5 038	11	80	3
1500	5 784	12	95	7
1600	6 580	13	112	2
1700	7 425	14	130	1

MEAN ASTRONOMICAL REFRACTIONS FOR EVERY DEGREE IN ALTITUDE.

Apt Alt.	Refraction.		Apt Alt.	Refraction		Apt Alt	Refraction	Apt Alt.	Refraction.
°	′	″	°	′	″	°	″	°	″
1	24	29	24	2	7	47	53	69	22
2	18	35	25	2	2	48	51	70	21
3	14	36	26	1	56	49	49	71	19
4	11	51	27	1	51	50	48	72	18
5	9	54	28	1	47	51	46	73	17
6	8	28	29	1	42	52	44	74	16
7	7	20	30	1	38	53	43	75	15
8	6	29	31	1	35	54	41	76	14
9	5	48	32	1	31	55	40	77	13
10	5	15	33	1	28	56	38	78	12
11	4	47	34	1	24	57	37	79	11
12	4	23	35	1	21	58	35	80	10
13	4	3	36	1	18	59	34	81	9
14	3	45	37	1	16	60	33	82	8
15	3	30	38	1	13	61	31	83	7
16	3	17	39	1	10	62	30	84	6
17	3	4	40	1	8	63	29	85	5
18	2	54	41	1	5	64	28	86	4
19	2	45	42	1	3	65	26	87	3
20	2	35	43	1	1	66	25	88	2
21	2	27	44	0	59	67	24	89	1
22	2	20	45	0	57	68	23	90	0
23	2	14	46	0	55				

Note.—The Horizontal Refraction is 33′, at a mean state of the Atmosphere.

DIP OF THE HORIZON

Height Feet	Dip. ′	″	Feet	Dip ′	″
1	0	58	11	3	10
2	1	21	12	3	19
3	1	40	13	3	27
4	1	56	14	3	36
5	2	9	15	3	42
6	2	21	16	3	50
7	2	33	17	3	57
8	2	44	18	4	4
9	2	53	19	4	11
10	3	2	20	4	17
			21	4	23
			22	4	30
			23	4	36
			24	4	42
			26	4	52
			28	5	5
			30	5	15
			35	5	39
			40	6	4
			45	6	27
			50	6	46
			60	7	25
			70	8	1
			80	8	34
			90	9	6
			100	9	35

DIP AT DIFFERENT DISTANCES FROM THE OBSERVER.

Miles.	Height of the Eye in Feet 5	10	15	20	25	30
¼	11′	23′	34′	45′	57′	68′
½	6	12	17	23	28	34
¾	4	8	12	15	19	23
1	3	6	9	12	15	17
1¼	3	5	7	10	12	14
1½	3	4	6	8	10	12
2	2	4	5	7	8	9
2½	2	3	4	6	7	8
3	2	3	4	5	6	7
3½	2	3	4	5	6	6
4	2	3	4	5	5	6
5	2	3	4	4	5	6
6	2	3	4	4	5	5

TABLE FOR REDUCING SLOPING LINES TO HORIZONTAL LINES.

Deduct,				Deduct.			
°	′	Links.	Inches.	°	′	Links	Inches.
4	0	¼	1 98	28	55	12½	99 00
5	44	½	3 96	29	30	13	102.96
7	6	¾	5 94	30	5	13½	106 92
8	10	1	7 92	30	40	14	110 88
11	30	2	15 84	31	15	14½	114 84
12	50	2½	19 80	31	45	15	118 80
14	4	3	23.76	32	20	15½	122.76
15	10	3½	27 72	32	50	16	126 72
16	15	4	31 68	33	25	16½	130 68
17	15	4½	35 64	33	55	17	134 64
18	10	5	39 60	34	25	17½	138.60
19	30	5½	43 56	34	55	18	142 56
19	55	6	47 52	35	25	18½	146.52
20	45	6½	51 48	35	55	19	150.48
21	35	7	55 44	36	25	19½	154 44
22	20	7½	59 40	36	55	20	158 40
23	5	8	63.36	37	20	20½	162.36
23	45	8½	67.32	37	50	21	166 32
24	30	9	71.28	38	15	21½	170 28
25	10	9½	75 24	38	45	22	174 24
25	50	10	79 20	39	15	22½	178.20
26	30	10½	83 16	39	40	23	182.16
27	10	11	87 12	40	5	23½	186.12
27	45	11½	91 08	40	40	23¾	188.10
28	20	12	95 04	41	0	24	190 08

POLYGON TABLES.

No. of sides.	Names.	Areas, or Multipliers.
3	Trigon, or equil △	0.4330127
4	Tetragon, or square	1 0000000
5	Pentagon	1.7204774
6	Hexagon	2 5980762
7	Heptagon	3.6339124
8	Octagon	4.8284271
9	Nonagon	6.1818240
10	Decagon	7 6942088
11	Undecagon	9.3656399
12	Dodecagon	11.1961524

No of sides.	Names.	Angle OAF	Tangents.
3	Trigon, or equil. △	30°	$0.5773503 = \frac{1}{3}\sqrt{3}$
4	Tetragon, or square	45°	$1.0000000 = 1$
5	Pentagon	54°	$1.3763819 = \sqrt{(1+\frac{2}{5}\sqrt{5})}$
6	Hexagon	60°	$1\ 7320508 = \sqrt{3}$
7	Heptagon	$64°\frac{2}{7}$	2.0765213
8	Octagon	$67°\frac{1}{2}$	$2.4142136 = 1+\sqrt{2}$
9	Nonagon	70°	2.7474774
10	Decagon	72°	$3.0776835 = \sqrt{(5+2\sqrt{5})}$
11	Undecagon	$74°\frac{7}{11}$	3.4056872
12	Dodecagon	75°	$3.7320508 = 2+\sqrt{3}$

SURFACES AND SOLIDITIES OF THE REGULAR BODIES, WHEN THE LINEAR EDGE IS 1.

No. of sides.	Names.	Surfaces.	Solidities.
4	Tetraedron	[illegible]	0.11785
6	Hexaedron	[illegible]	1.00000
8	Octaedron	3.46410	0.47140
12	Dodecaedron	20.64578	7.66312
20	Icosaedron	8.66025	2.18169

NUMBER OF MILES IN A DEGREE OF LONGITUDE, WHEN RECKONED ON ANY PARALLEL OF LATITUDE, FROM THE EQUATOR TO THE POLES.

Degr. Lat.	Miles.	Degr. Lat.	Miles.	Degr. Lat.	Miles.
0	60.00	31	51.43	62	28.17
1	59.99	32	50.88	63	27.24
2	59.96	33	50.32	64	26.30
3	59.92	34	49.74	65	25.36
4	59.86	35	49.15	66	24.41
5	59.77	36	48.54	67	23.45
6	59.67	37	47.92	68	22.48
7	59.56	38	47.28	69	21.50
8	59.42	39	46.62	70	20.52
9	59.26	40	45.95	71	19.54
10	59.08	41	45.28	72	18.55
11	58.89	42	44.59	73	17.54
12	58.68	43	43.88	74	16.53
13	58.46	44	43.16	75	15.52
14	58.22	45	42.43	76	14.51
15	57.95	46	41.68	77	13.50
16	57.67	47	40.92	78	12.47
17	57.37	48	40.15	79	11.45
18	57.06	49	39.36	80	10.42
19	56.73	50	38.57	81	9.38
20	56.38	51	37.76	82	8.35
21	56.01	52	36.94	83	7.32
22	55.63	53	36.11	84	6.28
23	55.23	54	35.26	85	5.23
24	54.81	55	34.41	86	4.18
25	54.38	56	33.55	87	3.14
26	53.93	57	32.68	88	2.09
27	53.46	58	31.79	89	1.05
28	52.97	59	30.90	90	0.00
29	52.47	60	30.00		
30	51.96	61	29.09		